Trend Analysis of Generalized Hypergeometric Functions

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ABSTRACT
An innovative attempt to analyze the trends of different generalized hypergeometric functions of the form $_pF_q\ a\ ;\ b\ ;x$ has been made in this research paper by using matlab and the nature of generalized hypergeometric functions like 4F3 (1, 2, 3, 4; 3,4,5; x) and 8F4 (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x) has been demonstrated. Comparison work of different generalized hypergeometric functions has been also undertaken.

Keywords
Generalized hypergeometric function, special function, matlab.

1. INTRODUCTION
The term "trend analysis" refers to the concept of collecting information and attempting to spot a pattern, or *trend*, in the information. In some fields of study, the term "trend analysis" has more formally-defined meaning. Trend analysis often refers to the science of studying changes in social patterns, including fashion, technology and the consumer behavior. Modern society is increasingly reliant on our capability to automatically detect patterns in vast masses of data. This is affecting not only the way we do business and run our industries, but also is changing the very nature of the scientific method. Every science now has an e-version (computational biology, computational chemistry, etc) and in many cases this involves automatisation of both the production and the analysis of experimental data. The use of computer simulations increases our reliance on automatic analysis of data even further this process is accelerating.

The distinct scientific communities that are working on various aspects of automatic analysis of data include Combinatorial Pattern Matching, Data Mining, Computational Statistics, Network Analysis, Text Mining, Image Processing, Syntactical Pattern Recognition, Machine Learning, Statistical Pattern Recognition, Computer Vision, and many others. We ca see the works on generalized Hypergeometric Functions of Dwork B. (1990) and Yoshida Masaaki (1997). Watanabe Y et al(1998) deals with a fast structural matching and its application to pattern analysis of 2-D electrophoresis images. We can see the work of Desmond J. Higham et al (2005) on MATLAB. Another important work of Rumshisky et al (2006) deals with inducing sense-discriminating context patterns from sense-tagged corpora. We can also quote the work of Bevel Tom et al (2008) on bloodstain pattern analysis with an introduction to crime scene reconstruction and of Hui Wang et al (2009) for high-definition metrology based spatial variation pattern analysis for the dynamic pattern analysis framework to the analysis of spatial-temporal crime relationships. The important work on MATLAB of Stormy Attaway (2009).We can see the work on generalised hypergeometric function is Askey, R. A. (2010), Daalhuis, Adri B. Olde (2010),

2. DATABASE 1 OF GENERALIZED HYPERGEOMATRIC FUNCTION 4F3 (1, 2, 3, 4; 3, 4, 5; X)
We have created the databases from generalized hypergeometric function 4F3 (1, 2, 3, 4; 3,4,5; x) and have plotted the graph between real and imaginary part of 4F3 (1, 2, 3, 4; 3,4,5; x) with the help of MATLAB, then interpreted the pattern of generalized hypergeometric function 4F3 (1, 2, 3, 4; 3,4,5; x) in different range of x.

X	$_4F_3$(1,2,3,4; 3,4,5; x)				
2 to 10 (step 2)	1 - 2.3562i	-0.52598 - 1.3254i	-0.59478 - 0.72722i	-0.52153 - 0.45099i	-0.44557 - 0.30536i
2 to 20 (step 2)	-0.38319 - 0.21998i	-0.33366 - 0.16585i	-0.29419 - 0.12943i	-0.26232 - 0.10379i	-0.23622 - 0.085059i
22to 30 (step 2)	-0.21453 - 0.070971i	-0.19628 - 0.060109i	-0.18073 - 0.051561i	-0.16735 - 0.044712i	-0.15573 - 0.039142i
32 to 40 (step 2)	-0.14555 - 0.034551i	-0.13657 - 0.030722i	-0.1286 - 0.027495i	-0.12147 - 0.024751i	-0.11506 - 0.022399i
42 to 50 (step 2)	-0.10927 - 0.020366i	-0.10402 - 0.018598i	-0.099233 - 0.01705i	-0.094855 - 0.015688i	-0.090837 - 0.014482i

Database 1.1

X	4F3(1,2,3,4;3,4,5;x)				
52 to 60 (step 2)	-0.08713 - 0.01341i	-0.08371 - 0.01245i	-0.08054 - 0.01159i	-0.07760 - 0.01082i	-0.07486 - 0.01012i
62to 70 (step 2)	-0.07230 - 0.009493i	-0.06991 - 0.008918i	-0.06766 - 0.008394i	-0.06556 - 0.007914i	-0.06357 - 0.007475i
72to 80 (step 2)	-0.06171 - 0.007071i	-0.05994 - 0.006699i	-0.05827 - 0.006356i	-0.05669 - 0.006038i	-0.0552 - 0.005744i
82 to 90 (step 2)	-0.05377 - 0.005470i	-0.05242 - 0.005216i	-0.05113 - 0.004979i	-0.04991 - 0.004758i	-0.04874 - 0.004551i
92 to 100 (step 2)	-0.04762 - 0.004357i	-0.04655 - 0.004176i	-0.04553 - 0.004005i	-0.04455 - 0.003845i	-0.04361 - 0.003694i

Database 1.2

3. MATLAB PROGRAM FOR PLOTTING GENERALIZED HYPERGEOMETRIC FUNCTION, 4F3 (1, 2, 3, 4; 3, 4, 5; X)

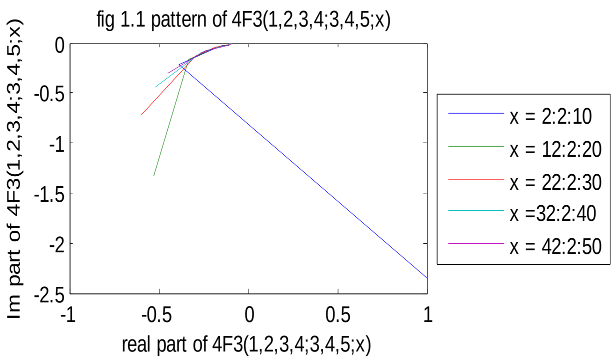
MATLAB program for fig 1.1:

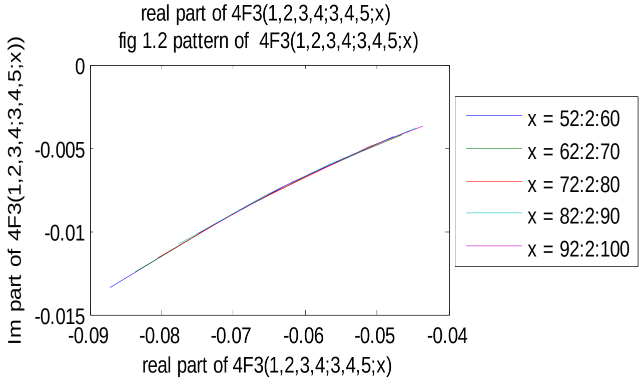
```
>> x = [2:2:10; 12:2:20; 22:2:30; 32:2:40; 42:2:50];
>> y = [hypergeom([1 2 3 4],[3 4 5],x)];
>> plot(y);
```

MATLAB program for fig 1.2:

```
>> x = [52:2:60; 62:2:70; 72:2:80; 82:2:90; 92:2:100];
>> y = [hypergeom([1 2 3 4],[3 4 5],x)];
>> plot(y);
```

Pattern of Generalized Hypergeometric Function ${}_4F_3$ (1, 2, 3, 4; 3, 4, 5; x)





4. INTERPRETATION

The databases 1.1 and 1.2 having values in complex numbers, indicate that when we move to higher hypergeometric function like ${}_4F_3$(1,2,3,4;3,4,5;x) the data turns into complex number form. It can be seen from figure 1.1 and figure 1.2 when the range of x increases the data converse quickly.

5. Database 2 of Generalized Hypergeometric Function ${}_8F_4$ (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x)

We have created two databases from generalized hypergeometric function 8F4 (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x) and have plotted the graph between real and imaginary part of 8F4 (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x) with the help of MATLAB which have helped us in interpretation the pattern of generalized hypergeomatric function ${}_8F_4$ (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x) in different ranges of x.

Database 2.1

X	${}_8F_4$(1,2,3,4,5,6,7,8; 3,5,7,9; x)				
2 to 10 (step 2)	-0.0014851 -1.2813e-005i	-0.0014296 -1.1884e-005i	-0.001378 - 1.1053e-005i	-0.00133 - 1.0307e-005i	-0.0012853 -9.6332e-006i
12to 20 (step 2)	-0.0012434 -9.0236e-006i	-0.0012042 -8.4701e-006i	-0.0011674 - 7.966e-006i	-0.0011328 -7.5056e-006i	-0.0011001 - 7.084e-006i
22to 30 (step 2)	-0.0010693 -6.6969e-006i	-0.0010402 -6.3407e-006i	-0.0010126 -6.0122e-006i	-0.00098642 -5.7086e-006i	-0.00096157 -5.4274e-006i
32 to 40 (step 2)	-0.00093794 -5.1665e-006i	-0.00091544 - 4.924e-006i	-0.000894 - 4.6981e-006i	-0.00087353 -4.4874e-006i	-0.00085399 -4.2906e-006i
42 to 50 (step 2)	-0.00083529 -4.1065e-006i	-0.0008174 - 3.934e-006i	-0.00080026 -3.7721e-006i	-0.00078382 - 3.62e-006i	-0.00076804 -3.4769e-006i

X	${}_8F_4$(1,2,3,4,5,6,7,8; 3,5,7,9; x)				
2 to 10 (step 2)	1 - 2.3562i	-0.52598 - 1.3254i	-0.59478 - 0.72722i	-0.52153 - 0.45099i	-0.44557 - 0.30536i
12to 20 (step 2)	-0.38319 - 0.21998i	-0.33366 - 0.16585i	-0.29419 - 0.12943i	-0.26232 - 0.10379i	-0.23622 - 0.085059i
22to 30 (step 2)	-0.21453 - 0.070971i	-0.19628 - 0.060109i	-0.18073 - 0.051561i	-0.16735 - 0.044712i	-0.15573 - 0.039142i
32 to 40 (step 2)	-0.14555 - 0.034551i	-0.13657 - 0.030722i	-0.1286 - 0.027495i	-0.12147 - 0.024751i	-0.11506 - 0.022399i
42 to 50 (step 2)	-0.10927 - 0.020366i	-0.10402 - 0.018598i	-0.099233 - 0.01705i	-0.09485 - 0.015688i	-0.090837 - 0.014482i

Database 2.2

6. MATLAB Program for Plotting Generalized Hypergeometric Function, 8F4 (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x) For Pattern Identification

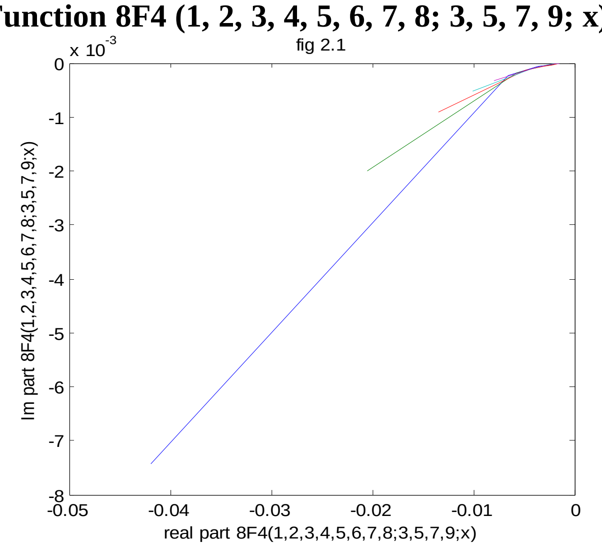
MATLAB program for fig 2.1:

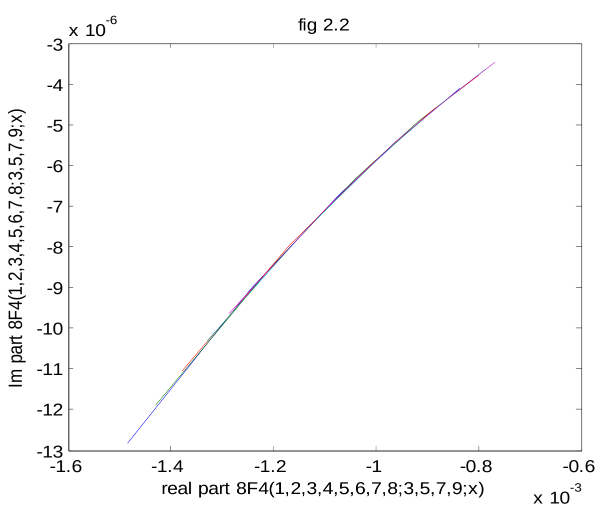
```
>> x = [2:2:10; 12:2:20; 22:2:30; 32:2:40; 42:2:50];
>> y = [hypergeom ([1 2 3 4 5 6 7 8], [3 5 7 9], x)];
>> plot(y);
```

MATLAB program for fig 2.2:

```
>> x = [52:2:60; 62:2:70; 72:2:80; 82:2:90; 92:2:100];
>> y = [hypergeom([1 2 3 4 5 6 7 8 ],[3 5 7 9],x)];
>> plot(y);
```

7. Pattern of Generalized Hypergeomatric Function 8F4 (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x)





8. INTERPRETATION

We observe that all values in databases 2.1 and 2.2 are in complex numbers which replicate that when we move to higher hypergeometric function like 8F4 (1, 2, 3, 4, 5, 6, 7, 8; 3, 5, 7, 9; x) the data turns into complex number form. It is evident from figures 2.1 and 2.2, where with the increase in the range of x, it can be seen that the data converse quickly.

9. Matlab Program for Pattern Identification of Special Function in Different Range

```
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.1 0.1]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.2 0.2]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.3 0.3]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.4 0.4]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.5 0.5]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.6 0.6]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.7 0.7]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.8 0.8]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-0.9 0.9]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[- 1 1]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-1.1 1.1]);
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-1.2 1.2]);
……………………………………………………………………
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-10 10]);
……………………………………………………………………
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-15 15]);
……………………………………………………………………..
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-100 100]);
……………………………………………………………………..
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-1000 1000]);
………………………………………………………………………
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-10000 10000]);
………………………………………………………………………
>> fplot(@(x) x.*hypergeom([1/2 1/2],3/2,x.^2),[-100000 100000]);
```

10. Trend Analysis of the Special Function x_2F_1 (1/2, 1/2; 3/2; x^2) in Different ranges

We have taken the function x2F1 (1/2, 1/2; 3/2; x2) and have plotted it in different ranges of values of x with the help of MATLAB . It has paved the way of analyzing the trend of the function x2F1 (1/2, 1/2; 3/2; x2) in different ranges of values of x.

fig 3.1

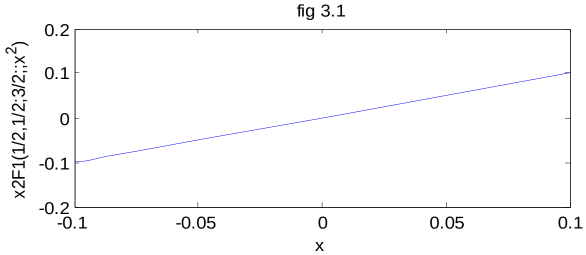


fig 3.2

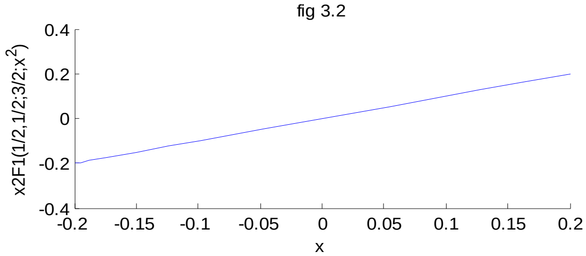


fig 3.3

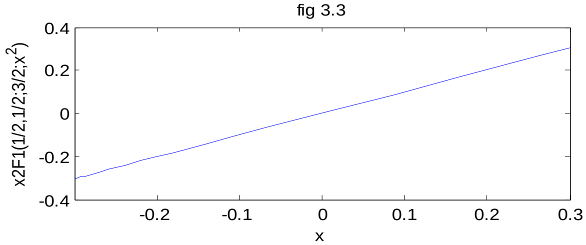


fig 3.4

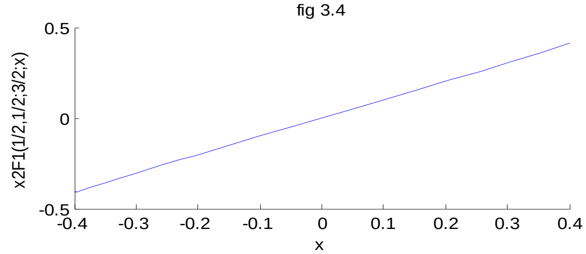


fig 3.5)

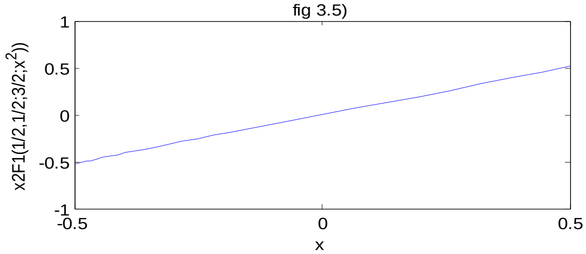


fig 3.6

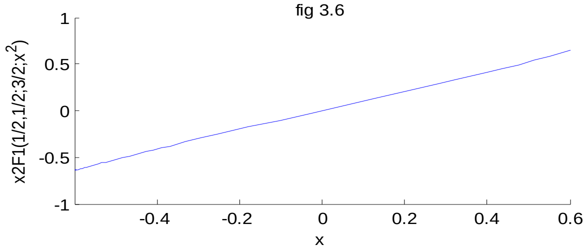


fig 3.7

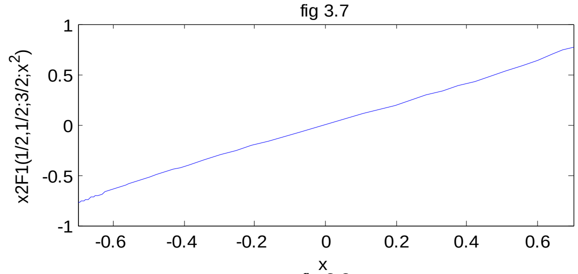


fig 3.8

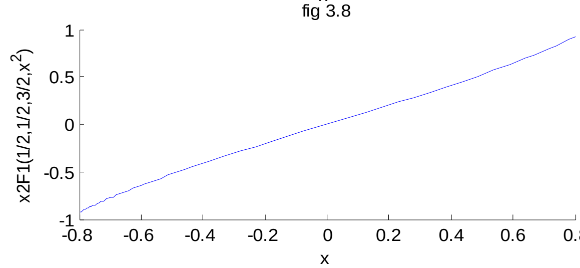


fig 3.9

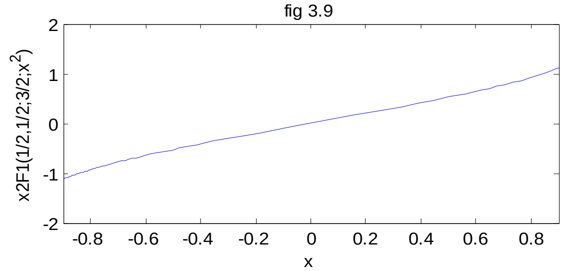


fig 3.10

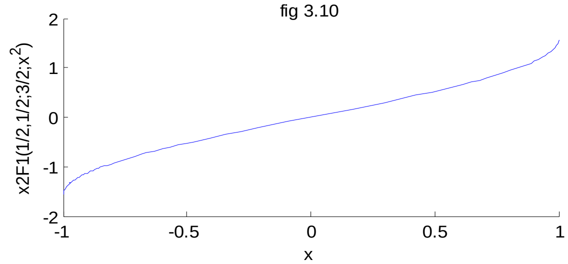


fig 3.11

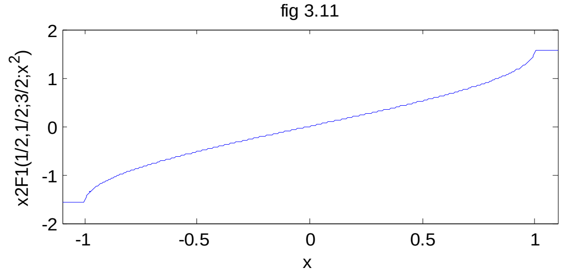


fig 3.12

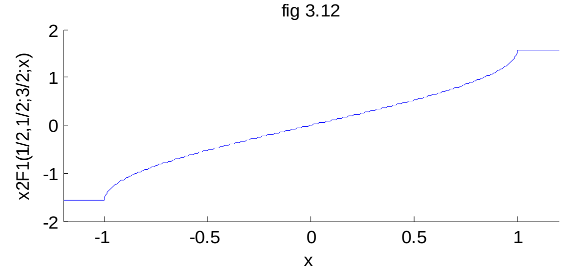


fig 3.13

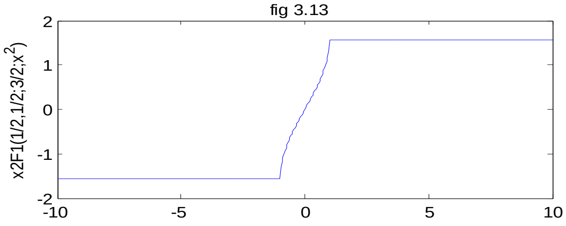


fig 3.14

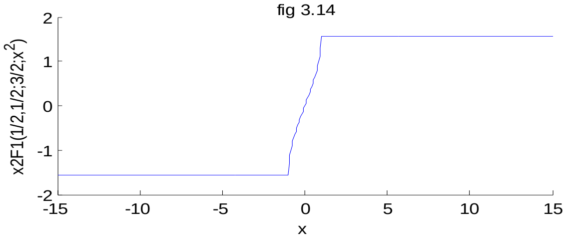


fig 3.15

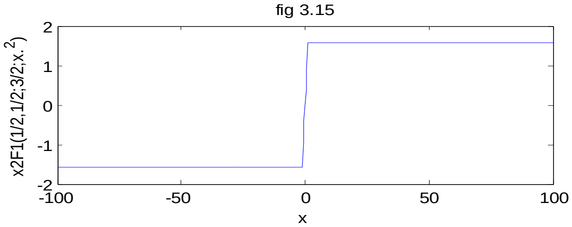


fig 3.16

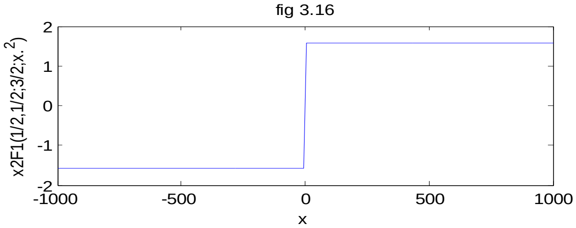


fig 3.17

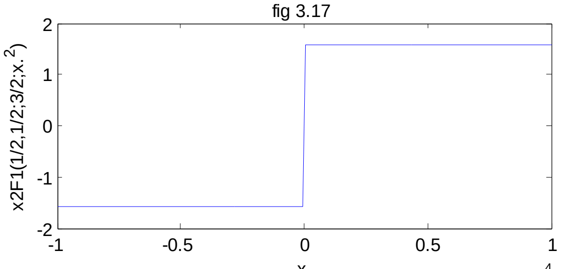
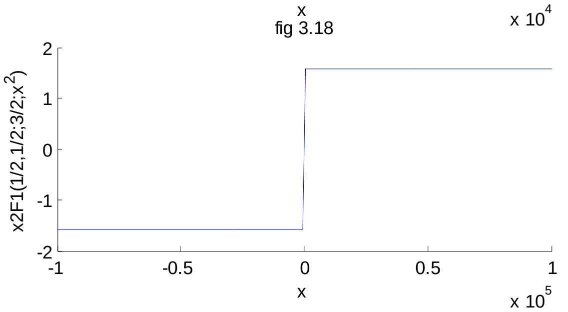


fig 3.18



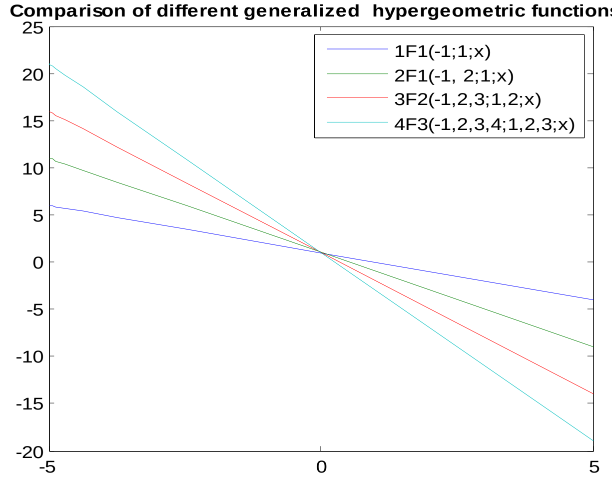
11. Interpretation of Trends x_2F_1 (1/2, 1/2; 3/2; x^2)

When we study the nature of a hypergeometric function x2F1 (1/2, 1/2; 3/2; x2) in different rages of x, as depicted in fig 3.1-3.18,we observe that the graph between x2F1 (1/2, 1/2; 3/2; x2) and x from fig 3.1 to fig 3.6, in the range of x from [-0.1:0.1] to [-0.6:0.6] remains approximately unchanged. While from fig 3.7 to fig 3.10, in the higher range of x from [-0.7:0.7] to [-1:1]),it can be seen that the graph between x2F1 (1/2, 1/2; 3/2; x2) and x shows variation at end points. In the same manner when we increase the range of x from [-1.1:1.1] to [-100:100] from fig 3.8 to fig 3.15 maximum part of the graph between x2F1 (1/2, 1/2; 3/2; x2) and x is parallel to x axis because the hypergeometric function x2F1 (1/2, 1/2; 3/2; x2) contains some complex number data together with real number data. The parallel line to x axis in the graph shows the real part of the complex number data. When we take higher range of x from [-1000:1000] to [-100000:100000], then from fig 3.16 to fig 3.18, it can be seen that the pattern of the graph between x2F1 (1/2, 1/2; 3/2; x2) and x is approximately same. The real part of all complex number data in the database is constant for negative values of x and near approx 0 for positive values of x.

12. Comparison of the Trends of Different Generalized Hypergeometric Functions

Here we have compared the different generalized hypergeometric functions specifically 1F1 (-1; 1; x), 2F1 (-1, 2; 1; x), 3F2 (-1, 2, 3; 1, 2; x) and 4F3 (-1, 2, 3, 4; 1, 2, 3; x). In the following figures, blue line represents the function 1F1 (-1; 1; x), green line represents the function 2F1 (-1, 2; 1; x), red line represents the function 3F2 (-1, 2, 3; 1, 2; x) and indigo line represents the function 4F3 (-1, 2, 3, 4; 1, 2, 3; x). All the above functions follow similar pattern, when we move from the lower hypergeometric function to higher hypergeometric function in the defined range of x, while the values of higher hypergeometric functions increase with the decrease in the value of x and the values of higher hypergeometric function decrease when we increase the value of x.

Fig 4
Comparison of different generalized hypergeometric functions



13. CONCLUSION

The results discussed in the present paper have enabled us to find the trends of different functions in various ranges and have paved the way for comparison of trends. This type of research work is of immense importance in the disciplines like Biological, Medical and Social Sciences where hypergeometric distributions find applications and can open new vistas of interest for research work on statistical decision theory and applications.

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