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Comparison of Predictive Capability of Software Reliability Growth Models with Exponentiated Weibull Distribution

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ABSTRACT
This study aims to compare the predictive capability of two popular software reliability growth models (SRGM), say exponential growth and inflection S-shaped growth models. We first review the exponentiated Weibull (EW) testing-effort functions and discuss exponential type and inflection S-shaped type SRGM with EW testing-effort. We then analyzed the actual data applications and compare the predictive capability of these two SRGM graphically. The findings reveal that inflection S-shaped type SRGM has better prediction capability as compare to exponential type SRGM.

Keywords
Testing-Effort Function, Exponentiated Weibull Distribution, Software Reliability Growth Models. Mean value function, non-homogeneous Poisson process, Estimation methods

1. INTRODUCTION
Software reliability is defined as the probability of failure-free operation of a computer program for a specified time in a specified environment (Musa et al., 1987; Lyu, 1996) and is a key factor in software development process. Numerous software reliability growth models (SRGMs) have been developed during the last three decades and they have applied successfully in practice to improve software reliability (Musa et al., 1987; Xie, 1991; Lyu, 1996; Pham, 2000).

In the past years, several SRGMs based on NHPP which incorporates the testing–effort functions (TEF) have been proposed by many authors (Yamada et al., 1984; 1986; 1993; Yamada and Ohtera, 1990; Huang et al., 2007; Kuo et al., 2001; Bokhari and Ahmad, 2006; Quadri et al., 2006). Recently, Bokhari and Ahmad (2007) and Ahmad et al. (2008; 2010) also proposed a new SRGM with the exponentiated Weibull (EW) testing-effort functions to predict the behavior of failure and fault of software.

This paper first reviews the EW testing-effort functions and then incorporate the EW testing-effort function into exponential and inflection S-shaped NHPP growth models. Actual data applications are analyzed and the predictive capability of these two SRGM is compared graphically.

2. REVIEW OF EW TESTING-EFFORT FUNCTIONS
Recently, Bokhari and Ahmad (2007) and Ahmad et al. (2008; 2010) proposed EW testing-effort function to predict the behavior of failure and fault of a software product. They have shown that EW testing-effort function is very suitable and more flexible testing resource for assessing the reliability of software products.

The cumulative testing-effort expenditure consumed in (0, t] is depicted in the following:

$$W(t) = \alpha(1 - e^{-\beta t^{\delta}})^{\theta}, \alpha > 0, \beta > 0, \delta > 0, \theta > 0 , \tag{1}$$

and the current testing-effort consumed at testing time t is

$$w(t) = \frac{dW(t)}{dt} = \alpha.\beta.\delta.\theta.t^{\delta-1}.e^{-\beta t^{\delta}}(1 - e^{-\beta t^{\delta}})^{\theta-1} \tag{2}$$

where α is the total amount of testing-effort expenditures; β is the scale parameter, and δ and θ are shape parameters.

3. SOFTWARE RELIABILITY GROWTH MODEL
Exponential growth model and inflection S-shaped growth model have been shown to be very useful in fitting software failure data.

3.1 Exponential Type SRGM with EW testing-effort
The exponential growth model proposed by Goel and Okumoto (1979) has been considered for comparative study. Based on the basic assumptions, if the number of detected errors by the current testing-effort expenditures is proportional to the number of remaining errors, then we obtain the following differential

equation (Yamada and Osaki, 1985; Yamada et al., 1986; 1993; Yamada and Ohtera, 1990; Bokhari and Ahmad, 2006):

$$\frac{dm(t)}{dt} / w(t) = b\left[a - m(t)\right], \; a > 0, \; 0 < b < 1 \quad , \tag{3}$$

where $m(t)$ represent the expected mean number of errors detected in time $(0,t]$ which is assumed to be a bounded non-decreasing function of t with $m(0) = 0$, $w(t)$ is the current testing-effort expenditure at time t, a is the expected number of initial error in the system, and b is the error detection rate per unit testing-effort at time t. Solving the above differential equation, we have

$$m(t) = a(1 - e^{-bW(t)}) \,. \tag{4}$$

Substituting $W(t)$ from (1), we get

$$m(t) = a(1 - e^{-b\alpha(1-e^{-\beta t^{\delta}})^{\theta}}) \,. \tag{5}$$

This is an NHPP model with mean value function considering the EW testing-effort expenditure.

3.2 Inflection S-shaped Type SRGM with EW testing-effort

Ohba (1984; 1984a) raised the inflection S-shaped NHPP model. Later, Ahmad et al. (2010) modified the inflection S-shaped model and incorporated the EW testing-effort in an NHPP growth model.

On the basis of assumptions, if the error detection rate with respect to current testing-effort expenditures is proportional to the number of detectable errors in the software and the proportionality increases linearly with each additional error removal, we obtain the following differential equation:

$$\frac{dm(t)}{dt} \times \frac{1}{w(t)} = \phi(t)\left(a - m(t)\right) \tag{6}$$

Where

$$\phi(t) = b\left[r + (1-r)\frac{m(t)}{a}\right], \; r\,(>0)$$

is the inflection rate and represents the proportion of independent errors present in the software. Solving (6) with the initial condition that, at $t = 0, W(t) = 0, m(t) = 0$, we obtain the mean value function

$$m(t) = \frac{a\left[1 - e^{-bW(t)}\right]}{1 + ((1-r)/r)e^{-bW(t)}} \tag{7}$$

Substituting $W(t)$ from (1), we get

$$m(t) = \frac{a\left[1 - e^{-b\alpha(1-e^{-\beta t^{\delta}})^{\theta}}\right]}{1 + ((1-r)/r)e^{-b\alpha(1-e^{-\beta t^{\delta}})^{\theta}}} \,. \tag{8}$$

4. COMPARISON OF PREDICTIVE CAPABILITY

The parameters of the SRGM are estimated based upon the data given below. Maximum Likelihood estimation (MLE) and Least Square estimation (LSE) techniques are used to estimate the model parameters (Musa et al., 1987; Musa, 1999; Lyu, 1996; Ahmad et al., 2008; 2010).

In order to compare predictive capability of exponential growth model and inflection S-shaped model, experiments on two actual software failure data are performed. The description of the data sets is given in Table I

Table I: Summary of studied actual data sets.

Data Set	References	Errors Removed	Observation Period	Software Project
DS1	Ohba (1984)	328, after 3.5 years: 188	19 weeks	PL/1 application software, Execution Time: 47.65CPU hours, Size: 1317000 line of code
DS2	Musa et al. (1987)	136, after a long time of testing: 358	21 weeks	Rome Air Development Center Project, Execution Time: 25.3 CPU hours, Size: 21700 line of code

DS 1: Table II lists the comparisons of exponential growth model and inflection S-shaped growth model SRGMs which reveal that the inflection S-shaped growth model has better performance.

Table II: Comparison results of exponential model and inflection S-shaped model

Model	a	r	b	AE (%)	MSE
Exponential model with EW	565.64		0.01964	57.98	113.10
Inflection S-shaped model with EW	388.48	0.381	0.06061	8.36	87.36

We compute the relative error in prediction of exponential growth model and inflection S-shaped growth model for this data set. Figures 1 and 2 show the relative error plotted against the percentage of data used (that is, t_e / t_q). Figures and table reveal that the inflection S-shaped growth model predicts the future behavior well as compare to exponential growth model.

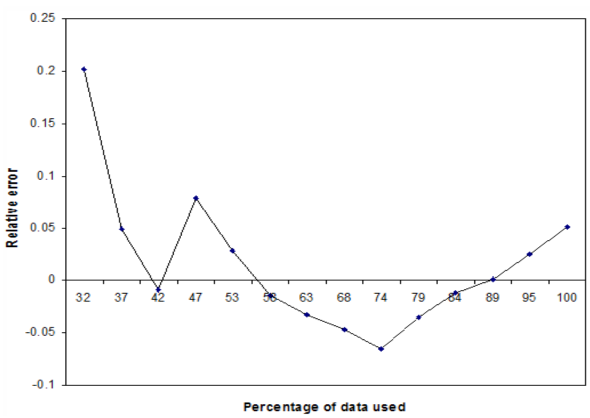

Figure 1: Predictive Relative Error Curve of exponential growth model

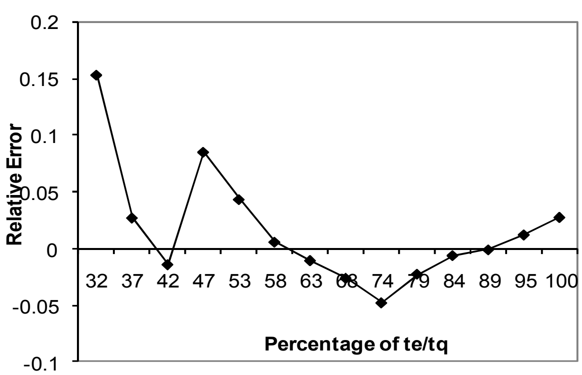

Figure 2: Predictive Relative Error Curve of inflection S-shaped growth model

DS 2: Table III shows the comparisons of exponential model and inflection S-shaped model with different SRGMs which reveal that the inflection S-shaped model has better performance for this data set.

Table III. Comparison results of exponential model and inflection S-shaped model

Model	a	r	b	AE (%)	MSE
Exponential model with EW	133.87		0.1546	28.79	78.55
Inflection S-shaped model with EW	161.69	47.275	0.0036	13.99	63.95

The relative error in prediction is calculated for exponential growth model and inflection S-shaped growth model and the results are shown graphically in Figures 3 and 4. Finally, from the Figures 3 to 4 and Table III, it can be concluded that the inflection S-shaped model gets reasonable prediction as compare to exponential model.

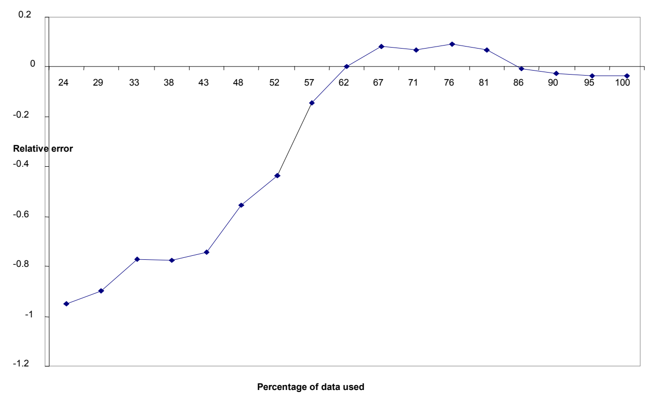

Figure 3: Predictive Relative Error Curve of exponential model

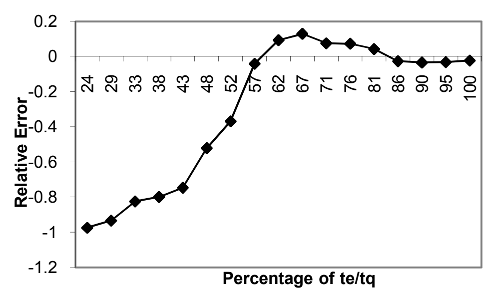

Figure 4: Predictive Relative Error Curve of inflection S-shaped model

5. CONCLUSION

In this paper we discuss exponential type and inflection S-shaped type SRGM with EW testing-effort. We analyzed the predictive capability of exponential growth and inflection S-shaped growth models for the actual data applications. We then compared its predictive capability graphically. The findings reveal that inflection S-shaped type SRGM has better prediction capability as compare to exponential type SRGM.

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