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Decision of Prognostics and Health Management under Uncertainty

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ABSTRACT
The decision making of Prognostics and Health management under uncertainty is addressed in this paper. Dempster-Shafer theory is adopted to tackle this problem and some modification about this method is made to accommodate with practice. The decision-making method and decision process are detailed.

Keywords
Prognostics and Health management, decision making, Dempster-Shafer theory

1. INTRODUCTION
The management of uncertainty is an important and often overlooked aspect in the estimation of remaining component life [1]. The extrapolation of trends based on recent observations is a common method for calculating remaining life. This calculation alone, however, does not provide sufficient information to form a decision or corrective action. Without corresponding measures of the uncertainty associated with the calculation, remaining life projections may have little practical value.In all but the simplest components, remaining life also depends on the destructive forces encountered during use (loads, temperature, humidity, corrosive exposure, shock etc.)[2].These are often governed by unpredictable events and circumstances that cannot be known a priori.

Major drawbacks of classical statistical estimation techniques (Bayesian theory) are that they require prior estimates and Competing hypotheses must be mutually exclusive. However it is usually difficult to define prior likelihoods. The interest in Dempster-Shafer theory stems from the richness of its uncertainty representation scheme. The Bayesian approach and Dempster-Shafer theory share fundamental ideas and produce identical results when uncertainties are not extreme. In fact, Bayesian analysis arises as a special case within the more generic Dempster-Shafer theory [3]. Disagreement between these two theories occurs when quantifying weak evidence and its associated uncertainties since in such situations the Dempster-Shafer theory offers greater flexibility than the Bayesian approach.

Probability theory lacks the ability to handle such situations when there is little information on which to evaluate a probability or when that information is nonspecific, ambiguous, or conflicting. Where it is not possible to characterize uncertainty with a precise measure such as a precise probability, it is reasonable to consider a measure of probability as an interval or a set.

Dempster-Shafer theory (DST) is a powerful theoretical tool which can be applied for the representation of incomplete knowledge, belief updating, and for combination of evidence through the Demspter-Shafer’s rule of combination. The Dempster-Shafer model of representation and processing of uncertainty has led to a huge number of practical applications in a wide range of domains.

The Dempster-Shafer theory is based on two ideas: the idea of obtaining degrees of belief for one question from subjective probabilities for a related question, and Dempster's rule for combining such degrees of belief when they are based on independent items of evidence.

Implementing the Dempster-Shafer theory in a specific problem generally involves solving two related problems. First, we must sort the uncertainties in the problem into a priori independent items of evidence. Second, we must carry out Dempster's rule computationally. These two problems and and their solutions are closely related. Sorting the uncertainties into independent items leads to a structure involving items of evidence that bear on different but related questions, and this structure can be used to make computations feasible.

Utilizing Dempster-Shafer theory to address the uncertainty problems, especially the decision level fusion of multiple resource data deserves to be researched. And it may involve tackling some practical problems that may be encountered in application.

Uncertainty Analysis

Prognosis is certainly the Achilles' heel of the Prognostics and Health Management (PHM) system and presents major challenges to the CBM/PHM system designer primarily because it entails large-grain uncertainty [4]. Uncertainty management of prognostics holds the key for a successful penetration of prognostics as a key enabler to health management in industrial applications [5]. Long-term prediction of a fault evolution to the point that may result in a failure requires means to represent and manage the inherent uncertainty. Bhaskar Saha et al [6] summarized soureces of uncertainty as multiple sources of error like modeling inconsistencies, system noise and degraded sensor fidelity. Irrespective of whether the diagnostic/prognostic algorithms are model-driven or datadriven it is not feasible to eliminate all of the above error factors.

One of the major challenges to the designers of modern PHM systems is the need to develop diagnostic and prognostic methods that are truly capable of handling real world uncertainties – as the real world is not deterministic. Such real world uncertainties cause havoc with deterministic approaches leading to high false alarm rates, inaccurate predictions, incorrect decisions and an overall PHM system that is not very robust. Some of the issues uncertainty presents to the designer are elaborated on below, including issues associated with various steps in the predictive process, the estimate of current condition, the prediction of time to failure (or time remaining) and the choice of appropriate lead times (how far ahead to predict); and on the choice of an overall prognostic methodology [7].

2. DECISION MAKING BASED ON DS THEORY [8-11]

2.1 DS theory

In a finite discrete space, Dempster-Shafer theory can be interpreted as a generalization of probability theory where probabilities are assigned to sets as opposed to mutually exclusive singletons. In traditional probability theory, evidence is associated with only one possible event. In DST, evidence can be associated with multiple possible events, e.g., sets of events. As a result, evidence in DST can be meaningful at a higher level of abstraction without having to resort to assumptions about the events within the evidential set. Where the evidence is sufficient enough to permit the assignment of probabilities to single events, the Dempster-Shafer model collapses to the traditional probabilistic formulation. One of the most important features of Dempster-Shafer theory is that the model is designed to cope with varying levels of precision regarding the information and no further assumptions are needed to represent the information. It also allows for the direct representation of uncertainty of system responses where an imprecise input can be characterized by a set or an interval and the resulting output is a set or an interval.

Let X be the universal set: the set of all states under consideration. The power set

$$2^X$$

is the set of all subsets of X, including the empty set $\varnothing$. For example, if:

$$X = \{a, b\}$$

then

$$2^X = \{\varnothing, \{a\}, \{b\}, X\}.$$

The elements of the power set can be taken to represent propositions that one might be interested in, by containing all and only the states in which this proposition is true.

The theory of evidence assigns a belief mass to each element of the power set. Formally, a function

$$m : 2^X \rightarrow [0, 1]$$

is called a basic belief assignment (BBA), when it has two properties. First, the mass of the empty set is zero:

$$m(\varnothing) = 0.$$

Second, the masses of the remaining members of the power set add up to a total of 1:

$$\sum_{A \in 2^X} m(A) = 1$$

The mass m(A) of a given member of the power set, A, expresses the proportion of all relevant and available evidence that supports the claim that the actual state belongs to A but to no particular subset of A. The value of m(A) pertains only to the set A and makes no additional claims about any subsets of A, each of which have, by definition, their own mass.

From the mass assignments, the upper and lower bounds of a probability interval can be defined. This interval contains the precise probability of a set of interest (in the classical sense), and is bounded by two non-additive continuous measures called belief (or support) and plausibility:

$$\mathrm{bel}(A) \le P(A) \le \mathrm{pl}(A).$$

The belief bel(A) for a set A is defined as the sum of all the masses of subsets of the set of interest:

$$\mathrm{bel}(A) = \sum_{B \mid B \subseteq A} m(B).$$

The plausibility pl(A) is the sum of all the masses of the sets B that intersect the set of interest A:

$$\mathrm{pl}(A) = \sum_{B \mid B \cap A \neq \varnothing} m(B).$$

The two measures are related to each other as follows:

$$\mathrm{pl}(A) = 1 - \mathrm{bel}(\overline{A}).$$

And conversely, for finite A, given the belief measure bel(B) for all subsets B of A, we can find the masses m(A) with the following inverse function:

$$m(A) = \sum_{B \mid B \subseteq A} (-1)^{|A-B|} \mathrm{bel}(B)$$

where |A − B| is the difference of the cardinalities of the two sets.

It follows from the last two equations that, for a finite set X, you need know only one of the three (mass, belief, or plausibility) to deduce the other two; though you may need to know the values for many sets in order to calculate one of the other values for a particular set. In the case of an infinite X, there can be well-defined belief and plausibility functions but no well-defined mass function.

2.2 Combination rule

Dempster's rule combines multiple belief functions through their basic probability assignments (m). These belief functions are defined on the same frame of discernment, but are based on independent arguments or bodies of evidence. The combination rule results in a belief function based on conjunctive pooled evidence. The Dempster rule of evidence theory provides the combination method of two bpa's m1 and m2 in the following manner:

$$M(C) = \begin{cases} \dfrac{\sum_{\substack{i,j \\ A_i \cap B_j = C}} M_1(A_i) M_2(B_j)}{1-K} & \forall C \subset \Omega, C \neq \phi \\ 0 & C = \phi \end{cases}$$

Where $K = \sum_{\substack{i,j \\ A_i \cap B_j = \phi}} M_1(A_i) M_2(B_j) < 1$.

When there are many evidences, they can be combined one by one based on Dempster combination rule.

3. DECISION MAKING OF PHM

Here we assume there is a decisions set D: (D1, D2 …Dn, U) for remaining life measurements to take. And there is a multi-information sources set I: (I1, I2 …Im), each information source have a corresponding belief allocation for decisions set D, thus they form a matrix B: (Bij) between multi-information sources set I and decisions set D.

$$B = \begin{Bmatrix} & D_1 & D_2 & \cdots & D_n & U \\ I_1 & b_{11} & b_{12} & \cdots & b_{1n} & b_{1u} \\ I_2 & b_{21} & b_{22} & \cdots & b_{2n} & b_{2u} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ I_m & b_{m1} & b_{m2} & \cdots & b_{mn} & b_{mu} \end{Bmatrix}$$

Then we adopt the combination rule to integrate the belief allocation from different information sources. Firstly the belief allocation from I1 and I2 are combined.

$$C_{12} = \begin{Bmatrix} & b_{11} & b_{12} & \cdots & b_{1n} & b_{1u} \\ b_{21} & c_{11} & c_{12} & \cdots & c_{1n} & c_{1u} \\ b_{22} & c_{21} & c_{21} & \cdots & c_{2n} & c_{2u} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ b_{2n} & c_{n1} & c_{n2} & \cdots & c_{nn} & c_{nu} \\ b_{2u} & c_{u1} & c_{u2} & \cdots & \cdots & c_{un} \end{Bmatrix}$$

Next step the combination result of I1 and I2 will be integrated with I3 according to the same combination rule. The previous will be continued till the belief allocations from all information sources are integrated.

Actually different sensors or information sources have different test ability, accuracy and reliability, however the above combination rule just take all information source with the same degree of confidence, which is not accordance to the engineering practice. And the above combination rule often encounters the problem with conflict data source and do not have ability to tackle that problem. Here we introduce a confidence factor set E for the information source as follows.

$$E=[e_1 \quad e_2 \quad \cdots \quad e_m]$$

Then we can get the new matrix B' : (Bij) between multi-information sources set I and decisions set D.

$$B' = \begin{Bmatrix} & D_1 & D_2 & \cdots & D_n & U \\ I_1 & e_1 b_{11} & e_1 b_{12} & \cdots & e_1 b_{1n} & e_1 b_{1u} \\ I_2 & e_2 b_{21} & e_2 b_{22} & \cdots & e_2 b_{2n} & e_2 b_{2u} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ I_m & e_m b_{m1} & e_m b_{m2} & \cdots & e_m b_{mn} & e_m b_{mu} \end{Bmatrix}$$

Also we can get the new combination of I1 and I2.

$$C'_{12} = \begin{Bmatrix} & e_1 b_{11} & e_1 b_{12} & \cdots & e_1 b_{1n} & e_1 b_{1u} \\ e_2 b_{21} & c_{11} & c_{12} & \cdots & c_{1n} & c_{1u} \\ e_2 b_{22} & c_{21} & c_{21} & \cdots & c_{2n} & c_{2u} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ e_2 b_{2n} & c_{n1} & c_{n2} & \cdots & c_{nn} & c_{nu} \\ e_2 b_{2u} & c_{u1} & c_{u2} & \cdots & \cdots & c_{un} \end{Bmatrix}$$

4. EXAMPLE AND RESULT

Here a brief example is offered to illustrate the decision making of PHM under uncertainty. Let Decision making frame D={D1, D2, D3, D4}, multi-information sources frame S={S1, S2, S3} and we assume they have equal degree of confidence, and the basic probability assignment space is as the table 1. U is used to denote the uncertainty. The combination of S1 and S2 are as table 2.

Table 1. Basic probability assignment space

	D1	**D2**	**D3**	**D4**	**U**
S1	0.05	0.4	0.05	0.35	0.15
S2	0.1	0.3	0.1	0.4	0.1
S3	0.15	0.2	0.1	0.45	0.1

Table 2. Combinations of S1 and S2

		S1				
		D1(0.05)	**D2(0.4)**	**D3(0.05)**	**D4(0.35)**	**U(0.15)**
S2	**D1(0.1)**	0.005	0.04	0.005	0.035	0.015
	D2(0.3)	0.015	0.12	0.015	0.105	0.045
	D3(0.1)	0.005	0.04	0.005	0.035	0.015
	D4(0.4)	0.02	0.16	0.02	0.14	0.06
	U(0.1)	0.005	0.04	0.005	0.035	0.015

Thus we can get the combination result of S1 and S2

Table 3. Combinations result of S1 and S2

D1	**D2**	**D3**	**D4**	**U**
0.05	0.41	0.05	0.46	0.03

We can see that the bpa of the uncertain falls distinctly from the result. But it is difficult to make the right judgment for the bpa of D2 and D4 is closely. So we continue to combine this result with S3.

Table 4. Combination of S1, S2 and S3

		S3				
		D1(0.15)	**D2(0.2)**	**D3(0.1)**	**D4(0.45)**	**U(0.1)**
S1, 2	**D1(0.05)**	0.0075	0.01	0.005	0.025	0.005
	D2(0.41)	0.0615	0.082	0.041	0.1845	0.041
	D3(0.05)	0.0075	0.01	0.005	0.025	0.005
	D4(0.46)	0.069	0.092	0.046	0.207	0.046
	U(0.03)	0.0045	0.006	0.003	0.0125	0.003

Thus we can get the combination result of S1, S2 and S3.

Table 5. Combinations result of S1, S2 and S3

D1	**D2**	**D3**	**D4**	**U**
0.04	0.30	0.03	0.62	0.01

We adopt the method of basic probability value based decision, and select $\varepsilon_1 = \varepsilon_2 = 0.1$, thus the decision result is D4.

5. CONCLUSIONS

The decision making of Prognostics and Health management under uncertainty can be addressed with Dempster-Shafer theory or modified Dempster-Shafer theory. The probability of achieving correct decision and reducing uncertainty is increased in decision making.

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