International Journal of Computer Applications (0975 – 8887)
Volume 11– No.1, December 2010

Structures on Q-Fuzzy Left N-Subgroups of Near Rings under Triangular Norms

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ABSTRACT
In this paper, we introduce the notion of Q-Fuzzification of left N-Subgroups in a near ring and investigate some related properties, characterization of Q-Fuzzy left N-Subgroups with respect to a triangular norm are given.

Ams subject classification (2000: 03F055, 03E72, 20N25
Keywords: Q-Fuzzy set, Q-Fuzzy left N-subgroup, sub near rings, Homomotphisms, Sup property, t-norm.

SECTION-1 INTRODUCTION
The theory of Fuzzy sets which was introduced by Zedah [7] is applied to many mathematical branches. Abou-Zoid [1] , introduced the notion of a fuzzy sub near ring and studied Fuzzy ideals of near ring. This concept discussed by many researchers among Cho, Davvaz, Dudek, Jun, Kim [2], [3], [4]. In [5], considered the intuitionistic Fuzzification of a right crisp left R.Subgroup in a near ring. A.Solairaju and R.Nagarajan [6} introduced a notion of Q-Fuzzy Groups. Also cho.et.al in [4] the notion of normal intuitionistic Fuzzy R-Sub group in a near ring is introduced and related are investigated. The notion of intuitionistic Q-Fuzzy semi primality in a semi group is given by Kim [3]. In this paper, we introduce the notion of Q-Fuzzification of left N-Subgroups in a near ring and investigate some related properties. Characterization of Q-Fuzzy left N-Subgroups are given.

1.1.1 Section-2 PRELIMINARIES:
Definition 2.1 **:** A non empty set with two binary operations '+' and '.' is called a near ring if it satisfies the following axioms;

(i) (S , +) is a group.
(ii) (S , .) is a semigroup.
(iii) x.(y + z) = x.y + x.z

for all x,y,z ∈ S.

Precisely speaking it is a left near ring. Because it satisfies the left distribution Low. As N-Subgroup of a near ring 'S' is a subset 'H' of 'S' such that

(i) (H , +) is a Sub group of (S, +)
(ii) SH ⊂ H
(iii) HS ⊂ H.

If 'H' satisfies (i) and (ii) then it is called left N=Subgroup of 'S' and if 'N' satisfies (i) and (iii) then it is called right N-Subgroup of S.
A map f : R→ S is called homomorphism if f(x+y) = f(x) + f(y) for all x,y in S.

Definition 2.2 :
Let 'S' be a near ring. A Fuzzy set I in 'S' is called Q-Fuzzy sub near ring in S if
(i) μ (x-y, a) ≥ min { μ(x,a), μ (y,a) }
(ii) μ(xy , a) ≥ min { μ(x,a), μ (y,a)} for all x,y in S.

Definition 2.3 :
A mapping μ : X → [0,1] , where X is an arbitrary non empty set and is called Fuzzy Set in X.

Definition 2.4 :
Let Q and N a set and group respectively. A mapping μ : N X Q → [0,1] is called Q-Fuzzy set in N.

Definition 2.5 : (T-norm)
A triangular norm is a function T : [0,1] X [0,1] → [0,1] that satisfies the following conditions for all x,y,z in [0,1].

(T_1) T (x,1) = x
(T_2) T (x,y) = T (y,x)
(T_3) T [x, T (y,z)] = T (T(x,y) , z)
(T_4) T (x, y) ≤ T (x,z) when y ≤ z.

Definition 2.6 :
A Q-Fuzzy Set 'μ' is called a Q-Fuzzy left N-Sub group of S over Q if μ satisfies
(i) μ (n(x-y,q) ≥T { μ(x,q), μ(y,q)}
(ii) μ (nx,q) ≥{ μ(x,q)}
for all x,y,n ∈ S and q∈ Q.

1.1.2 Section.3
Properties of Q-Fuzzy left N-Subgroups
Preposition 3.1 :
Let 'T' be a t-norm then every imaginable Q-Fuzzy left N-Subgroup 'μ' of a near ring 'S' is a Fuzzy left N-Subgroup of S.

Proof: Assume' μ' is imaginable Q-Fuzzy left N-Subgroup of S, then we have
$\mu(n(x-y,q) \geq T\{\mu(x,q), \mu(y,q)\}$ and $\mu(nx,q) \geq \{\mu(x,q)\}$ for all x,y in S.
Since 'μ' is imaginable, We have
$\text{Min}\{\mu(x,q), \mu(y,q)\} = T[\min\{\mu(x,q), \mu(y,q)\}, \min\{\mu(x,q), \mu(y,q)\}$

$\leq T\{\mu(x,q), \mu(y,q)\}$

$\leq \min\{\mu(x,q), \mu(y,q)\}$
and so
$T\{\mu(x,q), \mu(y,q)\} = \min\{\mu(x,q), \mu(y,q)\}$

It follows that $\mu(n(x-y),q) \geq T\{\mu(x,q), \mu(y,q)\} = \min\{\mu(x,q), \mu(y,q)\}$ for all $x,y \in S$.
Hence 'μ' is a Fuzzy left N-Subgroup of S.

Preposition 3.2 :

If 'μ' is Q-Fuzzy left N-Subgroups of a near ring 'S' and Q is a endomorphism of S, then $\mu_{(Q)}$ is a Q-Fuzzy left N-Subgroup of S.
Proof : For any $x,y \in S$, We have
(i) $\mu_{[Q]}(n(x-y,q) = \mu(Q\, n(x-y,q)$
$= \mu\{(Q(x,q),Q(y,q)\}$
$\geq T\{\mu(Q(x,q), \mu(Q(y,q)\}$
$= T\{\mu_{[Q]}((x,q), \mu_{[Q]}(y,q)\}$
(ii) $\mu_{[Q]}(nx,q) = \mu(Q(nx,q)]$
$\geq \mu(Q(x,q)]$
$\geq \mu_{[Q]}(x,q)]$
Hence $\mu_{(Q]}(x,q)$ is Q-Fuzzy left N-Subgroup of S.

Preposition 3.3 :

An onto homomorphism of a Q-Fuzzy left N-subgroup of near ring 'S' is G-Fuzzy left N Subgroup.

***Proof*:** Let $f : S \to S^1$ be an onto homomorphism of near rings and ' □ ' be a Q-Fuzzy left N- Subgroup of S^1 and 'μ' be the pre image of ' □ ' under f then We have
(i) $\mu((n(x-y),q)] = □(f(n(x-y),q)$
$= □(f(x,q), f(y,q)\}$
$\geq T\{\mu(f(x,q), \mu(f(y,q)\}$
$\geq T\{(f(x,q), (f(y,q)\}$

(ii) $\mu(nx, q) = □(f(nx,q)\}$
$\geq □(f(x,q)\}$
$\geq \mu(x,q)$

Preposition 3.4 :

An onto homomorphic image of a Q-Fuzzy left N-Subgroup with the supremum property is a Q-Fuzzy left N-subgroup.

Proof: Let $f: S \to S^1$ be an onto homomorphic of near rings and let ' μ ' be a supremum property of Q-Fuzzy N- Subgroup of S^1.
Let $x^1, y^1 \in S^1$ and $x_o \in f^{-1}(x^1)$, $y_o \in f^{-1}(y^1)$ be such that

$$\mu(xo,q) = \sup_{(h,a)\in f\text{-}1(x^1)} \mu(h,q),$$

$$\mu(y_o\, q) = \sup_{(h,q)\in f\text{-}1(y^1)} \{\mu(h,q)\}$$ respectively.

Then we can deduce that

(i) $\mu^f(n(x^1 - y^1), q) = \sup_{(z,a)\in f\text{-}1\,(n(x^1\text{-}y^1),q)} \mu(z,q)$
$\geq T\{\mu(x_o,q), \mu(y_o,q)\}$
$\geq T\{\sup_{(h,a)\in f'(x')} (x_o,q), \sup_{(h,a)\in f'(y')} (y_o q)\}$
$= T\{\mu^t(x',q), \mu^t(y',q)\}$

(ii) $\mu^f(nx,q) = \sup_{(z,q)\in f^{-1}(nx',q)} \mu(z,q)$

$\geq \mu(yo,q)$

$= \sup_{(h,a)\in f^{-1}(y',q)} \mu(t,q)$

$= \mu^t(y',q)$

Hence μ^t is a Q-Fuzzy left N-subset of S'.

Preposition 3.5:

Let ' T ' be a continuous t-norm and let ' f ' be a homomorphism on a near ring 'S' if ' μ ' is Q-Fuzzy left N-Subgroup of S, then μ^f is a Q-Fuzzy left N-Subgroup of f(S).

Proof : Let $A_1 = f^{-1}(y_1,q)$, and $A_2 = f^{-1}(y_2,q)$ and $A_{12} = f^{-1}(n(y_1\text{-}y_2,q))$ where $y_1, y_2 \in f(S)$, $q \in Q$.

Consider the set $A_1 - A_2 = \{x\in S / (x,q) = (a_1\text{-}q) - (a_2\text{-}q)\}$ for some $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$
If $(x,q) \in A_1 - A_2$, then
$(x,q) = (x_1\text{-}q) - (x_2\text{-}q)$ for some $(x_1\text{-}q) \in A_1$ and $(x_2,q) \in A_2$
so that We have
$f(x,q) = f(x_1,q) - f(x_2,q)$
$= y_1 - y_2$
$\therefore (x,q) \in f^{-1}[(y_1\text{-}q) - (y_2\text{-}q]) = f^{-1}(n(y_1 - y_2),q)$
$= A_{12}$ Thus
$A_1 - A_2 \subset A_{12}$
It follows that
$\mu^f[n(y_1 - y_2), q) = \sup\{\mu(x,q) / (x,q) \in f^{-1}(y_1 - q),(y_2 - q)\}$
$= \sup\{\mu(x,q) / (x,q) \in A_{12}\}$
$\geq \sup\{\mu(x,q) / (x,q)\in A_1 - A_2\}$
$\geq \sup\{\mu(x_1\text{-}q) - (x_2 - q) / (x_1,q) \in A_1$ and $(x_2,q) \in A_2\}$
Since T is continuous, For every $\in > 0$,
We see that If
$\sup\{\mu(x_1,q) / (x_1, q) \in A_1\} - (x_1^*,q) \in A_2\} \leq \delta$ and
$\sup\{\mu(x_2,.q)/(x_2, q) \in A_2\} - (x_2^*,q) \in A_2\} \leq \delta$
$T\{\sup\{\mu(x_1,q) / (x_1, q) \in A_1\},$
$\sup\{\mu(x_2,.q) /(x_2, q) \in A_2\} - T\{(x_1^*,q), (x_2^*,q)\} \leq \in$
Choose $(a_1,q) \in A_1$ and $(a_2,q) \in A_2$ such that $\sup\{\mu(x_1,q) / (x_1, q) \in A_1\} - \mu(a_1,q) \leq \delta$ then we have
$T\{\sup\{\mu(x_1,q) / (x_1, q) \in A_1\},$
$\sup\{\mu(x_2,.q)/ (x_2, q) \in A_2\}$

$-T\{\mu(a_1,q) - \mu(a_2,q)\} \leq \in$

Consequently We have

$\mu^t(n(y_1 - y_2, q) \geq \text{Sup}\{T(\mu(x_1,q), \mu(x_2, q) / (x_1,q) \in A_1 \text{ and } (x_2, q) \in A_2\}$

$\geq T\{\sup\{\mu(x_1,q) / (x_1,q) \in A_1 . \text{Sup}\{\mu(x_2, q) / (x_2,q) \in A_2\}$

$\geq T\{\mu t(y_1, q), \mu t(y_2, q)\}$

Similarly, We can show $\mu^f(nx, q) \geq \mu^f[(x, q)$. Hence μ^f is a Q-Fuzzy left N-Subgroup of f(S).

Preposition 3.6

Let 'μ' be a Q-Fuzzy left N-Subgroup of S. then the Q-Fuzzy subset <μ> is Q-Fuzzy left N-Subgroup of S generated by 'μ' more over <μ> is the smallest Q-Fuzzy left N-Subgroup containing it.

Proof: Let x,y ∈ N and let $\mu(x, q) = t_1$,

$\mu(y, q) = t_2$ and $\mu(n(x-y, q) = t$

Let it possible $t = \langle\mu\rangle(n(x-y),q)$

$\leq T\{\langle\mu\rangle(x, q), \langle\mu\rangle(n(y,q)\}$

$= T\{t_1, t_2\} = t_1$ (say)

Then $t_1 = \langle\mu\rangle(x,a) = \text{Sup}\{k / x \in \langle\mu_k\rangle\} \geq t$

Therefore there exist k_1, such that $x \in \langle\mu_{k1}\rangle$. Also

$t_2 = \langle\mu\rangle(y,q) = \text{Sup}\{k/y \in \langle\mu_k\rangle\} \geq t$,

Therefore there exist $k_2 > t$ such that $y \in \langle\mu_k\rangle$

without loss of generality, We may assume that k_1,k_2 so that $\langle\mu k_1\rangle \subset \langle\mu k_2\rangle$. Then $x,y \in \langle\mu_k\rangle$ that is n(x-y) which is a contradiction since $k_2 > t$ Therefore $t \geq t_1$

Consequently ,

$\mu(n(x-y,q) \geq T\{\langle\mu\rangle(x,q), \langle\mu\rangle((y,q)\}$---------❶

Now let, if possible,

$t_3 = \{\langle\mu\rangle(nx, q) \leq \langle\mu\rangle(x,q)\} = t_1$

Then $t_1 = \langle\mu\rangle(x,q) = \text{Sup}\{k/x \in \langle\mu_k\rangle\} > t_3$

Therefore there exists k such that $x \in \langle\mu_k\rangle$ and $t_1 > k > t_3$ so that $nx \in \langle\mu_k\rangle \subset \langle\mu_t\rangle$ which is a contradiction.

Hence $t_3 = \{\langle\mu\rangle(nx, q) \geq \{\langle\mu\rangle(x, q) = t_2$--❷

Consequently Conditions ❶❷yield that <μ> is Q-Fuzzy left N- Subgroup of S.

Finally to show that <μ> is the smallest Q-Fuzzy left N-Subgroup containing μ , let as assume that Q to be a Q-fuzzy left N-subgroup of S such that μ ⊂ Q and show that <μ> ⊂ Q .

Let it possible,

$t = \langle\mu\rangle(x,q) \geq Q(x,q)$ for some $x \in N$, $q \in Q$.

Let $\in > 0$ be given, then $t = \mu_t = \text{Sup}\{k / x \in \langle\mu_k\rangle$

and $t - \in \leq k \leq t$

so that $x \in \langle\mu_k\rangle \subset \langle\mu_{kt-\in}\rangle$, for all $\in > 0$.

Now $\alpha = \alpha_1x_1 + \alpha_2x_2 + \alpha_3x_3 + \ldots\ldots\ldots\alpha_nx_n$, $\alpha_I \in N$,

x_i belongs to t-∈.

$x_i \in \mu_{t-\in}$ implies $\mu(xi, q) \geq t-\in$,

i.e.) $Q(xi,q) \geq t-\in$ for all $\in > 0$

$\therefore$ $Q(x,q) \geq T\{Q(x_1,q) Q(x_2,q),\ldots\ldots Q(x_n,q)\}$

$\geq t - \in$ for all $\in > 0$

Hence $Q(x,q) = t$ which is a contradiction to our supposition.

Preposition 3.7 :

Let 'Q' be a Q-Fuzzy left N-Subgroup near ring S and let μ + be a Q-Fuzzy set in N defined by $\mu + (x,a) = \mu(x,a) + 1 - \mu(0,q)$ for x ∈ N. Then μ + is a normal Q-Fuzzy left N-subgroup of S containing 'μ'.

Proof *: For any x,y ∈ N and q ∈ Q,*

We have

(i)$\mu^{-1}(n(x-y),y) = \mu(n(x-y),q) + 1 - \mu(0,q)$

$\geq \{\mu(x,q), \mu(y,q)\} + 1 - \mu(0,q)\}$

$\geq T\{\mu(x,q)+1-\mu(0,q).\mu(y,q)+1-(0,q)\}$

$= T\{\mu^{-1}(x,q) . \mu^{-1}(y,q)\}$

(ii)$\mu + (nx,q) = \mu(nx,q) + 1 - \mu(0,q)$

$\geq \mu(x,q) + 1 - \mu(0,q)$

$= \mu + (x,q)$

CONCLUSION

OsmanKozanci, Sultanyamark and Serifeyilmaz (5) introduced the intuitconistic Q-Fuzzy R-Subgroups of near rings. In this paper we investigate the notion of Q-Fuzzy left N-Subgroup of near ring with respect to t-norm and characterization of them.

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