S-Product of Anti Q-Fuzzy Left M-N Subgroups of Near Rings under Triangular Conorms

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ABSTRACT
In this paper, we introduce the notion of Q- fuzzification of left M-N subgroups in a near-ring and investigate some related properties. Characterization of Anti Q- fuzzy left M-N subgroups with respect to s-norm is given.

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Index terms: Q- fuzzy set, Q- fuzzy M-N subgroup (sub near rings), anti Q- fuzzy left M-N subgroups, s-norm.

1. INTRODUCTION
The theory of fuzzy sets which was introduced by Zadeh [8] is applied to many mathematical branches. Abou-zoid [1], introduced the notion of a fuzzy sub near-ring and studied fuzzy ideals of near-ring. This concept discussed by many researchers among cho, Davvaz, Dudek, Jun, Kim [2],[3],[4]. In [5], considered the intuitionistic fuzzification of a right (resp left) R- subgroup in a near-ring. A.Solairaju and R.Nagarajan [7] introduced the new structures of Q- fuzzy groups and then they investigate the notion Q- fuzzy left R- subgroups of near rings with respect to T-norms in [6]. Also cho.at.al in [4] the notion of normal intuitionistic fuzzy R- subgroup in a near-ring is introduced and related properties are investigated. The notion of intuitionistic Q- fuzzy semi primality in a semi group is given by Kim [3]. In this paper, we introduce the notion of Q- fuzzification of left M-N subgroups in a near ring and investigate some related properties. Characterizations of Q- anti fuzzy left M-N subgroups are given.

2. PRELIMINARIES
Definition 2.1: A non empty set with two binary operations '+' and '.' is called a near-ring if it satisfies the following axioms

(i) (R,+) is a group.
(ii) (R,.) is a semi group.
(iii) x . (y+z) = x .y + x . z for all x,y,z ε R. Precisely speaking it is a left near-ring. Because it satisfies the left distributive law.

As R – subgroup of a near- ring 'S' is a subset 'H' of 'R' such that

(i) (H , +) is a subgroup of (R, +).
(ii) RH ⊂ H
(iii) HR ⊂ H. If 'H' satisfies (i) and (ii) then it is called left N- subgroup of 'R' and if 'N' satisfies (i) and (iii) then it is called a right N- subgroup of 'R'. A map f : R→ S is called homomorphism if f(x+y) = f(x) + f (y) for all x,y in R.

Definition 2.2 : Let M is a left operator sets of group G, N is right operator sets of group G. If (ma)n = m(an) for all a in G, m ε M, n ε N, then G is said to be an M-N group. If a subgroup of M-N group is also M-N group, then it is called M-N subgroup of G.

Definition 2.3 : Let G and G^1 both be M-N groups. f : G→ G^1 be a homomorphism's, If f(mx) = mf(x) and f(xn) = f(x)n for all x ε G, mε M, nε N, then f is called M-N homomorphism.

Definition 2.4: Let 'R' be a near ring. A fuzzy set 'μ' in R is called Q- fuzzy sub near ring in 'R' if

(i) $\mu(x-y,q) \geq \min \{ \mu(x,q) , \mu(y,q) \}$

(ii) $\mu(xy,q) \geq \min \{ \mu(x,q) , \mu(y,q) \}$ for all x,y in R.

Definition 2.5: A 'Q'-fuzzy set 'μ' is called a Anti Q-fuzzy left M-N subgroup of R over Q if 'μ' satisfies

(i) $\mu(m(x-y), q) \leq \max\{\mu(mx,q), \mu(my,q)\}$ (ii) $\mu(xn,q) \leq \mu(x,q)$ for all x,y,m,n ε R and q ε Q.

Definition 2.6 : By a s- norm 'S' , we mean a function S: $[0,1]\times[0,1]\rightarrow[0,1]$ satisfying the following conditions ;

(S1) $S(x,0) = x$

(S2) $S(x,y) \leq S(x,z)$ if $y \leq z$

(S3) $S(x,y) = S(y,x)$

(S4) $S(x, S(y,z)) = S(S(x,y),z)$, for all x,y,z ε [0,1].

Proposition 2.7: For a S-norm, then the following statement holds $S(x,y) \geq \max\{x,y\}$, for all x,y ε [0,1].

Definition 2.8: Let 'S' be a s-norm. A fuzzy set 'A' in 'R' is said to be sensible with respect to 'S' if Im(A) c Δs, where $\Delta s = \{ s(\alpha, \alpha) = \alpha / \alpha \in [0,1] \}$.

3. PROPERTIES OF ANTI Q- FUZZY LEFT M-N SUBGROUPS

Proposition 3.1: Let 'S' be a s- norm. Then every imaginable anti Q- fuzzy left M-N subgroup 'μ' of a near ring ' R' is a Q-fuzzy left M-Nsubgroup of R.

Proof: Assume 'μ' is imaginable anti Q- fuzzy left M-N subgroup of 'R', then we have

$\mu(m(x-y), q) \leq S\{ \mu(mx,q), \mu(my,q) \}$ and $\mu(xn, q) \leq \mu(x,q)$ for all x,y in R.

Since 'μ' is imaginable, we have

$\max\{ \mu(mx,q) , \mu(my,q) \}$

$= S\{\max\{\mu(mx,q), \mu(my,q)\}, \max\{ \mu(mx,q) , \mu(my,q) \}\}$

$\geq S(\mu(mx,q) , \mu(my,q))$

$\geq \max\{\mu(mx,q) , \mu(my,q)\}$

And so $S(\mu(mx,q) , \mu(my,q))$

$= \max\{ \mu(mx,q) , \mu(my,q) \}$. It follows that $\mu(m(x-y), q) \leq S(\mu(mx,q) , \mu(my,q))$

$= \max\{ \mu(mx,q) , \mu(my,q) \}$ for all x,y ε R. Hence 'μ' is a Q-fuzzy left M-N subgroup of R.

Proposition 3.2: If 'μ' is anti Q- fuzzy left M-N subgroups of a near ring 'R' and 'Θ' is an endomorphism of R, then μ[Θ] is a anti Q- fuzzy left M-N sub group of 'R'.

Proof: For any x,y ε R, we have

(i) $\mu[\Theta](m(x-y),q) = \mu(\Theta(m(x-y), q))$

$= \mu(\Theta(mx,q), \Theta(my,q))$

$\leq S\{ \mu(\Theta(mx,q)), \mu(\Theta(my,q)) \}$

$= S\{ \mu[\Theta](mx,q), \mu[\Theta](my,q) \}$

(ii) $\mu[\Theta](xn, q) = \mu(\Theta(xn, q))$

$\leq \mu(\Theta(x,q))$

$\leq \mu[\Theta](x,q)$.

Hence μ[Θ] is a anti Q- fuzzy left M-N subgroup of R.

Proposition 3.3: An onto homomorphism's of anti Q-fuzzy left M-N subgroup of near ring 'R' is anti Q- fuzzy left M-N subgroup.

Proof: Let $f : R\rightarrow R^1$ be an onto homomorphism of near rings and let 'ξ' be anti Q- fuzzy left M-N subgroup of R^1 and 'μ' be the pre image of 'ξ' under 'f', then we have

(i) $\mu(m(x-y), q) = \xi(f(m(x-y), q))$

$= \xi(f(mx,q), f(my,q))$

$\leq S(\xi(f(mx,q)),\xi(f(my,q)))$

$\leq S(\mu(mx,q), \mu(my,q))$

(ii) $\mu(xn,q) = \xi(f(xn,q))$

$\leq \xi(f(x,q))$

$\leq \mu(x,q)$.

Proposition 3.4: An onto homomorphic image of a anti Q-fuzzy left M-N subgroup with the inf property is anti Q-fuzzy left M-N- subgroup.

Proof: Let f: $R \rightarrow R^1$ be an onto homomorphism of near rings and let 'μ' be a inf property of anti Q-fuzzy left M-N subgroups of 'R'.

Let $x^1, y^1 \in R^1$, and $x_0 \in f^{-1}(x^1)$, $y_0 \in f^{-1}(y^1)$ be such that

$$\mu(x_0, q) = \inf_{(h,q)\varepsilon f^{-1}(x^1)} \mu(h,q), \quad \mu(y_0,q) = \inf_{(h,q)\varepsilon f^{-1}(y^1)} \mu(h,q)$$

Respectively, then we can deduce that

(i) $$\mu^f(m(x^1-y^1), q) = \inf_{(mz,q)\,\varepsilon\, f^{-1}(m(x^1-y^1),q)} \mu(mz,q)$$

$$\leq \max\{\mu(mx_0,q), \mu(my_0,q)$$

$$= \max\{\inf_{(h,q)\varepsilon f^{-1}(x^1,q)} \mu(mh,q), \inf_{(h,q)\varepsilon f^{-1}(y^1,q)} \mu(mh,q)\}$$

$$= \max\{\mu^f(mx^1,q), \mu^f(my^1,q)\}$$

(ii) $$\mu^f(xn,q) = \inf_{(zn,q)\varepsilon f^{-1}(r^1x^1n,q)} \mu(zn,q)$$

$$\leq \mu(y_0, q)$$

$$= \inf_{(h,q)\,\varepsilon\, f^{-1}(y^1,q)} \mu(hn,q)$$

$$= \mu^f(y^1,q).$$

Hence 'μ^f' is a anti Q- fuzzy left M-N subgroup of R^1.

Proposition 3.5: Let 'S' be a continuous s-norm and let 'f' be a homomorphism on a near ring 'R' . If 'μ' is anti Q-fuzzy left M-N subgroup of R, then μ^f is anti Q- fuzzy left M-N subgroup of f(R).

Proof: Let $A_1 = f^{-1}(y_1,q)$, $A_2 = f^{-1}(y_2,q)$ and $A_{12} = f^{-1}(n(y_1-y_2), q)$ where $y_1,y_2 \in f(S)$, $q \in Q$. Consider the set

$A_1 - A_2 = \{ x \in S / (x,q) = (a_1,q) - (a_2,q) \}$ for some $(a_1,q) \varepsilon A_1$ and $(a_2,q) \in A_2$.

If $(x,q) \in A_1-A_2$, then $(x,q) = (x_1,q) - (x_2,q)$ for some $(x_1,q) \in A_1$ and $(x_2,q) \in A_2$

so that we have

$$f(x,q) = f(x_1,q) - f(x_2,q)$$

$$= y_1 - y_2$$

$(x,q) \in f^{-1}((y_1,q) - (y_2,q))$

$= f^{-1}(n(y_1-y_2), q) = A_{12}$.

Thus $A_1-A_2 \subset A_{12}$.

It follows that

(i) $\mu^f(m(y_1-y_2), q)$

$= \inf\{\mu(mx,q)/(mx,q) \in f^{-1}(my_1,q)- (my_2,q))\}$

$= \inf\{ \mu(mx,q) / (x,q) \in A_{12} \}$

$\geq \inf \{ \mu(mx,q)/ (x,q) \in A_1-A_2\}$

$\geq \inf \{ \mu((mx_1,q)- (mx_2,q)) / (x_1,q) \in A_1$ and $(x_2,q) \in A_2\}$

$\geq \inf \{ S(\mu(mx_1,q) , \mu(mx_2,q))/ (x_1,q) \in A_1$ and $(x_2,q) \in A_2\}$

Since 'S' is continuous. For every $\varepsilon > 0$, we see that if

$\inf \{\mu(mx_1,q) / (x_1,q) \in A_1\} - (mx_1*, q) \} \geq \delta$ and

$\inf \{ \mu(mx_2,q) / (x_2,q) \in A_2\} - (mx_2*,q)\} \geq \delta$

$S\{\inf\{\mu(mx_1,q) / (x_1,q) \in A_1\} , \inf \{ \mu(mx_2,q) / (x_2,q) \in A_2 \}$
$- S((mx_1*,q), (mx_2*,q) \geq \varepsilon$

Choose (a_1,q) ε A_1 and (a_2,q) ε A_2 such that

inf { μ(mx_1,q) / (x_1.q) ε A_1 } - μ(ma_1,q) ≥ δ and

inf { μ(mx_2,q) / (x_2,q) ε A_2} - μ(ma_2,q) ≥ δ. Then we have

S{inf{ μ(mx_1,q) / (x_1,q) ε A_1}, inf {μ(mx_2,q) / (x_2,q) ε A_2 } – S(μ(ma_1,q), μ(ma_2,q) ≥ ε consequently, we have μ^f(m(y_1-$y_)$, q) ≤ inf{ S(μ(mx_1,q), μ(x_2,q)) / (x_1,q) ε A_1 ,(x_2,q) ε A_2}

≤ S (inf{μ(mx_1,q) / (x_1,q) ε A_1}, inf{μ(mx_2,q) / (x_2,q)εA_2}

≤ S (μ^f(my_1,q) , μ^f(my_2,q) }

Similarly we can show μ^f(xn,q) ≤ μ^f(y,q). Hence 'μ^f' is anti Q- fuzzy left M-N subgroup of 'f(R)'.

Proposition 3.6: Let μ be anti Q fuzzy M-N subgroup of R. Then the Q- fuzzy subset <μ> is a anti Q- fuzzy M-N subgroup of S generated by. More over <μ> is the smallest anti Q- fuzzy M-N subgroup containing μ.

Proof; Let x,y ε N and let μ (x,q) = t, μ(y,q) = t_2 and μ(m(x-y) , q) = t

Let it possible t = < μ >(m(x-y) , q) ≥ S { <μ> (mx,q) , <μ> (my,q) } S {t_1,t_2} = t_1 (say)

Then t_1 = <μ > (mx,q) = inf { k / x ε <μ_k> } ≤ t, therefore there exist k_1, such that x ε <μ_{k1}> . Also t_2 = <μ >(my,q) = sup {k / yε <μ_k > } t_1 ≤ t. Therefore there exists k_2 ≤ t such that y ε <μ_k > without loss of generality, we may assume that k_1 k_2, so that <μ_{k1}>C <μ_{k2}>. Then x,y ε <μ_k> that is x-y which is a contradiction since k_2 ≤ t. therefore t ≤ t_1. Consequently, μ(m(x-y),q) ≤ S {<μ>(mx,q) , <μ> (my,q)}

------ (1)

Now let , if possible, t_3 = <μ> (xn,q) ≤ <μ> (xn,q) = t_1

Then t_1 = <μ> (xn,q) = inf {k / x ε <μ_k>} ≤ t_3, therefore there exists k such that x ε <μ_k> and t_1 ≤ k ≤ t_3 so that xn ε <μ_k> C {μ_{t1}} which is a contradiction.

Hence t_3 = <μ> (xn,q) ≤ <μ> (xn,q) = t_1

------ (2)

Consequently conditions (1) and (2) yield that <μ> is a anti Q- fuzzy M-N subgroup of R. Finally, to show that <μ> is the smallest anti Q- fuzzy M-N subgroup containing μ , let us assume that θ to be anti Q- fuzzy M-N subgroup of R such that μ C θ and show that <μ> C θ.

Let it possible, t = <μ> (x,q) ≥ θ (x,q) for some x ε N , q ε Q. Let ε > 0 be given, then t = μ_t = sup { k / x ε <μ_k> }. Therefore there exists K such that x ε ,μ_k> and t-ε ≥ k ≥ t so that x ε <μ_k> C < μt-ε >, for all ε >0. Now x = $ά_1x_1$ + $ά_2$ x_2+ ………$ά_nx_n$, $ά_i$ ε N, x_i belongs to t-ε. X_i ε $\mu_{t-ε}$ implies μ (x_i,q) ≤ t-ε, that is θ(x_i,q) ≤ t-ε for all ε > 0. Therefore

θ (x,q) ≤ S { θ(x_1,q) , θ (x_2,q) … θ (x_n,q)}

≤ t-ε for ε > 0

Hence θ (x,q) = t which is a contradiction to our supposition.

Proposition3.7: Let 'μ' be a anti Q- fuzzy M-N subgroup of a near ring R and let μ* be a Q- fuzzy set in N defined by μ*(x,q) = μ(x,q) +1- μ(0,q) for all x, ε N. Then μ* is a normal anti Q- fuzzy M-N subgroup of R containing μ.

Proof : For any x, y ε R and q ε Q we have

μ*(m(x-y),q) = μ(m(x-y),q) +1 – μ(0,q) ≤ S(μ(mx,q)+1- μ(0,q), (μ(my,q)+1- μ(0,q))

= T (μ*(mx,q) , μ*(my,q)).

μ*(xn,q) = μ(xn,q) +1 – μ(0,q)

≤ μ(x,q) +1- μ(0,q)

= μ(x,q)

Proposition 3.8: Let μ be anti Q- fuzzy left M-N subgroup of near ring R. Let μ^+ be a fuzzy α–cut set in R defined by $\mu^+(x,q) = \mu(x,q) + 1 - \mu(0,q)$ for x ε R , q ε Q. Then μ^+ is α-cut normal anti Q- fuzzy left M-N subgroup of R which contains μ.

Proof: For any x,y ε R, we have $\mu^+(x,q) + 1 - \mu(0,q)$ and $\mu^+(x,q) \le \alpha$ for all x εR, m ε M.

$\mu^+(m(x-y), q) = \mu(m(x-y), q) + 1 - \mu(0,q)$

$\le \max\{ \mu(m_x,q), \mu(m_y,q)\} + 1 - \mu(0,q)$

$= \max\{\mu(m_x,q)+1-\mu(0,q), \mu(m_y,q)+1-\mu(0,q)\}$

$= \max\{ \mu^+(m_x,q), \mu^+(m_y,q)\}$

$\le \max\{ \alpha, \alpha\} \le \alpha$

$\mu^+(xn,q) = \mu(xn,q) + 1 - \mu(0,q)$

$\le \mu(x,q) + 1 - \mu(0,q)$

$= \mu^+(x,q)$

$\le \alpha$

Therefore, μ^+ is a α-cut normal Anti Q-fuzzy left M-N subgroup of R.

Definition 3.9: Let u and v be Q-fuzzy subsets in R. Then the S-product of u and v written as $[u,v]_S(x,q) = S(u(x,q),v(x,q))$ for all x ε R, q ε Q.

Proposition 3.10 : If u and v be Anti Q-fuzzy left M-N subgroups of R, then the S-product of Anti Q-fuzzy left M-N subgroups of R is Anti Q-fuzzy left M-N subgroups of R.

Proof: For any x,y ε R, q ε Q

$[u,v]_S(m(x-y),q)$

$=S\{u(m(x-y),q),v(m(x-y),q)\}$

$\le S\{\max\{u(m_x,q),u(m_y,q)\},\max\{v(m_x,q), v(m_y,q)\}\}$

$\le \max\{S\{u(m_x,q),v(m_x,q)\},S\{v(m_y,q), v(m_y,q)\}\}$

$\le \max\{[u,v]_S(m_x,q),[u,v]_S(m_y,q)\}$

$[u,v]_S(x_n,q) = S\{u(x_n,q), v(x_n,q)\}$

$\le S\{u(x,q), v(x,q)\}$

$\le [u,v]_S(x,q)$

Hence S-product of Anti Q-fuzzy left M-N subgroups of R is Anti Q-fuzzy left M-N subgroups of R.

Definition3.11: Anti Q-fuzzy left M-N subgroup near ring R is said to Anti Q-fuzzy characteristic, if $A^f(x,q) = A(x,q)$ for all x εR, qεQ.

Proposition 3.12 : Let f : R→ R' be an epimorphism of 'A' is anti Q-fuzzy left M-N subgroups of R the A^f is anti Q-fuzzy left M-N subgroups of R'.

Proof: Let x,y εR and qεQ

$A^f(m(x-y),q) = A\,f(m(x-y),q)$

$= A\,(f(mx) - f(my), q)$

$\le \max\{ A\,(f(mx),q), A\,(f(my),q)\}$

$\le \max\{ A^f(mx,q), A^f(my,q)\}$

$A^f(xn,q) = Af(xn,q)$

$\le Af(x,q)$

$\le A^f(x,q)$

Therefore, A^f is anti Q-fuzzy left M-N subgroup of R'.

Proposition 3.13 : Let f : R→ R' be epimorphism. If A^f is anti Q-fuzzy left M-N subgroup of R', then A is anti Q-fuzzy left M-N subgroup of R.

Proof: Let x,y ε R, q ε Q, then there exists a,b ε X such that f(a,q) = x and f(b,q) = y.

It follows that $A(x,q) = A\,f(a,q) = A^f(a,q)$

$A(m(x-y),q) = A\,f(a,q) = A^f(a,q)$

$\le \max\{A^f(a,q), A^f(b,q)\}$

$= \max \{A(a,q), A(b,q)\}$

$\leq \max \{A(x,q), A(y,q)\}$

$A(xn,q) = A\ f(a,q) = A^f(a,q)$

$\leq A\ f(a,q) \leq A\ f(x,q)$

There fore A is anti Q-fuzzy left M-N subgroup of R.

4. CONCLUSION

Osman kazanci , Sultanyamark and Serifeyilmaz introduced the intutionistic Q- fuzzy R-subgroups of near rings. A.Solairaju and R.Nagarajan investigate the notion of Q- fuzzy left R- subgroup of near rings with respect to T-norms.In this paper we investigate the notion of anti Q-fuzzy left M-N subgroup of near ring with respect to s-norm and characterization of them.

5. REFERENCES

1. S. Abou-Zoid , " On fuzzy sub near rings and ideals" , Fuzzy sets. Syst. 44 (1991), 139-146.

2. Y.U. Cho, Y.B.Jun, " On intuitionistic fuzzy R-subgroup of near rings" , J. Appl. Math. and Computing , 18 (1-2) (2005), 665-677.

3. K.H.Kim , Y.B. Jun, "On fuzzy R- subgroups of near rings, J. fuzzy math 8 (3) (2000), 549-558.

4. K.H.Kim, Y.B.Jun, " Normal fuzzy R- subgroups of near rings", J. fuzzy sets. Syst.121(2001), 341-345.

5. Osman Kazanci, Sultan Yamark and Serife Yimaz " On intuitionistic Q- fuzzy R- subgroups of near rings, International mathematical forum, 2, 2007, 59, (2899-2910).

6. A. Solairaju and R. Nagarajan, "Q-fuzzy left R-subgroups of near rings with respect to T-norm", Antarctica Journal of Mathematics, 5(2)(2008), 59-63.

7. A. Solairaju and R. Nagarajan, "A New structure and construction of Q-fuzzy groups", Advanced Fuzzy Mathematics, 4(1)(2009), 23-29.

8. L.A.Zadeh, Fuzzy set, inform.control 8 (1965) 338-353.