Fuzzy Sub-Bi HX Group and its Bi Level Sub-Bi HX Groups

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ABSTRACT
The definition of a fuzzy HX group and define a new bialgebraic structure of fuzzy sub-bi HX group of a bi-hx group and some related properties are investigated and redefined in this paper.

Keywords
Bi HXgroup, Fuzzy set , fuzzy subgroup , fuzzy HX group, fuzzy sub-bi HX group of a bi HX group,

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1. INTRODUCTION
The concept of fuzzy sets was initiated by Zadeh. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld gave the idea of fuzzy subgroups. Li Hongxing introduce the concept of HX group and the authors Luo Chengzhong , Mi Honghai , Li Hongxing introduce the concept of fuzzy HX group. The notion of bigroup was first introduced by P.L.Maggu in 1994. W.B. Vasantha Kandasamy and D.Meiyappan introduced concept of fuzzy sub-bigroup of a bigroup and fuzzy sub-bigroup of a group. In this paper we define a new bialgebraic structure of fuzzy sub-bi HX group of a bi HX group and study some of their related properties. we also discuss the image and pre image of a fuzzy sub-bi HX group of a bi HX group under homomorphism and anti-homomorphism.

2. PRELIMINARIES
In this section we site the fundamental definitions that will be used in the sequel. Through out this paper, G = (G , *) is a group, e is the identity element of G, and xy, we mean x * y.

2.1 Definition
In $2^G-\{\phi\}$, a nonempty set $\vartheta \subset 2^G-\{\phi\}$ is called a HX group on G, if ϑ is a group with respect to the algebraic operation defined by AB = { ab /a $\in$ A and b $\in$ B}, which its unit element is denoted by E.

2.2 Definition

A set $(\vartheta , + , \bullet)$ with two binary operation + and $\bullet$ is called a bi HX group if there exist two proper subsets ϑ_1 and ϑ_2 of ϑ such that

i. $\vartheta = \vartheta_1 \cup \vartheta_2$
ii. $(\vartheta_1 , +)$ is a HX group.
iii. $(\vartheta_2 , \bullet)$ is a HX group.

A non-empty subset H of a bi HX group $(\vartheta , + , \bullet)$ is called a sub-bi HX group, if H itself is a bi HX group under ‘ + ‘ and ‘$\bullet$ ‘ operations defined on ϑ.

2.3 Definition

Let X be any non empty set. A fuzzy subset λ of X is a function $\lambda : X \to [0,1]$.

2.4 Definition

A fuzzy set λ is called fuzzy HX subgroup of a HX group ϑ if for A , B $\in \vartheta$,

(i) $\lambda(AB) \geq \min\{\lambda(A), \lambda(B)\}$
(ii) $\lambda(A^{-1}) = \lambda(A)$.

2.5 Definition

Let λ be a fuzzy HX subgroup of a HX group ϑ. For any t $\in[0,1]$, we define the set $\lambda_\alpha = \{ A \in \vartheta / \lambda(A) \geq \alpha \}$ is called the level subset of λ .

2.6 Definition

Let $\vartheta = (\vartheta , + , \bullet)$ be a bi HX group. Then $\lambda : \vartheta \to [0 ,1]$ is said to be fuzzy sub-bi HX group of the bi HX group ϑ if there exist two fuzzy subsets λ_1 of ϑ_1 and λ_2 of ϑ_2 such that

i. $\lambda = \lambda_1 \cup \lambda_2$

ii. $(\lambda_1 , +)$ is a fuzzy HX group of $(\vartheta_1 , +)$.

iii. $(\lambda_2 , \bullet)$ is a fuzzy HX group of $(\vartheta_2 , \bullet)$.

2.1 Example

Let $G_1 = \{ 0 \}$ and $G_2 = \{ 1 , -1 \}$.

Then $(G_1 , +)$ and $(G_2 , \bullet)$ are groups.

Let $\vartheta_1 = \{ \{0\} \}$ and $\vartheta_2 = \{ \{1\} ,\{-1\} \}$, then $(\vartheta_1 , +)$ and $(\vartheta_2 , \bullet)$ are HX groups and hence $\vartheta = (\vartheta_1 \cup \vartheta_2 ,+, \bullet)$ is a bi HX group.

Define $\lambda_1: \vartheta_1 \to [0,1]$ by $\lambda_1(\{0\}) = 0.7$ and

$\lambda_2: \vartheta_2 \to [0,1]$ by $\lambda_2(\{1\}) = 0.8$ and

$\lambda_2(\{-1\}) = 0.5$.

Clearly $(\lambda_1 , +)$ is a fuzzy HX group of $(\vartheta_1 , +)$ and $(\lambda_2 , \bullet)$ is a fuzzy HX group of $(\vartheta_2 , \bullet)$ and hence $\lambda = (\lambda_1 \cup \lambda_2 , + , \bullet)$ be a fuzzy sub-bi HX group of the bi HX group ϑ.

2.7 Definition

Let $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$ and $\vartheta' = (\vartheta'_1 \cup \vartheta'_2, \oplus, o)$ be any two bi HX groups. We say the map $f : \vartheta \to \vartheta'$ is said to be a bi HX group homomorphism if f restricted to ϑ_1 (that is, f / ϑ_1) is a HX group homomorphism from ϑ_1 to ϑ'_1 and f restricted to ϑ_2 (i.e. f / ϑ_2) is a HX group homomorphism from ϑ_2 to ϑ'_2.

3. BI-LEVEL SUBSETS OF A FUZZY SUB-BI HX GROUP OF A BI HX GROUP

3.1 Definition

Let $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$ be a bi HX group and $\lambda = (\lambda_1 \cup \lambda_2, +, \bullet)$ be a fuzzy sub-bi HX group of the bi HX group ϑ. The bi-level subset of the fuzzy sub-bi HX group λ of the bi HX group ϑ is defined as $\lambda_\alpha = \lambda_{1\alpha} \cup \lambda_{2\alpha}$ for every $\alpha \in [0, \min\{\lambda_1(E_1), \lambda_2(E_2)\}]$ where E_1 denotes the identity element of the HX group $(\vartheta_1, +)$ and E_2 denotes the identity element of the HX group $(\vartheta_2, \bullet)$.

Remark:

The condition $\alpha \in [0, \min\{\lambda_1(E_1), \lambda_2(E_2)\}]$ is essential for the bi-level subset to be a sub-bi HX group, for if $\alpha \notin [0, \min\{\lambda_1(E_1), \lambda_2(E_2)\}]$, the bi- level subset need not in general be a sub-bi HX group of the bi HX group ϑ.

3.1 Example

Consider **Example 2.1**, the bi-level subset λ_α for $\alpha = 0.8$ of the fuzzy sub-bi HX group λ is given by $\lambda_\alpha = \{1\}$ which is not a sub-bi HX group of the bi HX group ϑ. Therefore the bi-level subset λ_α, for $\alpha = 0.8$ is not a sub-bi HX group of the bi HX group ϑ.

3.1 Theorem

Every bi-level subset of a fuzzy sub-bi HX group λ of a bi HX group ϑ is a sub-bi HX group of the bi HX group ϑ.

Proof

Let $\lambda = (\lambda_1 \cup \lambda_2, +, \bullet)$ be a fuzzy sub-bi HX group of a bi HX group $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$. Consider the bi-level subset λ_α of a fuzzy sub-bi HX group λ for every $\alpha \in [0, \min\{\lambda_1(E_1), \lambda_2(E_2)\}]$ where E_1 denotes the identity element of the HX group $(\vartheta_1, +)$ and E_2 denotes the identity element of the HX group $(\vartheta_2, \bullet)$.

Then $\lambda_\alpha = \lambda_{1\alpha} \cup \lambda_{2\alpha}$ where $\lambda_{1\alpha}$ and $\lambda_{2\alpha}$ are sub HX groups of ϑ_1 and ϑ_2 respectively. Hence by the definition of sub-bi HX group λ_α is a sub-bi HX group of the bi HX group $(\vartheta, +, \bullet)$.

3.2 Theorem

Let ϑ be a bi HX group and λ be a fuzzy sub-bi HX group of ϑ. Two bi-level subgroups λ_α, λ_β with $\alpha < \beta$ of λ are equal if and only if there is no $A \in \vartheta$ such that $\alpha \le \lambda(A) < \beta$.

Proof

Let $\lambda_\alpha = \lambda_\beta$.

Suppose that there exists $A \in \vartheta$ such that $\alpha < \lambda(A) < \beta$ then, $\lambda_\beta \subset \lambda_\alpha$. Since $A \in \lambda_\alpha$ but not in λ_β which contradicts the hypothesis. Hence there exists no $A \in \vartheta$ such that $\alpha \le \lambda(A) < \beta$.

Conversely, let there be no $A \in \vartheta$ such that $\alpha \le \lambda(A) < \beta$.

Since $\alpha < \beta$, we have, $\lambda_\beta \subseteq \lambda_\alpha$.

Let $A \in \lambda_\alpha$, then $\lambda(A) \ge \alpha$.

Since there exists no $A \in \vartheta$ such that $\alpha \le \lambda(A) < \beta$, we have $\lambda(A) \ge \beta$ which implies $\lambda_\alpha \subseteq \lambda_\beta$.

Hence $\lambda_\alpha = \lambda_\beta$.

3.3 Theorem

Let ϑ be a bi HX group. Let λ be a fuzzy subset of ϑ such that the bi-level subset $\lambda_\alpha = \lambda_{1\alpha} \cup \lambda_{2\alpha}$ is a sub-bi HX group of the bi HX group ϑ where $\alpha \in [0, \min\{\mu_{A1}(E_1), \mu_{A2}(E_2)\}]$, then λ is a fuzzy sub-bi HX group of ϑ.

Proof

Let $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$ be a bi HX group.

Let the bi-level subset λ_α is a sub-bi HX group of the bi HX group ϑ. Then $\lambda_\alpha = \lambda_{1\alpha} \cup \lambda_{2\alpha}$ and $(\lambda_{1\alpha}, +)$ is a sub HX group of $(\vartheta_1, +)$ and $(\lambda_{2\alpha}, \bullet)$ is a sub HX group of $(\vartheta_2, \bullet)$. We have to prove that λ is a fuzzy sub-bi HX group of the bi HX group ϑ.

$(\lambda_{1\alpha}, +)$ is a sub HX group of $(\vartheta_1, +)$, then $(\lambda_1, +)$ is a fuzzy sub HX group of $(\vartheta_1, +)$.

$(\lambda_{2\alpha}, \bullet)$ is a sub HX group of $(\vartheta_2, \bullet)$, then $(\lambda_2, \bullet)$ is a fuzzy sub HX group of $(\vartheta_2, \bullet)$. Clearly $\lambda = \lambda_1 \cup \lambda_2$.

This implies that λ is a fuzzy sub-bi HX group of the bi HX group ϑ.

3.4. Theorem

A fuzzy subset λ of ϑ is a fuzzy sub-bi HX group of a bi HX group ϑ if and only if the bi-level subsets λ_α, $\alpha \in [0, \min\{\lambda(E_1), \lambda(E_2)\}]$, are sub-bi HX groups of ϑ.

Proof

It is clear.

Remark

As a consequence of the Theorem 3.2 , the bi-level sub-bi HX groups of a fuzzy sub-bi HX group λ of a bi HX group ϑ form a chain. Since $\lambda(A) \le \lambda(E_1)$ or $\lambda(A) \le \lambda(E_2)$ for all λ in ϑ. Therefore , $\lambda_{\alpha 0}$, $\alpha_0 \in [0, \min\{\lambda(E_1), \lambda(E_2)\}]$, where $\alpha_0 = \min\{\lambda(E_1), \lambda(E_2)\}$ is the smallest sub-bi HX group and we have the chain :

$\{E_1, E_2\} \subseteq \lambda_{\alpha 0} \subset \lambda_{\alpha 1} \subset \lambda_{\alpha 2} \subset \lambda_{\alpha 3} \subset \ldots\ldots \subset \lambda_{\alpha n}$, where $\alpha_0 > \alpha_1 > \alpha_2 > \ldots > \alpha_n$.

4. FUZZY SUB-BI HX GROUP OF A BI HX GROUP UNDER HOMOMORPHISM AND ANTI HOMOMORPHISM

We now discuss the properties of a fuzzy sub-bi HX group of a bi HX group under homomorphism and anti homomorphism.

4.1 Theorem

Let f be a homomorphism from a bi HX group ϑ into a bi HX group ϑ'. If λ is a fuzzy sub-bi HX group of ϑ and λ is f-invariant, then $f(\lambda)$, the image of λ under f, is a fuzzy sub-bi HX group of ϑ'.

Proof

Let $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$ and $\vartheta' = (\vartheta'_1 \cup \vartheta'_2, \oplus, o)$ be any two bi HX groups. Let $\lambda = (\lambda_1 \cup \lambda_2, +, \bullet)$ be a fuzzy sub-bi HX group of a bi HX group ϑ. Then $(\lambda_1, +)$ is a fuzzy sub HX group of $(\vartheta_1, +)$ and $(\lambda_2, \bullet)$ is a fuzzy sub HX group of $(\vartheta_2, \bullet)$.

Let f: $\vartheta \to \vartheta'$ be a homomorphism.
That is , f (AB) = f (A) f (B) for all A, B in ϑ.
We have to prove that $f(\lambda) = \eta$ is a fuzzy sub-bi HX group of ϑ'.

$\eta = f(\lambda) = f(\lambda_1 \cup \lambda_2) = f(\lambda_1) \cup f(\lambda_2)$.

Since f is a homomorphism, $(f(\lambda_1), \oplus)$ is a fuzzy sub HX group of $(\vartheta'_1, \oplus)$ and $(f(\lambda_2), o)$ is a fuzzy sub HX group of (ϑ'_2, o).

Hence $\eta = f(\lambda_1) \cup f(\lambda_2)$ is a fuzzy sub-bi HX group of ϑ'.

4.2 Theorem

The homomorphic pre-image of a fuzzy sub-bi HX group η of a bi HX group ϑ' is a fuzzy sub-bi HX group of a bi HX group ϑ.

Proof

Let $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$ and $\vartheta' = (\vartheta'_1 \cup \vartheta'_2, \oplus, o)$ be any two bi HX groups.

Let $\eta = (\eta_1 \cup \eta_2, \oplus, o)$ be a fuzzy sub-bi HX group of a bi HX group ϑ'. Then $(\eta_1, \oplus)$ is a fuzzy sub HX group of $(\vartheta'_1, \oplus)$ and (η_2, o) is a fuzzy sub HX group of (ϑ'_2, o).

Let f: $\vartheta \to \vartheta'$ be a homomorphism.
That is , f (AB) = f (A)f (B) for all A , B in ϑ.
We have to prove that $f^{-1}(\eta) = \lambda$ is a fuzzy sub-bi HX group of ϑ.

$\lambda = f^{-1}(\eta) = f^{-1}(\eta_1 \cup \eta_2) = f^{-1}(\eta_1) \cup f^{-1}(\eta_2)$.

Since f is a homomorphism, $(f^{-1}(\eta_1), +)$ is a fuzzy sub HX group of $(\vartheta_1, +)$ and $(f^{-1}(\eta_2), \bullet)$ is a fuzzy sub HX group of $(\vartheta_2, \bullet)$.

Hence $\lambda = f^{-1}(\eta_1) \cup f^{-1}(\eta_2)$ is a fuzzy sub-bi HX group of ϑ.

4.3 Theorem

Let f be an anti homomorphism from a bi HX group ϑ into a bi HX group ϑ'. If λ is a fuzzy sub-bi HX group of ϑ and λ is f-invariant, then $f(\lambda)$, the image of λ under f, is a fuzzy sub-bi HX group of ϑ'.

Proof

Let $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$ and $\vartheta' = (\vartheta'_1 \cup \vartheta'_2, \oplus, o)$ be any two bi HX groups. Let $\lambda = (\lambda_1 \cup \lambda_2, +, \bullet)$ be a fuzzy sub-bi HX group of a bi HX group ϑ. Then $(\lambda_1, +)$ is a fuzzy sub HX group of $(\vartheta_1, +)$ and $(\lambda_2, \bullet)$ is a fuzzy sub HX group of $(\vartheta_2, \bullet)$.

Let f: $\vartheta \to \vartheta'$ be an anti homomorphism.
That is , f (AB) = f (B) f (A) for all A, B in ϑ.
We have to prove that $f(\lambda) = \eta$ is a fuzzy sub-bi HX group of ϑ'.

$\eta = f(\lambda) = f(\lambda_1 \cup \lambda_2) = f(\lambda_1) \cup f(\lambda_2)$.

Since f is an anti homomorphism, $(f(\lambda_1), \oplus)$ is a fuzzy sub HX group of $(\vartheta'_1, \oplus)$ and $(f(\lambda_2), o)$ is a fuzzy sub HX group of (ϑ'_2, o).

Hence $\eta = f(\lambda_1) \cup f(\lambda_2)$ is a fuzzy sub-bi HX group of ϑ'.

4.4 Theorem

The anti homomorphic pre-image of a fuzzy sub-bi HX group η of a bi HX group ϑ' is a fuzzy sub-bi HX group of a bi HX group ϑ.

Proof

Let $\vartheta = (\vartheta_1 \cup \vartheta_2, +, \bullet)$ and $\vartheta' = (\vartheta'_1 \cup \vartheta'_2, \oplus, o)$ be any two bi HX groups.

Let $\eta = (\eta_1 \cup \eta_2, \oplus, o)$ be a fuzzy sub-bi HX group of a bi HX group ϑ' .Then $(\eta_1, \oplus)$ is a fuzzy sub HX group of $(\vartheta'_1, \oplus)$ and (η_2, o) is a fuzzy sub HX group of (ϑ'_2, o).

Let f: $\vartheta \to \vartheta'$ be an anti homomorphism.
That is , f (AB) = f (B)f (A) for all A , B in ϑ.
We have to prove that $f^{-1}(\eta) = \lambda$ is a fuzzy sub-bi HX group of ϑ.

$\lambda = f^{-1}(\eta) = f^{-1}(\eta_1 \cup \eta_2) = f^{-1}(\eta_1) \cup f^{-1}(\eta_2)$.

Since f is an anti homomorphism, $(f^{-1}(\eta_1), +)$ is a fuzzy sub HX group of $(\vartheta_1, +)$ and $(f^{-1}(\eta_2), \bullet)$ is a fuzzy sub HX group of $(\vartheta_2, \bullet)$.

Hence $\lambda = f^{-1}(\eta_1) \cup f^{-1}(\eta_2)$ is a fuzzy sub-bi HX group of ϑ.

5. CONCLUSION

In this paper , we define a new bialgebraic structure of fuzzy sub-bi Hx group and studied some of its properties.Futher, we wish to define the relation between fuzzy (normal) HX group and fuzzy (normal) sub bi HX group in HX group also the same in Intuitionistic fuzzy and other some groups are in progress.

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