A Moderate Algorithm for Generalized Radial basis Function Neural Networks

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ABSTRACT

A Radial Basis Function (RBF) neural network can be regarded as a feed forward network composed of multiple layers of neurons with entirely different roles. The input layer made of sensory units that connect the network to its environment. A radial basis function neural network depends mainly upon an adequate choice of the number and positions of its basis function centers. In case of generalized RBF neural network, the output layer is linear and supplying each layer response as the linear combination of the hidden responses. In this paper we have proposed a moderate algorithm for most generalized form of RBF neural network and the results may be reduced for various forms of RBF and other artificial neural networks as particular cases.

Key Words – Radial Basis Function, Neural Networks.

1. INTRODUCTION

The radial basis function neural networks are powerful function approximators for multivariate nonlinear continuous mappings. They have a simple architecture and the learning algorithm corresponds to the solution of a linear regression problem. The RBF network behavior strongly depends upon the number and the position of the basis functions at the hidden layer. Traditional method of determining the centers are randomly choose input vectors from the training data set or to obtain vectors from unsupervised clustering algorithms applied to input data or vectors obtain through a supervised learning scheme. In this paper we have proposed a moderate algorithm for most generalized RBF neural network and the results may be reduced for various forms of RBF and other artificial neural networks as particular cases.

Let $x_{11},..., x_{1r_1}$ is a set of inputs and $c_{11},..., c_{1r_1}$ be the set of centers , then

$$\| x_{1k} - c_{1k} \| \quad ; \quad k = 1,2, \ldots, r_1 \ .$$

be the radial vectors in Euclidean sense. If the corresponding weights are

$$w_{1k} \quad ; \quad k = 1,2, \ldots, r_1$$

the radial strength of x_{1k} will be

$$w_{1k} \ \| x_{1k} - c_{1k} \| \quad ; \quad k = 1,2, \ldots, r_1$$

Then the combined probabilistic strength of type x_{1k} will be

$$\prod_{k=1}^{r_1} \frac{\{ w_{1k} \| x_{1k} - c_{1k} \| \}^{m_{1k}}}{m_{1k} !}$$

for m_{1k} external input lines and n units of RBF neural network.

If we choose an activation function

$$f_1(\sum_{k=1}^{r_1} m_{1k})$$

the system output for first stage of generalized RBF neural network with the weight vector

$$W = \{ w_{11}, w_{12}, \ldots, w_{1r_1} \}$$

may be taken as $(0 \le w_{1i} \le 1)$

$$f_1 (\sum_{k=1}^{r_1} m_{1k}) \ \prod_{k=1}^{r_1} \frac{\{ w_{1k} \| x_{1k} - c_{1k} \| \}^{m_{1k}}}{m_{1k} !}$$

Similarly for other stages of generalized RBF neural network, we have the outputs respectively as

$$f_2 (\sum_{k=1}^{r_2} m_{2k}) \ \prod_{k=1}^{r_2} \frac{(w_{2k} \| x_{2k} - c_{2k} \|)^{m_{2k}}}{m_{2k} !} \ \ldots$$

$$\ldots, \ f_n (\sum_{k=1}^{r_s} m_{nk}) \prod_{k=1}^{r_s} \frac{(w_{nk} \| x_{nk} - c_{nk} \|)^{m_{nk}}}{m_{nk} !}$$

Finally, the total output Z of generalized multilayer RBF neural network with an unified activation function

$$f (\sum_{k=1}^{r_1} m_{1k} +...+ \sum_{k=1}^{r_s} m_{nk})$$

may be taken as below (Because in case of indefinite large numbers the probabilistic models are best suited for infinity)

$$Z = \sum_{(m_{1r})=0}^{\infty} \cdots \sum_{(m_{ns})=0}^{\infty}$$

$$f (\sum_{k=1}^{r_1} m_{1k} +...+ \sum_{k=1}^{r_s} m_{nk})$$

$$f_1 (\sum_{k=1}^{r_1} m_{1k}) \quad f_n (\sum_{k=1}^{r_s} m_{nk}) \quad \ldots \qquad X$$

$$X \ \prod_{k=1}^{r_1} \frac{(w_{1k} \| x_{1k} - c_{1k} \|)^{m_{1k}}}{m_{1k} !} \quad \ldots$$

$$\ldots \ \prod_{k=1}^{r_s} \frac{(w_{nk} \| x_{nk} - c_{nk} \|)^{m_{nk}}}{m_{nk} !}$$

$$= \sum_{m_1,m_2,...,m_n=0}^{\infty} f (m_1 + m_2 + ... + m_n)$$

$$\prod_{i=1}^{n} f_i(m_i) \; \frac{(\sum_{j=1}^{r_1} w_{1j} \| x_{1j} - c_{1j} \|)^{m_1}}{m_1!} \; \dots$$

$$\dots \; \frac{(\sum_{j=1}^{r_s} w_{nj} \| x_{nj} - c_{nj} \|)^{m_n}}{m_n!} \qquad \dots\dots(1)$$

by using the Identity [9].

$$\sum_{m_1,m_2,\dots,m_n=0}^{\infty} f(m_1 + m_2 + \dots + m_n) \prod_{i=1}^{n} f_i(m_i)$$

$$\frac{(\sum_{j=1}^{r_1} x_{1j})^{m_1}}{m_1!} \; \dots \; \frac{(\sum_{j=1}^{r_s} x_{nj})^{m_n}}{m_n!}$$

$$= \sum_{(m_{1r})=0}^{\infty} \dots \sum_{(m_{ns})=0}^{\infty}$$

$$f(\sum_{k=1}^{r_1} m_{1k} + \dots + \sum_{k=1}^{r_s} m_{nk})$$

$$f_1(\sum_{k=1}^{r_1} m_{1k}) \; \dots \; f_n(\sum_{k=1}^{r_s} m_{nk}) \quad X$$

$$X \; \prod_{k=1}^{r_1} \frac{(x_{1k})^{m_{1k}}}{m_{1k}!} \; \dots \; \prod_{k=1}^{r_s} \frac{(x_{nk})^{m_{nk}}}{m_{nk}!}$$

In case of symmetrical layers $f(\sum_{i=1}^{n} m_i)$ in R.H.S. of (1) may be taken as constant say, δ. Also a group of neurons are similar in nature so $f_i(m_i)$ may be taken as modular parameters $\lambda_i^{m_i}$, say. So we have

$$Z = \delta \sum_{m_1,m_2,\dots,m_n=0}^{\infty} \left\{ \frac{(\lambda_1 \sum_{j=1}^{r_1} w_{1j} \| x_{1j} - c_{1j} \|)^{m_1}}{m_1!} \right.$$

$$\dots \; \left. \frac{(\lambda_n \sum_{j=1}^{r_s} w_{nj} \| x_{nj} - c_{nj} \|)^{m_n}}{m_n!} \right\}$$

$$= \delta \, Exp\,(\; \lambda_1 \sum_{j=1}^{r_1} w_{1j} \| x_{1j} - c_{1j} \| + \; \dots$$

$$\dots + \; \lambda_n \sum_{j=1}^{r_s} w_{nj} \| x_{nj} - c_{nj} \|)$$

$$\propto Exp\,(\; \lambda_1 \sum_{j=1}^{r_1} w_{1j} \| x_{1j} - c_{1j} \| + \; \dots$$

$$\dots + \; \lambda_n \sum_{j=1}^{r_s} w_{nj} \| x_{nj} - c_{nj} \|) \qquad \dots (2)$$

That is in generalized RBF neural network, the out put is directly proportional to exponentially weighted sum of inputs and middle stages outcomes taking radially along with constraints λ_1 , … , λ_n ; which may be suitably defined as per network configuration.

Particularly when we shift centers to origin , we have

$$Z \propto Exp\,(\; \lambda_1 \sum_{j=1}^{r_1} w_{1j} \, x_{1j} \; + \; \dots$$

$$\dots + \; \lambda_n \sum_{j=1}^{r_s} w_{nj} \, x_{nj})$$

Or

$$Z \propto Exp^{(\lambda_1 W_1 X_1 + \dots + \lambda_n W_n X_n)}$$

Where $\quad W_i X_i = \sum_{j=1}^{r_i} w_{ij} x_{ij}$

Also if the network configures with $\lambda_i = 1$; i = 1,2,…,n : then
We have

$$Z \propto Exp^{(W_1 X_1 + \dots + W_n X_n)}$$

Which resembles with generalized artificial neural network , and is the generalization of the author's result [10]

$$Z \propto Exp\,(WX)$$

CONCLUSION

In this paper we have proposed a moderate algorithm for most generalized RBF neural network, further work on application is being carried out and some fruitful results are expected in near future.

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