Even Vertex Graceful of Path, Circuit, Star, Wheel, some Extension-friendship Graphs and Helm Graph

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ABSTRACT
Even vertex gracefulness of path, circuit, star and wheel are obtained. Also even vertex gracefulness of the connected graphs $C_n \nabla F(2nC_3)$, $C_n \nabla F(3nC_3)$ and C(4, n) are got.

INTRODUCTION
A.Solairaju, and A.Sasikala [2008] got gracefulness of a spanning tree of the graph of product of P_m and C_n, A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A.Solairaju, and C. Vimala [2008] also got the gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n .

A.Solairaju and P.Muruganantham [2009] proved that ladder P_2 x P_n is even-edge graceful (even vertex graceful). They found [2010] the connected graphs P_n o nC_3 and P_n o nC_7 are both even vertex graceful, where n is any positive integer. They also obtained [2010] that the connected graph $P_n \Delta nC_4$ is even vertex graceful, where n is any even positive integer.

Section I: Preliminaries
Definition 1.1: Let G = (V,E) be a simple graph with p vertices and q edges.

A map f : V(G) $\rightarrow$ {0,1,2,…,q} is called a graceful labeling if

(i) f is one – to – one

(ii) The edges receive all the labels (numbers) from 1 to q where the label of an edge is the absolute value of the difference between the vertex labels at its ends.

A graph having a graceful labeling is called a graceful graph.

Definition 1.2: A graph is if there exists an injective map f : E (G) $\rightarrow$ {1, 2 ,…, 2q} so that the induced map f^+: V(G) $\rightarrow$ {0, 2, 4,…, 2k-2} defined by $f^+(x) = \sum f(xy)$ (mod 2k) where k = max { p, q } makes all distinct.

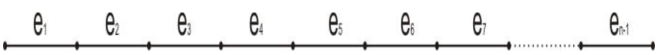
Definition 1.3: C_n is a circuit with n vertices. S_n is a star with n vertices. W_n is a wheel with n vertices.

Section II: Even vertex graceful of standard graphs
The following result is first started.

Theorem 2.1: A path with n vertices is even vertex graceful.

Proof: A path P_n is a connected graph with n vertices. It has (n-1) edges as follows:

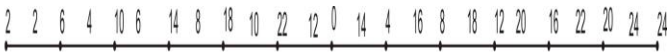
Some arbitrary labeling of edges of the path Pn is given below:



Define f: $E(P_n) \rightarrow$ { 1,2, …, (n-1)} by $f(e_i) = 2i$, i varies from 1 to (n-1).

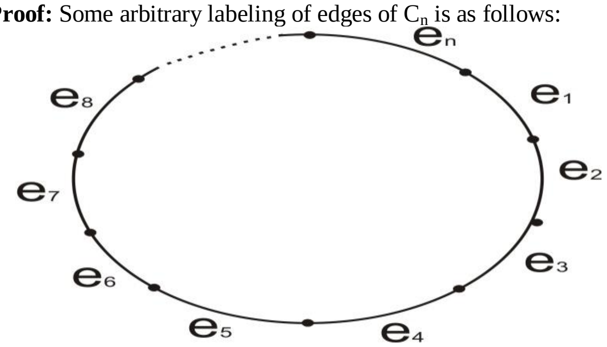
Then the induced map $f^+(u) = \sum f(uv)$ (mod 2q) where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling. The path P_n with n vertices is even vertex graceful.

Example 2.1: The path P_{13} is even vertex graceful.



Theorem 2.2: A circuit C_n with n vertices is even vertex graceful.

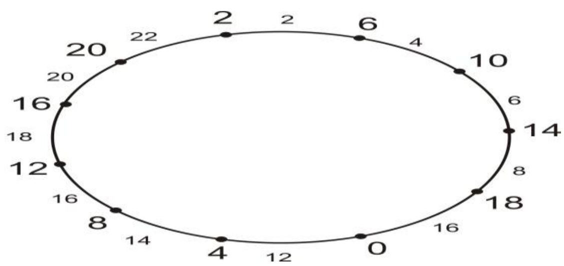
Proof: Some arbitrary labeling of edges of C_n is as follows:



Define f: $E(C_n) \rightarrow$ { 1,2, …, n} by $f(e_i) = 2i$, i varies from 1 to n.

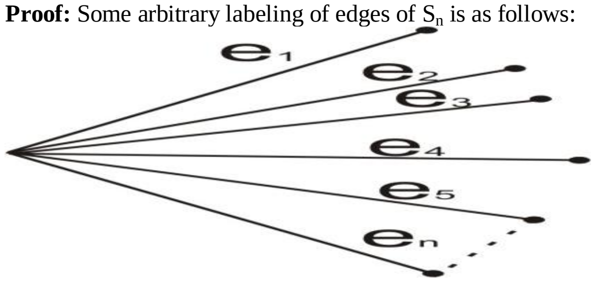
Then the induced map $f^+(u) = \sum f(uv)$ (mod 2q) where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling. The path C_n with n vertices is even vertex graceful.

Example 2.2: The path C_{11} is even vertex graceful.



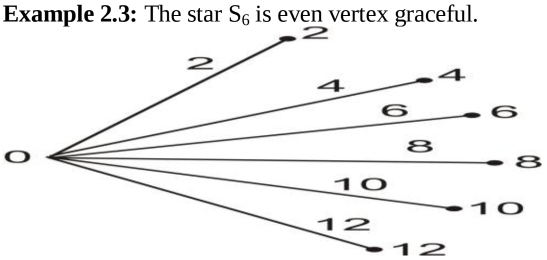
Theorem 2.3: A star with n vertices (S_n) is even vertex graceful.

Proof: Some arbitrary labeling of edges of S_n is as follows:



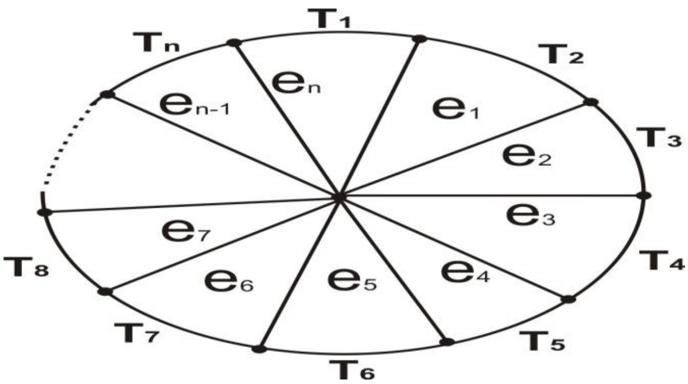
Define f: $E(S_n) \rightarrow \{1,2,\ldots,n\}$ by $f(e_i) = 2i$, i varies from 1 to n. Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling. The path S_n with n vertices is even vertex graceful.

Example 2.3: The star S_6 is even vertex graceful.



Theorem 2.4: A star with n vertices (W_n) is even vertex graceful.

Proof: Some arbitrary labeling of edges of W_n is as follows:



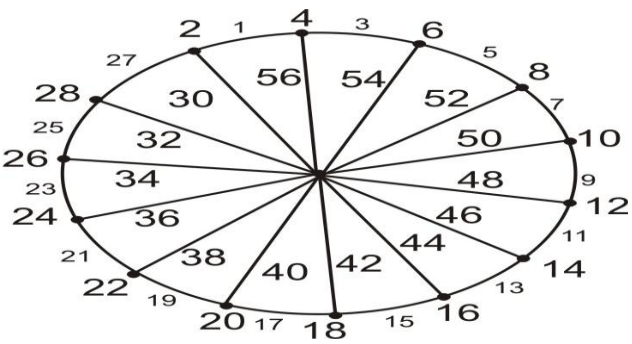
Define f: $E(C_n) \rightarrow \{1,2,\ldots,n\}$ by $f(T_j) = (2j-1)$, i varies from 1 to n;

n is even : $f(e_i) = 2q - 2(i-1)$, i varies from 1 to n.

n is odd and $n \equiv 3 \pmod 4$; $f(e_{n-i}) = 2i + 2$, i varies from 1 to (n-1); $f(e_n) = f(e_1) + 2$.

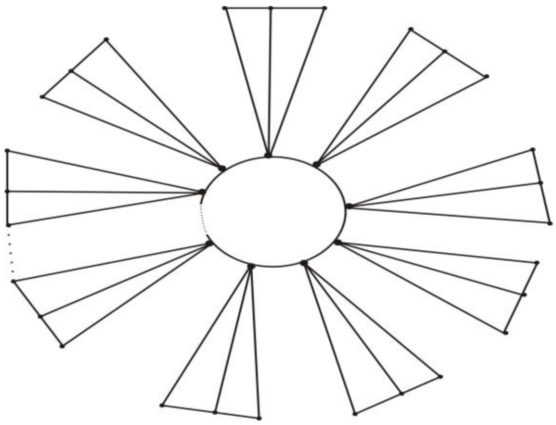
n is odd and $n \equiv 1 \pmod 4$; $f(e_{n-i}) = 2i$, i varies from 1 to (n-1); $f(e_n) = f(e_1) + 2$. Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. So f and f^+ both satisfy even vertex graceful labeling. The path S_n with n vertices is even vertex graceful.

Example 2.4: The path W_{14} is even vertex graceful.



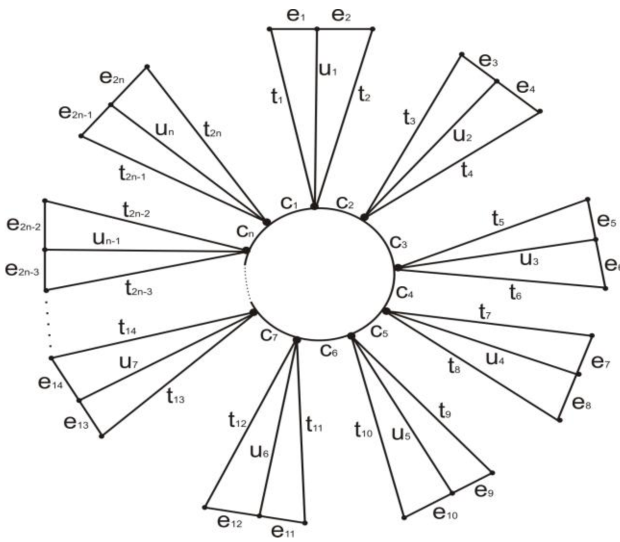
Section 3 - Even vertex graceful of extension-friendship graph

Definition 3.1: A fan graph or an extension-friendship graph $C_n \nabla F(2nC_3)$ is defined as the following connected graph such that every vertex of C_n is merged with one copy of $2C_3$.



Theorem 3.1: The connected graph $C_n \nabla F(2nC_3)$ is even vertex graceful.

Proof: The graph $C_n \nabla F(2nC_3)$ is chosen with some arbitrary labeling of edges as in definition (1.5).

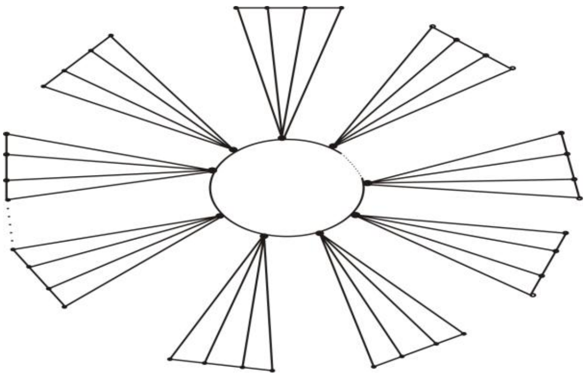


Define a map f: $E[C_n \nabla F(2nC_3)] \rightarrow \{0, 1,2,\ldots, 2q\}$ by

$f(e_i) = 2i-1$,	$i = 1,2,\ldots 2n$
$f(t_i) = (2i) + f(e_{2n})$	$i = 1,2,\ldots 2n$
$f(u_1) = 2q$	
$f(u_i) = 2q-4i+4$	$i = 2,\ldots n$
$f(c_1) = f(u_n) - 2$	
$f(c_2) = f(c_1) - 6$	
$f(c_i) = f(c_2) - 4(i-2)$	$i = 3,4,\ldots n$

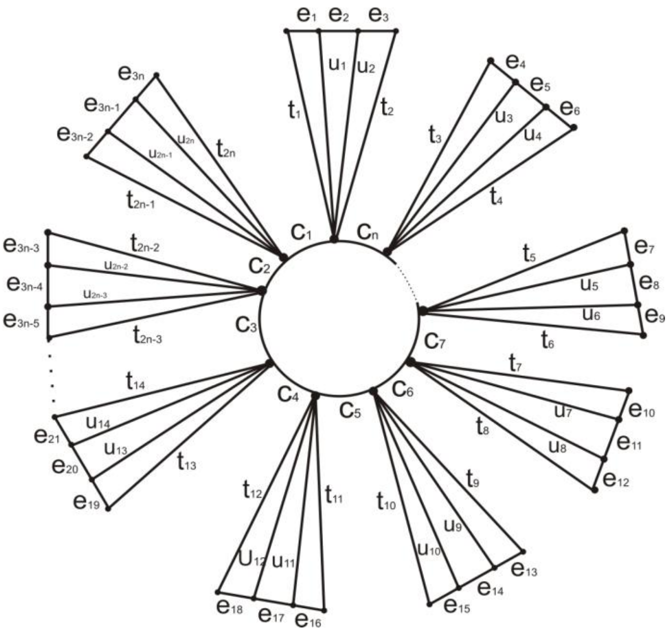
Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling as well as edge–odd graceful labeling. Thus the connected graph $C_n \nabla F(2nC_3$ is both even vertex graceful and odd-edge graceful.

Definition 3.2: A fan graph or an extension-friendship graph $\mathbf{C_n \nabla F(3nC_3)}$ is defined as the following connected graph such that every vertex of C_n is merged with one copy of $3C_3$.



Theorem 3.2: The connected graph $C_n \nabla F(3nC_3)$ is even vertex graceful.

Proof: The graph $C_n \nabla F(3nC_3)$ is chosen with some arbitrary labeling of edges as in definition (1.6).

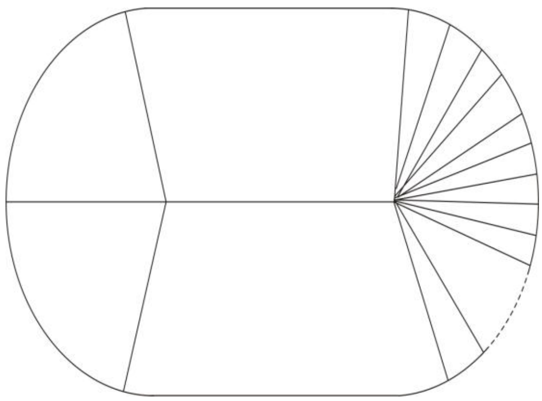


Define a map f: $E[C_n \nabla F(3nC_3)] \rightarrow \{0, 1,2,\ldots, 2q\}$ by

$f(e_i) = 2i-1$, $i=1,2,\ldots,3n$
$f(t_i) = f(e_{2n}) + 2i$, $i=1,2,\ldots,2n$
$f(u_1) = (2q-4)$
$f(u_2) = (2q-6)$
$f(u_i) = f(u_1)- 4(i-1)$; $i=3,5,7,\ldots,2n-1$
$f(u_i) = f(u_2)- 4(i-2)$, $i=4,6,\ldots 2n$
$f(c_1) = f(u_{2n}) - 2$; $f(c_2) = f(c_1) - 2$;
$f(c_i) = f(c_1) - 3(i-1)$ where i varies $3,5,7,\ldots,n$ if n is odd; i varies $3,5,7,\ldots,n-1$ if n is even.
$f(c_n) = f(c_{n-1}) - 4$ if n is even;
$f(c_i) = f(c_2)- 3(i-2)$ where i varies $2,4,6,\ldots,n$ if n is even; i varies $2,4,6,\ldots,n-1$ if n is odd;
$f(c_n) = f(c_{n-1}) - 4$ if n is odd

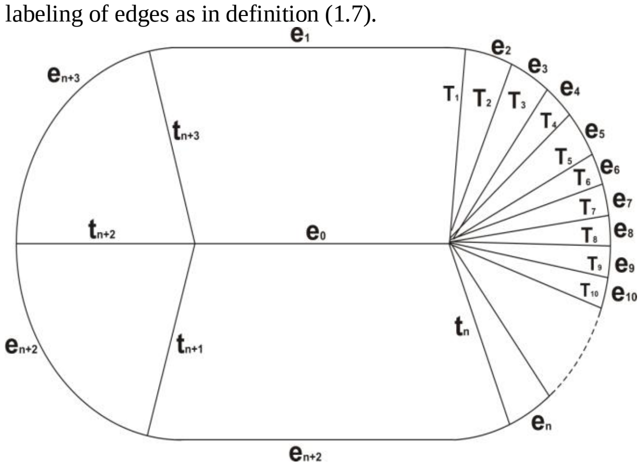
Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling Thus the connected graph $C_n \nabla F(3nC_3)$ is even vertex graceful

Definition 3.3: The graph C(4, n) is a connected graph defined by merging C_4 and C_n as follows:



Theorem 3.3: The connected C(4, n) is even vertex graceful.

Proof: The graph $C_n \nabla F(3nC_3)$ is chosen with some arbitrary labeling of edges as in definition (1.7).



Define a map f: $E[C(4, n)] \rightarrow \{0, 1,2,\ldots, 2q\}$ by

$f(e_i) = 2i-1$, $i=1,2,\ldots, (n+3)$;
$f(T_i) = 2q - 2(i-1)$, $i=1,2,\ldots, (n+3)$;
$f(e_0) = (q -15) = (2n-8)$

Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling Thus the connected graph C(4, n) is even vertex graceful.

CONCLUSION

Even vertex graceful of friendship graphs $F(nC_3)$, $F(nC_5)$, and $F(2nC_3)$ are obtained in [7]. For further investigations. Path, circuit, star and wheel are all even vertex graceful. Also the connected graphs $C_n \nabla F(2nC_3)$, $C_n \nabla F(3nC_3)$ and C(4, n) are all even vertex graceful..

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