

Anti L-Fuzzy Normal M-Subgroups

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ABSTRACT

This paper contains some definitions and results of anti L-fuzzy normal M-subgroup is being given. Using homomorphism and anti-homomorphism, anti L-fuzzy normal M-subgroups is studied. Some properties of anti L-fuzzy normal M-subgroups are also established.

Keywords: L-fuzzy subset, anti L-fuzzy M-subgroup, anti L-fuzzy normal M-subgroup, anti L-fuzzy characteristic M-subgroup, homomorphism, anti-homomorphism

INTRODUCTION

The notion of fuzzy sets was introduced by L.A. Zadeh [10]. Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. In 1971, Rosenfield [1] introduced the concept of fuzzy subgroup. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings. In [2], Biswas introduced the concept of anti-fuzzy subgroups of groups. Palaniappan, N and Muthuraj, [6] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy groups. In this paper we define a new algebraic structure of anti L-fuzzy Normal M-subgroups and study some their related properties.

1. PRELIMINARIES

1.1 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be an **anti L-fuzzy M-subgroup** (ALFMSG) of G if it satisfies the following axioms:

- (i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$,
- (ii) $\mu_A(x^{-1}) \leq \mu_A(x)$, for all x and y in G.

1.2 Definition: Let A and B be two anti L-fuzzy M-subgroups of a M-group G. Then A and B are said to be conjugate anti L-fuzzy M-subgroups of G if for some $g \in G$, $\mu_A(x) = \mu_B(g^{-1}xg)$, for every x in G.

1.3 Definition: Let G be a M-group. An anti L-fuzzy M-subgroup A of G is said to be **anti L-fuzzy normal M-subgroup** (ALFNMSG) of G if $\mu_A(xy) = \mu_A(yx)$, for all x and y in G.

1.4 Definition: Let G be a M-group. An anti L-fuzzy M-subgroup A of G is said to be an **anti L-fuzzy characteristic M-subgroup** (ALFCMSG) of G if $\mu_A(x) = \mu_A(f(x))$, for all x in G and f in AutG.

2. SOME PROPERTIES OF ANTI L-FUZZY NORMAL M-SUBGROUPS

2.1 Theorem: Let G be a M-group. If A and B are two anti L-fuzzy normal M-subgroups of G, then their intersection $A \cap B$ is an anti L-fuzzy normal M-subgroup of G.

Proof: Let x and y in G. Let $A = \{ \langle x, \mu_A(x) \rangle / x \in G \}$ and $B = \{ \langle x, \mu_B(x) \rangle / x \in G \}$

be an anti L-fuzzy normal M-subgroups of a M-group G.

Let $C = A \cap B$ and $C = \{ \langle x, \mu_C(x) \rangle / x \in G \}$.

Then, we know that C is an anti L-fuzzy M-subgroup of a M-group G,

since A and B are two anti L-fuzzy M-subgroups of a M-group G.

And, $\mu_C(xy) = \mu_A(xy) \wedge \mu_B(xy)$, as A and B are an ALFNMSGs of a M-group G.

$$= \mu_A(yx) \wedge \mu_B(yx) = \mu_C(yx).$$

Therefore, $\mu_C(xy) = \mu_C(yx)$.

Hence $A \cap B$ is an anti L-fuzzy normal M-subgroup of a M-group G.

2.2 Theorem: Let G be a M-group. The intersection of a family of anti L-fuzzy normal M-subgroups of G is an anti L-fuzzy normal M-subgroup of G.

Proof: Let $\{A_i\}_{i \in I}$ be a family of anti L-fuzzy normal M-subgroups of a M-group G and let $A = \bigcap A_i$. Then for x and y in G. We know that, the intersection of a family of anti L-fuzzy M-subgroups of a M-group G is an anti L-fuzzy M-subgroup of a M-group G.

$$\text{Now, } \mu_A(xy) = \inf_{i \in I} \mu_{A_i}(xy)$$

$$= \inf_{i \in I} \mu_{A_i}(yx), \text{ as } \{A_i\}_{i \in I} \text{ are}$$

ALFNMSG of a M-group G

$$= \mu_A(yx).$$

Therefore, $\mu_A(xy) = \mu_A(yx)$.

Hence the intersection of a family of anti L-fuzzy normal M-subgroups of a M-group G is an anti L-fuzzy normal M-subgroup of a M-group G.

2.3 Theorem: If A is an anti L-fuzzy characteristic M-subgroup of a M-group G, then A is an anti L-fuzzy normal M-subgroup of a M-group G.

Proof: Let A be an anti L-fuzzy characteristic M-subgroup of a M-group G and let x and y in G. Consider the map $f : G \rightarrow G$ defined by $f(x) = yxy^{-1}$.

We know that, $f \in \text{Aut}G$.

Now,

$$\begin{aligned} \mu_A(xy) &= \mu_A(f(xy)), \\ &\text{as A is an ALFCMSG of a M-group G} \\ &= \mu_A(y(xy)y^{-1}) \\ &= \mu_A(yx). \end{aligned}$$

Therefore, $\mu_A(xy) = \mu_A(yx)$.

Hence A is an anti L-fuzzy normal M-subgroup of a M-group G.

2.4 Theorem: An anti L-fuzzy M-subgroup A of a M-group G is an anti L-fuzzy normal M-subgroup of G if and only if A is constant on the conjugate classes of G.

Proof: Suppose that A is an anti L-fuzzy normal M-subgroup of a M-group G and let x and y in G. Now,

$$\begin{aligned} \mu_A(y^{-1}xy) &= \mu_A(xy y^{-1}), \\ &\text{since A is an ALFNMSG of G} \\ &= \mu_A(x). \end{aligned}$$

Therefore, $\mu_A(y^{-1}xy) = \mu_A(x)$.

Hence $(x) = \{y^{-1}xy / y \in G\}$.

Hence A is constant on the conjugate classes of G.

Conversely, suppose that A is constant on the conjugate classes of G.

$$\begin{aligned} \text{Then, } \mu_A(xy) &= \mu_A(xyxx^{-1}) \\ &= \mu_A(x(yx)x^{-1}), \text{ as A is constant} \\ &\quad \text{on the conjugate classes of G} \\ &= \mu_A(yx). \end{aligned}$$

Therefore, $\mu_A(xy) = \mu_A(yx)$.

Hence A is an anti L-fuzzy normal M-subgroup of a M-group G.

2.5 Theorem: Let A be an anti L-fuzzy normal M-subgroup of a M-group G. Then for any $y \in G$ we have $\mu_A(yxy^{-1}) = \mu_A(y^{-1}xy)$, for every $x \in G$.

Proof: Let A be an anti L-fuzzy normal M-subgroup of a M-group G.

For any $y \in G$, we have,

$$\begin{aligned} \mu_A(yxy^{-1}) &= \mu_A(y^{-1}yx) \\ &= \mu_A(x), \\ &\quad \text{since A is an ALFNMSG of G} \\ &= \mu_A(xy y^{-1}), \\ &\quad \text{since A is an ALFNMSG of G} \\ &= \mu_A(y^{-1}xy). \end{aligned}$$

Therefore, $\mu_A(yxy^{-1}) = \mu_A(y^{-1}xy)$, for all x and y in G.

2.6 Theorem: Let A and B be anti L-fuzzy M-subgroups of M-groups G and H, respectively. If A and B are anti L-fuzzy normal M-subgroups, then AxB is an anti L-fuzzy normal M-subgroup of GxH .

Proof: Let A and B be an anti L-fuzzy normal M-subgroups of the M-groups G and H respectively. We know that AxB is an anti L-fuzzy M-subgroup of GxH .

Let x_1 and x_2 be in G, y_1 and y_2 be in H.

Then (x_1, y_1) and (x_2, y_2) are in GxH .

$$\begin{aligned} \text{Now, } \mu_{AxB}[(x_1, y_1)(x_2, y_2)] &= \mu_{AxB}(x_1x_2, y_1y_2) \\ &= \mu_A(x_1x_2) \vee \mu_B(y_1y_2) \\ &= \mu_A(x_2x_1) \vee \mu_B(y_2y_1), \\ &\quad \text{since A and B are ALFNMSGs of the} \\ &\quad \text{groups G and H} \end{aligned}$$

$$\begin{aligned} &= \mu_{AxB}(x_2x_1, y_2y_1) \\ &= \mu_{AxB}[(x_2, y_2)(x_1, y_1)]. \end{aligned}$$

Therefore, $\mu_{AxB}[(x_1, y_1)(x_2, y_2)] =$

$$\mu_{AxB}[(x_2, y_2)(x_1, y_1)].$$

Hence AxB is an anti L-fuzzy normal M-subgroup of GxH .

2.7 Theorem: Let an anti L-fuzzy normal M-subgroup A of a M-group G be conjugate to an anti L-fuzzy normal M-subgroup M of G and an anti L-fuzzy normal M-subgroup B of a M-group H be conjugate to an anti L-fuzzy normal M-subgroup N of H. Then an anti L-fuzzy normal M-subgroup AxB of a M-group GxH is conjugate to an anti L-fuzzy normal M-subgroup MxN of GxH .

Proof: Let A and B be anti L-fuzzy normal M-subgroups of the M-groups G and H respectively. Let x, x^{-1} and f be in G and y, y^{-1} and g be in H.

Then (x, y) , (x^{-1}, y^{-1}) and (f, g) are in GxH .

$$\begin{aligned} \text{Now, } \mu_{AxB}(f, g) &= \mu_A(f) \vee \mu_B(g) \\ &= \mu_M(xf x^{-1}) \vee \mu_N(yg y^{-1}), \end{aligned}$$

(since ALFNMSGs A and B of M-groups G and H are conjugate to ALFNMSGs M and N of G and H.)

$$\begin{aligned} &= \mu_{MxN}(xf x^{-1}, yg y^{-1}) \\ &= \mu_{MxN}[(x, y)(f, g)(x^{-1}, y^{-1})] \\ &= \mu_{MxN}[(x, y)(f, g)(x, y)^{-1}]. \end{aligned}$$

Therefore, $\mu_{AxB}(f, g) = \mu_{MxN}[(x, y)(f, g)(x, y)^{-1}]$.

Hence an anti L-fuzzy Normal M-subgroup AxB of a M-group GxH is conjugate to an anti L-fuzzy Normal M-subgroup MxN of GxH .

2.8 Theorem: Let A and B be L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and e' are the identity element of G and H, respectively. If AxB is an anti L-fuzzy normal M-subgroup of GxH , then at least one of the following two statements must hold.

(i) $\mu_B(e') \leq \mu_A(x)$, for all x in G,

(ii) $\mu_A(e) \leq \mu_B(y)$, for all y in H.

Proof: Let AxB is an anti L-fuzzy Normal M-subgroup of GxH .

By contraposition, suppose that none of the statements (i) and (ii) holds.

Then we can find a in G and b in H such that $\mu_A(a) < \mu_B(e^l)$ and $\mu_B(b) < \mu_A(e)$.

$$\begin{aligned} \text{We have, } \mu_{AxB}(a,b) &= \mu_A(a) \vee \mu_B(b) \\ &< \mu_A(e) \vee \mu_B(e^l) \\ &= \mu_{AxB}(e, e^l). \end{aligned}$$

Thus AxB is not an anti L-fuzzy Normal M-subgroup of GxH . Hence either $\mu_B(e^l) \leq \mu_A(x)$, for all x in G or $\mu_A(e) \leq \mu_B(y)$, for all y in H .

2.9 Theorem: Let A and B be L-fuzzy subsets of the M-groups G and H , respectively and AxB is an anti L-fuzzy normal M-subgroup of GxH .

Then the following are true:

- (i) if $\mu_A(x) \geq \mu_B(e^l)$, then A is an anti L-fuzzy normal M-subgroup of G .
- (ii) if $\mu_B(x) \geq \mu_A(e)$, then B is an anti L-fuzzy normal M-subgroup of H .
- (iii) either A is an anti L-fuzzy normal M-subgroup of G or B is an anti L-fuzzy normal M-subgroup of H .

Proof: Let AxB be an anti L-fuzzy Normal M-subgroup of GxH and x, y in G .

Then (x, e^l) and (y, e^l) are in GxH . Now, using the property $\mu_A(x) \geq \mu_B(e^l)$, for all x in G ,

$$\begin{aligned} \text{we get, } \mu_A(xy^{-1}) &= \mu_A(xy^{-1}) \vee \mu_B(e^le^l) \\ &= \mu_{AxB}(xy^{-1}, e^le^l) \\ &= \mu_{AxB}[(x, e^l)(y^{-1}, e^l)] \\ &\leq \mu_{AxB}(x, e^l) \vee \mu_{AxB}(y^{-1}, e^l) \\ &= \{\mu_A(x) \vee \mu_B(e^l)\} \\ &\quad \vee \{\mu_A(y^{-1}) \vee \mu_B(e^l)\} \\ &= \mu_A(x) \vee \mu_A(y^{-1}) \\ &\leq \mu_A(x) \vee \mu_A(y). \end{aligned}$$

Therefore, $\mu_A(xy^{-1}) \leq \mu_A(x) \vee \mu_A(y)$.

We know that $\mu_A(mx) \leq \mu_A(x)$.

$$\begin{aligned} \mu_A(xy) &= \mu_A(xy) \vee \mu_B(e^le^l) \\ &= \mu_{AxB}(xy, e^le^l) \\ &= \mu_{AxB}[(x, e^l)(y, e^l)] \\ &= \mu_{AxB}((y, e^l)(x, e^l)) \\ &= \mu_{AxB}(yx, e^le^l) \\ &= \mu_A(yx) \vee \mu_B(e^le^l) \\ &= \mu_A(yx) \end{aligned}$$

Hence A is an anti L-fuzzy Normal M-subgroup of G . Thus (i) is proved.

Now, using the property $\mu_B(x) \geq \mu_A(e)$, for all x in G , we get,

$$\begin{aligned} \mu_B(xy^{-1}) &= \mu_B(xy^{-1}) \vee \mu_A(ee) \\ &= \mu_{AxB}(ee, (xy^{-1})) \\ &= \mu_{AxB}[(e, x)(e, y^{-1})] \\ &\leq \mu_{AxB}(e, x) \vee \mu_{AxB}(e, y^{-1}) \\ &= \{\mu_B(x) \vee \mu_A(e)\} \vee \{\mu_B(y^{-1}) \vee \mu_A(e)\} \end{aligned}$$

$$\begin{aligned} &= \mu_B(x) \vee \mu_B(y^{-1}) \\ &\leq \mu_B(x) \vee \mu_B(y). \end{aligned}$$

Therefore, $\mu_B(xy^{-1}) \leq \mu_B(x) \vee \mu_B(y)$, for all x and y in H .

We know that $\mu_B(mx) \leq \mu_B(x)$.

$$\begin{aligned} \mu_B(xy) &= \mu_A(ee) \vee \mu_B(xy) \\ &= \mu_{AxB}(ee, xy) \\ &= \mu_{AxB}[(e, x)(e, y)] \\ &= \mu_{AxB}[(e, y)(e, x)] \\ &= \mu_{AxB}(ee, yx) \\ &= \mu_A(ee) \vee \mu_B(yx) \\ &= \mu_B(yx) \end{aligned}$$

Hence B is an anti L-fuzzy Normal M-subgroup of H . Thus (ii) is proved.

(iii) is clear.

3. ANTI L-FUZZY NORMAL M-SUBGROUPS OF AN M-GROUP G UNDER HOMOMORPHISM AND ANTI-HOMOMORPHISM

3.1 Theorem: The homomorphic image of an anti L-fuzzy M-subgroup of G is an anti L-fuzzy M-subgroup of G^l

3.2 Theorem: Let G and G^l be any two M-groups. The homomorphic image of an anti L-fuzzy normal M-subgroup of G is an anti L-fuzzy normal M-subgroup of G^l .

Proof: Let G and G^l be any two M-groups. Let $f: G \rightarrow G^l$ be a homomorphism.

That is $f(xy) = f(x)f(y)$, $f(mx) = mf(x)$, for all x and y in G and m in M .

Let $V = f(A)$, where A is an anti L-fuzzy normal M-subgroup of G .

We have to prove that V is an anti L-fuzzy normal M-subgroup of G^l .

Now, for $f(x)$ and $f(y) \in G^l$, we have V is an anti L-fuzzy M-subgroup of a M-group G^l ,

since A is an anti L-fuzzy M-subgroup of a M-group G .

Now, $\mu_V(f(x)f(y)) = \mu_V(f(xy))$,

as f is a homomorphism

$$\leq \mu_A(xy) = \mu_A(yx),$$

as A is an ALFNMSG of G

$$\geq \mu_V(f(yx))$$

$$= \mu_V(f(y)f(x)),$$

as f is a homomorphism,

which implies that $\mu_V(f(x)f(y)) = \mu_V(f(y)f(x))$.

Hence V is an anti L-fuzzy normal M-subgroup of a M-group G^l .

3.3 Theorem: The homomorphic pre-image of an anti L-fuzzy M-subgroup of G^l is an anti L-fuzzy M-subgroup of G .

3.4 Theorem: Let G and G^1 be any two M-groups. The homomorphic pre-image of an anti L-fuzzy normal M-subgroup of G^1 is an anti L-fuzzy normal M-subgroup of G .

Proof: Let G and G^1 be any two M-groups. Let $f : G \rightarrow G^1$ be a homomorphism.

That is $f(xy) = f(x)f(y)$, $f(mx) = mf(x)$, for all x and y in G and m in M .

Let $V=f(A)$, where V is an anti L-fuzzy normal M-subgroup of G^1 .

We have to prove that A is an anti L-fuzzy normal M-subgroup of G .

Let x and y in G . Then, we know that, A is an anti L-fuzzy M-subgroup of a M-group G ,

since V is an anti L-fuzzy M-subgroup of a M-group G^1 .

$$\begin{aligned} \text{Now, } \mu_A(xy) &= \mu_V(f(xy)), \\ &\text{since } \mu_A(x) = \mu_V(f(x)) \\ &= \mu_V(f(x)f(y)), \\ &\text{as } f \text{ is a homomorphism} \\ &= \mu_V(f(y)f(x)), \\ &\text{as } V \text{ is an ALFNMSG of } G^1 \\ &= \mu_V(f(yx)), \\ &\text{as } f \text{ is a homomorphism} \\ &= \mu_A(yx), \\ &\text{since } \mu_A(x) = \mu_V(f(x)), \end{aligned}$$

which implies that $\mu_A(xy) = \mu_A(yx)$.

Hence A is an anti L-fuzzy normal M-subgroup of a M-group G .

3.5 Theorem: The anti-homomorphic image of an anti L-fuzzy normal M-subgroup of G is an anti L-fuzzy M-subgroup of G^1 .

3.6 Theorem: Let G and G^1 be any two M-groups. The anti-homomorphic image of an anti L-fuzzy normal M-subgroup of G is an anti L-fuzzy normal M-subgroup of G^1 .

Proof: Let G and G^1 be any two M-groups. Let $f : G \rightarrow G^1$ be an anti-homomorphism.

That is $f(xy) = f(y)f(x)$, $f(mx) = f(x)m$, for all x and y in G and m in M .

Let $V = f(A)$, where A is an anti L-fuzzy normal M-subgroup of G .

We have to prove that V is an anti L-fuzzy normal M-subgroup of G^1 .

For $f(x)$ and $f(y) \in G^1$, we know that, V is an anti L-fuzzy M-subgroup of a M-group G^1 ,

since A is an anti L-fuzzy M-subgroup of a M-group G .

Now,

$$\begin{aligned} \mu_V(f(x)f(y)) &= \mu_V(f(yx)), \\ &\text{as } f \text{ is an anti-homomorphism} \\ &\leq \mu_A(yx) = \mu_A(xy), \\ &\text{as } A \text{ is an ALFNMSG of } G \\ &\geq \mu_V(f(xy)) = \mu_V(f(y)f(x)), \\ &\text{as } f \text{ is an anti-homomorphism,} \end{aligned}$$

which implies that $\mu_V(f(x)f(y)) = \mu_V(f(y)f(x))$.

Hence V is an anti L-fuzzy normal M-subgroup of a M-group G^1 .

3.7 Theorem: Let G and G^1 be any two M-groups. The anti-homomorphic pre-image of an anti L-fuzzy M-subgroup of G^1 is an anti L-fuzzy M-subgroup of G .

3.8 Theorem: Let G and G^1 be any two M-groups. The anti-homomorphic pre-image of an anti L-fuzzy normal M-subgroup of G^1 is an anti L-fuzzy normal M-subgroup of G .

Proof: Let G and G^1 be any two M-groups. Let $f : G \rightarrow G^1$ be anti-homomorphism.

That is $f(xy) = f(y)f(x)$, $f(mx) = f(x)m$, for all x and y in G and m in M .

Let $V=f(A)$, where V is an anti L-fuzzy normal M-subgroup of G^1 .

We have to prove that A is an anti L-fuzzy normal M-subgroup of G .

Let x and y in G , we have, A is an anti L-fuzzy M-subgroup of a M-group G ,

since V is an anti L-fuzzy M-subgroup of a M-group G^1 .

$$\begin{aligned} \text{Now, } \mu_A(xy) &= \mu_V(f(xy)), \\ &\text{since } \mu_A(x) = \mu_V(f(x)) \\ &= \mu_V(f(y)f(x)), \\ &\text{as } f \text{ is an anti homomorphism} \\ &= \mu_V(f(x)f(y)), \\ &\text{as } V \text{ is an ALFNMSG of } G^1 \\ &= \mu_V(f(yx)), \\ &\text{as } f \text{ is an anti homomorphism} \\ &= \mu_A(yx), \text{ since } \mu_A(x) = \mu_V(f(x)), \end{aligned}$$

which implies that $\mu_A(xy) = \mu_A(yx)$.

Hence A is an anti L-fuzzy normal M-subgroup of a M-group G .

In the following Theorem ◦ is the composition operation of functions:

3.9 Theorem: Let A be an anti L-fuzzy M-subgroup of a M-group H and f is an isomorphism from a M-group G onto H . If A is an anti L-fuzzy normal M-subgroup of a M-group H , then $A \circ f$ is an anti L-fuzzy normal M-subgroup of a M-group G .

Proof: Let x and y in G and A be an anti L-fuzzy normal M-subgroup of a M-group H .

We know that, $A \circ f$ is an anti L-fuzzy M-subgroup of a M-group G .

Then we have,

$$\begin{aligned} (\mu_{A \circ f})(xy) &= \mu_A(f(xy)) \\ &= \mu_A(f(x)f(y)), \\ &\text{as } f \text{ is an isomorphism} \\ &= \mu_A(f(y)f(x)), \\ &\text{as } A \text{ is an ALFNMSG of a group } H \\ &= \mu_A(f(yx)), \text{ as } f \text{ is an isomorphism} \\ &= (\mu_{A \circ f})(yx), \end{aligned}$$

which implies that $(\mu_{A \circ f})(xy) = (\mu_{A \circ f})(yx)$.

Hence $A \circ f$ is an anti L-fuzzy normal M-subgroup of a M-group G.

3.10 Theorem: Let A be an anti L-fuzzy M-subgroup of a M-group H and f is an anti-isomorphism from a M-group G onto H. If A is an anti L-fuzzy normal M-subgroup of a M-group H, then $A \circ f$ is an anti L-fuzzy normal M-subgroup of a M-group G.

Proof: Let x and y in G and A be an anti L-fuzzy normal M-subgroup of a M-group H.

We know that, $A \circ f$ is an anti L-fuzzy M-subgroup of a M-group G.

Then we have,

$$(\mu_{A \circ f})(xy) = \mu_A(f(xy))$$

$$= \mu_A(f(y)f(x)),$$

as f is an anti-isomorphism

$$= \mu_A(f(x)f(y)),$$

as A is an ALFNMSG of a group H

$$= \mu_A(f(yx)),$$

as f is an anti-isomorphism

$$= (\mu_{A \circ f})(yx),$$

which implies that $(\mu_{A \circ f})(xy) = (\mu_{A \circ f})(yx)$.

Hence $(A \circ f)$ is an anti L-fuzzy normal M-subgroup of a M-group G.

4. CONCLUSIONS

Further work is in progress in order to develop the Intuitionistic Fuzzy normal M-subgroup and Intuitionistic Anti L-Fuzzy normal M-subgroup.

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