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Fuzzy Dot Groups

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ABSTRACT:

H.Sherwood (4) introduced the concept of product of fuzzy groups (Fuzzy sets & fuzzy systems) and its properties. We define a new class of fuzzy groups called fuzzy dot group which is weaker than the standard fuzzy group defined by Rosenfeld (2) and characterize some properties of fuzzy dot groups.

Keyword: fuzzy group, fuzzy dot group standard fuzzy group, d-fuzzy groupoid, homomorphism's, direct product.

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1. INTRODUCTION

The concept of fuzzy sets was first introduced by Zadeh (9). Rosenfeld (2) used this concept to formulate the notion of fuzzy groups. Since then, many other fuzzy algebraic concepts based on the Rosenfeld's fuzzy groups were developed. Anthony and Sherwood (1) redefined fuzzy groups in terms of t- norm which is replaced the min operations of Rosenfeld's definition. Some properties of these redefined fuzzy groups, which we call t- fuzzy groups, have been developed by Sherwood (4), sessa (3), sidky and misherf (5). However the definition of t- fuzzy groups seems to be too general. A.Solairaju and R.Nagarajan (6) investigate the characterizations of weaker groups in terms of Rosenfeld groups. We define a new class of fuzzy group called fuzzy dot group which is weaker than the fuzzy groups defined by Rosenfeld's (2), and characterize some properties of fuzzy dot groups.

2. PRELIMINARIES

Definition 2.1: A function 'A' from a set X to the closed unit interval [0,1] in U is called a fuzzy set in X, for every $x \varepsilon A$, A(x) is called membership grade of x in A. The set $\{ x \varepsilon A / A(x) > 0 \}$ is called the support of A and it is denoted by supp(A). For fuzzy sets λ and μ in a set X, then $\lambda \circ \mu$ has been defined in most articles by

$$(\lambda \circ \mu)(x) = \sup_{ab=x} \min \{ \lambda(a), \mu(b) \}, \quad \text{if } ab=x$$
$$= 0 \quad \text{if } ab \neq x$$

We weaker this definition as follows.

Definition 2.2: Let X be a set and let λ, μ be two fuzzy sets in X, $\lambda \circ \mu$ is defined by

$$(\lambda \circ \mu)(x) = \begin{cases} \sup \{ \lambda(a), \mu(b) \} & \text{if } ab = x \\ 0 & \text{if } ab \neq \mathbf{x} \end{cases}$$

Definition 2.3: Let X be a group, we define λ^{-1} by $\lambda^{-1}(x) = \lambda(x^{-1})$ for $x \varepsilon X$. The standard definition of a fuzzy group by Rosenfeld(2) is that a fuzzy set 'A' in a group X is a fuzzy group iff $A(xy) \geq \min \{A(x), A(y)\}$ and $A(x^{-1}) = A(x)$ for all $x,y \varepsilon X$ we weaken this definition as follows.

Definition 2.4: Let 'S' be a groupoid. A function $A : S \rightarrow [0,1]$ is a dot groupoid in S iff for every x, y in S , (FDG1) $A(xy) \geq A(x) \bullet A(y)$, we denote a dot fuzzy groupoid by a d- fuzzy groupoid. If X is a group, a fuzzy dot groupoid 'A. in X iff for $x \varepsilon X$, (FDG2) $A(x^{-1}) = A(x)$, we denote a dot fuzzy group X by a d- fuzzy group .Since $\min (a,b) \geq ab$, our definition of a d- fuzzy group is weaker than the standard definition by Rosenfeld(2). It is easy to see that if G is fuzzy group in a group X and 'e' is the identity of X , $G(e) \geq$

$G(x)$ for all $x \varepsilon X$. If 'G' is a d-fuzzy group in a group X, $G(e) = G(xx^{-1}) \geq G(x)\, G(x^{-1}) = [G(x)]^2$ for all $x \varepsilon X$

3. SOME PROPERTIES OF D- FUZZY GROUPS

Proposition 3.1: Let 'A' be a fuzzy subset in a group X such that $A(e) = 1$, where 'e' is the identity of X then 'A' is a d-fuzzy group iff $A(xy^{-1}) \geq A(x) \bullet A(y)$ for all $x,y \varepsilon X$.

Proof: suppose 'A' is d- fuzzy group. Then $A(xy^{-1}) \geq A(x) \bullet A(x^{-1}) = A(x) \bullet A(y)$ .Suppose $A(xy^{-1}) \geq A(x) \bullet A(y)$. Then $A(x^{-1}) = A(ex^{-1}) \geq A(e) \bullet A(x^{-1}) \geq A(x) = A(ex) \geq A(e) \bullet A(x) = A(x^{-1})$That is $A(x) = A(x^{-1})$ and $A(xy^{-1}) = A(x(y^{-1})) \geq A(x) \bullet A(y^{-1}) = A(x) \bullet A(y)$.

Definition 3.2: Let 'A' be a d- fuzzy groupoid in a group X such that $A(a) = A(a^{-1})$. Let $e\lambda : X \to X$ be identity defined by $e_\lambda(x) = x\lambda$ for all $x \varepsilon A$. similar we can define the left identity of A.

Proposition 3.3: If the d- fuzzy groupoid 'A' on X has left identity e_λ and a right identity e_μ then $e_\lambda = e_\mu$.

Proof: $e_\lambda(A)(x) = \sup_{z \varepsilon e_\lambda^{-1}(x)} A(z) = A(x^{-1}a)$

$$\geq A(x) \bullet A(a^{-1})$$
$$= A(x)$$
$$= A(xa^{-1}a) \geq A(xa^{-1}) \bullet A(a) = A(xa^{-1})$$
$$= e_\lambda(A)(x)$$

Thus $e_\lambda(A)(x) \geq A(x) \geq e_\lambda(A)(x)$. That is $e_\lambda(A) = A$. similarly we may show $e_\mu(A) = A$.

Definition 3.4: Let $f: G \to G^1$ be a homomorphism of d-fuzzy groups. For any fuzzy set $A \varepsilon G^1$ we define a new fuzzy set A^f in G by $A^f(x) = Af(x)$ for all $x \varepsilon G$, and $f(x^{-1}) = f(x) = (f(x))^{-1}$.

Definition 3.5. Let A and B be two fuzzy subsets of X then the direct product $A \times B$ is defined by $(A\times B)(x,y) = \min\{A(x), B(y)\}$ and $(x,y) \bullet (z,p) = (xz, yp)$ for all x,y,z,p in X.

Proposition 3.6: Let G and G^1 be groups and f a homomorphism from G onto G^1, (i) if A is d- fuzzy group of G^1 then A^f is d- fuzzy group of G .(ii) if A^f is d- fuzzy group of G then A is d- fuzzy group of G^1.

Proof : (i) Let $x,y \varepsilon G$, we have (FDG1) $A^f(xy) = Af(xy) = A(f(x)\, f(y)) \geq Af(x) \bullet Af(y) = A^f(x) \bullet A^f(y)$.

(FDG2) $A^f(x^{-1}) = Af(x^{-1}) = Af(x) = A^f(x)$

A^f is d- fuzzy group of G^1.

(ii) For any $x,y \varepsilon G^1$, There exists a ,b ε G such that $f(a) = x$ and $f(b) = y$.

(FDG1) $A(xy) = A(f(a)\, f(b)) = Af(ab) = A^f(ab$

$\geq A^f(a) \bullet A^f(b) = Af(a) \bullet Af(b) = A(x) \bullet A($

(FDG2) $A(x^{-1}) = A(f(a^{-1})) = Af(a^{-1}) = A^f(a^{-1}) = A^f(a) = Af(a) = A(x)$

A is d- fuzzy group of G^1.

Proposition 3.7: If A and B be d- fuzzy groups of G_1 and G_2 respectively then $A\times B$ is d- fuzzy group of $G_1 \times G_2$.

Proof: Let (a_1,b_1) , (a_2,b_2) $\varepsilon\, G_1\times G_2$

(FDG1) $A\times B\, ((a_1,b_1)(a_2,b_2)) = (A\times B)\, (a_1a_2,b_1 = \min\{ A(a_1a_2)$, $B(b_1b$

$\geq \min\{ A(a_1) \bullet A(a_2) , B(b_1) \bullet B(b_2)\}$

$\geq \min\{ A(a_1) \bullet B(b_1) , A(a_2) \bullet B(b_2)\}$

$\geq \min\{ \{A(a_1) ,B(b_1)\}, \min\{A(a_2), B(b_2)\}\}$

$\geq A\times B(a_1,b_1)\ A\times B(a_2,b_2)$

(FDG2) $A\times B(a_1,b_1)^{-1} = A\times B(a_1^{-1},b_1^{-1})$

$= \min\{A(a_1^{-1}) , B(b_1^{-1})\}$

$= \min(A(a_1) , B(b_1)\}$

Proposition 3.8: If A and B be d- fuzzy groups of G_1 and G_2 respectively then A×B is d- fuzzy group of $G_1 \times G_2$.

Proof: Let (a_1,b_1) , (a_2,b_2) ε $G_1 x G_2$

(WF1) $A\times B\ ((a_1,b_1)(a_2,b_2)) = (A\times B)\ (a_1a_2,b_1b_2)$

$= \min \{ A(a_1a_2) , B(b_1b_2)\}$

$\geq \min \{ A(a_1) \bullet A(a_2) , B(b_1) \bullet B(b_2)\}$

$\geq \min \{ A(a_1) \bullet B(b_1) , A(a_2) \bullet B(b_2)\} \geq \min \{ \{A(a_1) ,B(b_1)\}, \min \{A(a_2), B(b_2)\}\}$

$\geq A\times B(a_1,b_1)\ A\times B(a_2,b_2)$

(WF2) $A\times B(a_1,b_1)^{-1} = A\times B(a_1^{-1},b_1^{-1})$

$= \min \{A(a_1^{-1}) , B(b_1^{-1})\}$

$= \min (A(a_1) , B(b_1)\}$

$= A\times B(a_1,b_1)$

Corollary 3.9: If $A_1,A_2,\ldots A_n$ are d- fuzzy groups of G_1, $G_2\ldots.G_n$ respectively then $A_1\times A_2\times\ldots\ldots..A_n$ is d- fuzzy groups of $G_1\times G_2\times\ldots\ldots..G_n$.

Proof: This result can easily show by induction method.

Proposition 3.10: Let A and B be fuzzy subsets of G_1 and G_2 respectively such that A×B is a d- fuzzy group of $G_1\times G_2$ then A and B is d- fuzzy group of G_1 and G_2 respectively.

Proof:

$(A\times B)\ (e_1,e_2) = \min (A(e_1), A(e_2)\} \geq (A\times B)(x,y)$ for all (x,y) ε $G_1 \times G_2$,then

$A(x) \leq A(e_1)$ or $B(y) \leq B(e_2)$. If $A(x) \leq A(e_1)$ then $A(x) \leq B(e_2)$ or $B(y) \leq B(e_2)$.

Let $A(x) \leq B(e_2)$. Then for all x, y ε G $(A\times B)\ (x,e_2) = A(x)$

(FDG1) $A(xy) = (A\times B)(xy,e_2)$

$= (A\times B)((x,e_2)(y,e_2))$

$\geq (A\times B)(x,e_1) \bullet (A\times B)(y,e_2)$

$\geq A(x) \bullet A(y)$

(FDG2) $A(x^{-1}) = (A\times B)(x^{-1},e_2)$

$= (A\times B)(x^{-1},e_2^{-1})$

$= (A\times B)\ (x,e_2)^{-1}$

$= (A\times B)\ (x,e_2)$

$= A(x)$ Therefore A is w- fuzzy group of G.

Now suppose that $A(x) \leq B(e_2)$ is not true for all x ε G_1. If $A(x) \geq B(e_2)$ there exists x ε G_1, then $B(y) \leq B(e_2)$ for all y ε G_2. Therefore

$(A\times B)(e_1,y) = B(y)$ for all y ε G_2. Similarly for all x,y ε G_2,

$B(xy) = (A\times B)(e_1,xy)$

$= (A\times B)((e_1,x)\ (e_2,y))$

$\geq (A\times B)(e_1,x) \bullet (A\times B)(e_1,y)$

$= B(x) \bullet B(y)$

And

$B(x^{-1}) = (A\times B)(e^1,x^{-1})$

$= (A\times B)(e_1^{-1},x^{-1})$

$= (A\times B)(e_1,x)^{-1}$

$= (A\times B)(e_1,x) = B(x)$ hence B is d- fuzzy group of G_2 consequently either A or B is d- fuzzy group of G_1or G_2 respectively.

Definition 3.11: Let f: $G \rightarrow G^1$ be a group homomorphisms and 'A' be d- fuzzy group of G^1 then $Af(x) = (A \text{ o } f)(x) = f^{-1}(A)\ (x)$

Proposition3.12: Let f: $G \rightarrow G^1$ be a group homomorphism and let 'A' be a d- fuzzy group of G^1 then $f^{-1}(A)$ is d- fuzzy group of G.

Proof: Let x,y ε G , we have

(FDG1) $f^{-1}(A)(xy) = (Aof)(xy)$

$= A\, f(xy)$

$= A(f(x) f(y))$

$\geq Af(x) \bullet Af(y)$

$\geq (Aof)(x) \bullet (Aof)(y)$

$\geq f^{-1}(A)(x) \bullet f^{-1}(A)(y)$

(FDG2) $f^{-1}(A)(x^{-1}) = (Aof)(x^{-1}) = Af(x^{-1}) = A(f(x)) = f^{-1}(A)(x)$

Proposition 3.13: Let A be a d- fuzzy group of group G and A* be a fuzzy set in G defined by $A^*(x) = A(x) + 1 - A(e)$ for all x ε G. Then A* is d- fuzzy group of G containing A.

Proof: for x,y ε G, we have

(FDG1) $A^*(xy) = A(xy) + 1 - A(e)$

$\geq (A(x) \bullet A(y)) + 1 - A(e)$

$\geq (A(x)+1-A(e)) \bullet (A(y)+1-A(e))$

$\geq A^*(x) \bullet A^*(y)$

(FDG2) $A^*(x^{-1}) = A(x^{-1}) + 1 - A(e)$

$= A(x) + 1 - A(e)$

$= A^*(x)$

A* is d- fuzzy group of G containing A.

Proposition3.14 : Let $< A > = \{ A, A^1, A^2, \text{----} A^P \text{----} E\}$ then $\bigcup_{p=1}^{\infty} A^P = A$ and $\bigcap_{p=1}^{\infty} A^P = E$

Proof: Let x ε G we have $A \subset \bigcup_{p=1}^{\infty} A^P$ ----------------------- (i)

(since $A(x) \leq \bigcup A^P(x)$)

$\bigcup A^P(x) = \max\{ A(x), A^2(x), \text{-----} \}$

$= A(x)$ (since $A(x) \leq (A(x))^p$ for all natural number p.

$\bigcup_{p=1}^{\infty} A^P \subset A$ --------------- (ii) from (i) and (ii) $\bigcup_{p=1}^{\infty} A^P = A$. Also we have to

Show that $E = \bigcap_{p=1}^{\infty} A^P$. Let e ε A^P implies that (e,1) ε A^P for all p.

Implies that (e,1) $\bigcap_{p=1}^{\infty} A^P$

Therefore $E \subset \bigcap_{p=1}^{\infty} A^P$ ------------------------ (iii)

Let x ε $\bigcap_{p=1}^{\infty} A^P$ implies that x ε A^P for all p.

$$\bigcap_{p=1}^{\infty} A^P(x) = \min\{ A(x), A^2(x) \text{-----} \}$$

$$= 0 \quad \text{if } x \neq e$$

$$= 1 \quad \text{if } x = e$$

x ε $\bigcap_{p=1}^{\infty} A^P$ implies that x = e thus $\bigcap_{p=1}^{\infty} A^P \subset E$ -------------- (iv) hence

From (iii) and (iv) $E = \bigcap_{p=1}^{\infty} A^P$

Proposition3.15: Let A be a d-fuzzy group, then $A \supset A^2 \supset A^3 \ldots\ldots\ldots \supset A^p \ldots\ldots E$.

Proof: It is known that A(a) ε [0,1], hence $A(a) \geq A(a)^2$, $A(a^2) \geq (A(a^2))^2, \ldots\ldots\ldots\ldots\ldots A(a^n) \geq (A(a^n))^2$. by using the definition of fuzzy subsets , this gives that $A \supset A^2$. By generalizing it for any natural numbers i and j with $i \leq j$. we obtain $(A_i(a))^i \geq (A_j(a))^j$, $(A_i(a^2))^i \geq (A_j(a^2))^j, \ldots\ldots\ldots (A_i(a^n))^i \geq A_j(a^n))^j$.

So $A^i \supset A^j$ for any natural numbers i and j with $i \leq j$ which means that $A \supset A^2 \supset A^3 \ldots\ldots \supset A^{pn} \ldots$ finally we get $E = \bigcap_{p=1} A^p$ which is immediate from proposition (3.14). Since

$$\lim_{n\to\infty} A(a)^p = 1 \quad \text{if } a = e.$$
$$= 0 \quad \text{if } a \neq e$$

we then obtain the required relations.

4. CONCLUSION

H.Sherwood introduced the concept of product of fuzzy groups We define a new class of fuzzy groups called fuzzy dot group which is weaker than the standard fuzzy group defined by Rosenfeld and characterize some properties of fuzzy dot groups.

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