Anti-Homomorphism in Fuzzy Sub Groups

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ABSTRACT

In this paper, a new concept of anti-homomorphism between two fuzzy groups G and G^I is defined many results analogous to homomorphism of groups are established

KEYWORDS

Fuzzy set, fuzzy level subset, fuzzy groups, fuzzy level subgroup, fuzzy normal subgroup, fuzzy abelian subgroup, fuzzy cyclic subgroup homomorphism, Anti-homomorphism, Anti–automorphism.

SUBJECT CLASSIFICATION:- 20 N 25

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh.L.A [10], several researchers explored on the generalization of the notion of fuzzy set. Choudhury.F.P, Chakraborty .A. B and Khare.S.S.[3] defined fuzzy subgroups and fuzzy homomorphisms, Dobritsa.V.P and Yakh Yaeva.G.E discussed homomorphism between two fuzzy groups [4]. Chandrasekhara Rao.K and Gopalakrishnamoorthy.G[1] defined the anti homomorphisms in groups and obtained some results .We define the concept of anti homomorphisms in fuzzy subgroups and normal fuzzy subgroups and establish some results in this Paper.

2. PRELIMINARIES :

2.1 Definition :

Let X be a non–empty Universal set. A fuzzy subset A of X is a function $A : X \to [0,1]$.

2.2 Definition:

Let G be a group. A fuzzy subset A of G is said to be a fuzzy subgroup of G if it is satisfying the following axioms:

(i) $A(x\,y) \geq \min \{ A(x), A(y) \}$,

(ii) $A(x^{-1}) \geq A(x)$ for all $x, y \in G$.

2. 3 Definition :

Let A be a fuzzy subset of a set X. For $t \in [0, 1]$, the level subset of A is the set, $A_t = \{ x \in X : \mu_A(x) \geq t \}$. This is called a fuzzy level subset of A.

2.4 Definition :

Let A be a fuzzy subgroup of a group G. The subgroup A_t of G, for $t \in [0,1]$ such that $t \leq \mu_A(e)$ is called a level subgroup of A.

2.5 Definition :

A fuzzy subgroup A of a group G is called fuzzy normal if $A(x\,y)=A(y\,x)$

2.6 Definition :

If (G, **.**) and (G^I, **.**) are any two groups, then the function $f : G \to G^I$ is called a group homomorphism if $f (x\, y) = f (x) f (y)$, for all x and $y \in G$.

2.7 Definition :

If (G, **.**) and (G^I, **.**) are any two groups, then the function $f : G \to G^I$ is called a group anti-homomorphism if

$f (x\, y) = f (y) f (x)$, for all x and $y \in G$.

2.8 Definition :

Let X and X^1 be any two sets. Let $f : X \to X^1$ be any function and let A be a fuzzy subset in X, V be a fuzzy subset in $f(X)= X^1$, defined by $\mu_V(y) = \sup_{x \in f^{-1}(y)} \mu_A(x)$, for all $x \in X$ and $y \in X^1$. A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

*2.9 Definition***:**

Let $f : G \to G$ be an anti automorphism if

$f (x\, y) = f (y) f (x)$, for all $x ,y \in G$

2.10 Definition:

Let μ is a fuzzy characteristic subgroup of a group G if μ(f(x)) = μ(x)

3.MAIN RESULTS

3.1 Theorem :

Let f : G → G^I be an anti-homomorphism, If μ^I is a fuzzy sub group of G^I.

Then $f^{-1}(\mu^I)$ is an fuzzy subgroup of G.

Proof.

Let x, y ∈G.

Then

$f^{-1}(\mu^I)(x\,y) = \mu^I(f(x\,y))$

$= \mu^I(f(y)\,f(x))$

$\geq \min[(\mu^I f(y)), (\mu^I(f(x))]$

$= \min[(f^{-1}\mu^I)(y), (f^{-1}\mu^I)(x)]$ …………(1)

and

$(f^{-1}(\mu^I))(x^{-1}) = \mu^I(f(x^{-1}))$

$= \mu^I(f^{-1}(x))$

$= f^{-1}(\mu^I)(x)$ …………(2)

form (1) and (2), the result follows.

Hence $f^{-1}(\mu^I)$ is an anti subgroup of G.

3.2 Theorem :

Let f : G → G^I be an anti-homomorphism. If μ is a fuzzy group G , Then $f^{-1}(\mu)$ a fuzzy normal subgroup of G^I .

Proof.

For all x, y∈G ,We have

$f^{-1}(\mu)(x\,y) = \mu(f(x\,y)) = \mu[f(y)\,f(x)]$

$= \mu(f(y\,x))$

$= f^{-1}(\mu)(y\,x)$

Which implies that

$f^{-1}(\mu)$ is a fuzzy normal subgroup of G^I

3.3 Theorem :

Let f : G → G^I be a map, Let μ be a fuzzy subgroup of G. Let f be a surjective anti homomorphism, Then f(μ) is a normal subgroup of G^I

Proof.

Each level subgroup of μ is a subgroup in G.

But f is a surjective anti homomorphism.

Hence $(f(\mu))_t$, t ∈Im (μ), is anti homomorphic image of some level subgroup of μ .

It follows that $(f(\mu))_t$ is a normal subgroup of G.

A second application of theorem2.3.2 of Rajesh kumar "Fuzzy Algebra" gives that

f (μ) is a fuzzy normal subgroup of G'.

3.4 Theorem :

A fuzzy characteristic sub group of a fuzzy group is a normal fuzzy sub group.

Proof.

Let f be an anti automorphism of G

Which implies

f (y x) = f (y)(x) for all x, y∈G

Which implies

μ (x y) = μ (f (y x)) because μ is a characteristic fuzzy sub group G

Which implies

μ (x y) = μ (f (y) f(x)) because f is a anti automorphism of G

= μ (f (y x)

= μ (y x) since μ is a characteristic fuzzy sub group G

Hence μ is a normal fuzzy sub group of G

3.5 Definition :

Let μ be a fuzzy subgroup of a group G.

Let H = { x∈G/ μ(x)=μ(e) }. Then μ is a fuzzy abelian subgroup of G.

3.6 Theorem:

Anti-homomorphism pre image of a fuzzy abelian subgroup is a fuzzy abelian subgroup.

Proof:

let μ is a fuzzy subgroup of G

To prove μ is a fuzzy abelian subgroup of G

Let γ be a fuzzy abelian subgroup of G^I

Since μ be a fuzzy subgroup of G and γ is a fuzzy abelian subgroup of G^I

Then U= { y∈ G^I / γ (y) = γ (e^I)} is an fuzzy abelian group of G^I ,

where e^I is the identity of G^I .

Consider the set

H = {x∈G/ μ (x) = μ (e)} where e is the identity of G

Let x y∈ H ⊆G.

Then μ(x y)=μ(e)

γ(f(x y))=γ(f(e))

$\gamma(f(x\ y))=\gamma(e^I)$

$\gamma(f(y)f(x))=\gamma(e^I)$

$f(y)f(x) \in U$ and U is abelian,

then $(f(y)f(x))= (f(x)f(y))$

i.e., $\gamma(f(y)f(x))= \gamma(f(x)f(y))$

$\gamma(f(x\ y))=\gamma(f(y\ x))$ since f is anti-homomorphism.

$\mu(x\ y)=\mu(y\ x)$

$\mu(e)=\mu(y\ x)$

i.e., $\mu(y\ x)=\mu(e) \Rightarrow y\ x \in H$

For all $x\ y \in H$ implies $y\ x \in H$ and $x\ y = y\ x$.

H is an abelian group.

There fore, μ is a fuzzy abelian subgroup of G.

3.7 Theorem :

Anti homomorphism image of a fuzzy abelian subgroup is a fuzzy abelian subgroup.

Proof:

Let γ is a fuzzy subgroup of G^I

To prove γ is a fuzzy abelian subgroup of G^I .

Let f be an anti homomorphism from G to G^I .

Since μ be an fuzzy abelian subgroup of G

Then $H = \{x\in G/ \mu(x)=\mu(e)\}$ is an abelian subgroup of G^I where e is the identity element of G .

Let γ be the fuzzy subgroup of G^I .

Let $U = \{ y\in G^I / \gamma(y) = \gamma(e^I)\}$ where e' is the identity element G^I .

Let $x\ y \in U \subseteq G^I$

$\gamma(x\ y) = \gamma(e^I)$

$$\sup_{t\in f^{-1}(x\ y)} \mu(t) = \sup_{t\in f^{-1}(e^I)} \mu(t)$$

$\mu(x\ y) = \mu(e)$

Then $x\ y \in H$ and H is abelian

$x\ y = y\ x$

$\mu(x\ y) = \mu(y\ x)$

$$\sup_{t\in f^{-1}(x\ y)} \mu(t) = \sup_{t\in f^{-1}(y\ x)} \mu(t)$$

$\gamma(x\ y) = \gamma(y\ x)$

$\gamma(e') = \gamma(y\ x)$

ie, $\gamma(x\ y) = \gamma(e^I) \Rightarrow y\ x \in U$

For all $x\ y \in U$ implies $y\ x\in U$ and $x\ y=y\ x$.

Then U is abelian subgroup of G^I

Therefore γ is a fuzzy abelian subgroup of G^I .

3.8 Definition:

Let μ be a fuzzy subgroup of G then μ is called a cyclic fuzzy subgroup if μ_t is a cyclic subgroup for all $t\in[0,1]$, where $\mu_t = \{ x\in G / \mu(x)\geq t \}$.

3.9 Theorem:

Anti homomorphism pre-image of cyclic fuzzy subgroup is a cyclic fuzzy subgroup

Proof:

Let μ is a fuzzy subgroup of G

To prove μ is a cyclic fuzzy subgroup of G

Let f be an anti homomorphism from G into G^I

Let γ be a cyclic fuzzy subgroup of G^I and Since μ be a fuzzy subgroup of G .

Now ,for any $t\in[0,1]$,

$\mu_t = \{ x\in G / \mu(x)\geq t \}$.Since γ be a cyclic fuzzy subgroup of G^I ,

then μ_λ is a cyclic subgroup of G^I .

Clearly $f^{-1}(\mu_\lambda) \subseteq \mu_t$

Then μ_t is cyclic and hence μ is a cyclic fuzzy subgroup of G .

3.10 Theorem:

Anti homomorphism image of cyclic fuzzy subgroup is a cyclic fuzzy subgroup

Proof :

Let γ is a fuzzy subgroup of G^I

To prove γ is a cyclic fuzzy subgroup of G^I

Let f be an anti homomorphism from G into G^I

Let μ be a cyclic fuzzy subgroup of G and γ be a fuzzy subgroup of G^I

Clearly μ_t is a cyclic subgroup of G for $t\in[0,1]$,

and $f(\mu_t) \subseteq \mu_\lambda$ for $\lambda\in[0,1]$,

Then γ_λ is a cyclic

Hence γ is a cyclic fuzzy subgroup of G^I .

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