On Fuzzy Soft Groups

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ABSTRACT

Molodtsov introduced the concept of soft set theory which can be used as generic mathematical tool for dealing with uncertainty. In this paper, we discuss fuzzy soft sets algebraic structure and give the definition of fuzzy soft group. We define operations on fuzzy soft groups and prove some results on them. Furthermore, definitions of fuzzy soft function and fuzzy soft homomorphism are defined and the theorems on homomorphic image and homomorphic pre image are given. After that, the definition of normal fuzzy soft group is given and some of its basic properties are studied.

Keywords: Soft Set, Fuzzy Soft Group, Soft Homomorphism, Normal Fuzzy Soft Group, Pre-image, Fuzzy Soft Function.

1. INTRODUCTION

Researchers in economics, engineering, environmental science, sociology, medical science and many other fields deal daily with the complexities of modeling uncertain data. Classical methods are not always successful, because the uncertainties appearing in these domains may be of various types. While Probability theory, fuzzy sets [15], rough sets [12], and other mathematical tools are well-known and often useful approaches to describing uncertainty, each of these theories has its inherent difficulties as pointed out by Molodtsov [7]. Consequently, Molodtsov [7] proposed a completely new approach for modeling vagueness and uncertainly. This is so-called soft theory is free from the difficulties affecting existing methods. In soft set theory the problem of setting the membership function, among other related problems, simply does not arise. This makes the theory very convenient and easy to apply in practice soft set theory has potential applications in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Person integration, Probability theory and Measurement theory. Most of these

applications have already demonstrated in Molodtsov's book [7]. At present, work on the soft set theory is progressing rapidly. Maji et al [8] describe the application of soft set theory to a decision making problem using rough sets. The same authors have also published a detailed theoretical study on soft sets. The algebraic structure of set theories dealing with uncertainties has also been studied by some authors. Rosenfeld [14] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets. Rough groups were defined by Biswas et al [4], and some other author (e.g.,[2] and [8]) have studied the algebraic properties of Rough sets as well. The main purpose of this paper is to introduce a basic version of soft group theory, which extends the notion of a group to include the algebraic structures of soft sets. Our definition of soft groups is similar to the definition of rough groups, but is constructed using different methods. This paper begins by introducing the basic concepts of fuzzy soft set theory then we discuss a basic version of fuzzy soft group theory, which extends the notion of a group to the algebraic structures of fuzzy soft sets. A fuzzy soft group, on the other hand, is a parameterized family of fuzzy subgroups.

2. PRELIMINARIES

Definition 2.1: A pair (f,A) is called a soft set over X, if f:A:$\to$ P(X). Here let 'X' be the initial Universe and E be the set of parameters. Let P(X) denotes the power set X. To illustrate this idea, Molodtsov considered several examples in [7]. Let I^x denote the set of all fuzzy sets on X.

Definition 2.2: A pair (f,A) is called a soft set over X, where f:A:$\to I^x$ (ie) for each a ε A, $f(a) = f_a : X \to I$ is a fuzzy set in X.

Definition 2.3: Let 'X' be group and (f, A) be a soft set over X, then (f, A) is said to be a soft group over X iff f(a) $\le$ x, $\forall$ a ε A. Union of two fuzzy soft sets (f, A) and (g, B) over

a common Universe X is the fuzzy soft set (h, C) where C = A ∪ B. We write (f, A) ∪ (g, B) = (h, C). Intersection of two fuzzy soft sets (f, A) and (g, B) over a common Universe X is the fuzzy soft set (h, C), where C=A∩B. We write (f, A) ∩ (g, B) = (h, C). Product of two fuzzy soft sets (f, A) and (g, B) denoted (f, A) ∩ (g, B) is defined as (h, AXB) where h(a, b) = f(a) ∩ g(b).

Definition 2.4: Let 'X' be a group and (f, A) be a fuzzy soft set over X, then (f, A) is said to be fuzzy soft group if ;

(i) $f_a(xy) \geq \min \{f_a(x), f_a(y)\}$

(ii) $f_a(x^{-1}) \geq f_a(x)$

(iii) $f_a(x) = 1 \ \forall \ a \ \varepsilon \ A, x,y \ \varepsilon \ X.$

Definiition 2.5: Let 'f_a' be a fuzzy soft set in X.

Let $\phi : X \to Y$ be a map and define $f_a^{\phi} : X \to [0, 1]$ by $f_a^{\phi}(x) = f_a(\phi(x)), \forall x \ \varepsilon \ X.$

Definition 2.6: Let (f,A) be a fuzzy soft groups over X and $\lambda \ \varepsilon \ [0, 1]$ then

(i)   (f, A) is said to be λ-identity fuzzy soft group over X if $f_a(x) = \lambda$ if x=e $\forall$ xεX
        = 0 otherwise

(ii)   (f, A) is said to be λ-absolute fuzzy soft group over X if $f_a(x) = \lambda \ \forall$ xεX, aεA.

Definition 2.7: Let (f,A) be a fuzzy soft set over X. The soft set $(f,A)_\alpha = \{ (f_a)_\alpha : a\varepsilon A \}$ for each $\alpha \ \varepsilon \ [0, 1]$ is called an α - level soft set.

Definition 2.8: Let 'X' be a group and $f_{a_1}, f_{a_2}, \ldots, f_{a_n}$ is n-fuzzy soft subgroups of $X_1, X_2,\ldots X_n$, define a map

$f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi \ : \ (X_1, \ X_2,..X_n) \to [0, 1]$ by

$(f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi) \quad (x_1,x_2,\ldots x_n) = \min$

$(f_{a_1}(x_1) f_{a_2}(x_2) \ldots f_{a_n}(x_n)\}$ (this is called generalized product of fuzzy soft groups).

Definition 2.9: Fuzzy Soft group

Let 'X' be a group and (f, A) be a fuzzy soft set over X. Then (f, A) is said to be fuzzy soft group if

(i) $f_a(xy) \geq \min \{f_a(x), f_a(y)\}$

(ii) $f_a(x^{-1}) \geq f_a(x)$

(iii) $f_a(x) = 1 \ \forall \ a \ \varepsilon \ A, x,y \ \varepsilon \ X.$

Throughout this chapter, fuzzy soft set (f,A) is Re called as f_a.

3. PROPERTIES OF FUZZY SOFT GROUPS

Proposition 3.1: Let f_a be a fuzzy soft group of G then $f_a(x) \leq f_a(e) \ \forall \ x \ \varepsilon \ G$. The subset $G_{fa} = \{x\varepsilon G/f_a(x) = f(e)\}$ is fuzzy soft group of G.

Proof: Let 'x' be any element of G, then $f_a(xy) = \min \{f_a(x), f_a(y)\}= \min \{f_a(x), f_a(x^{-1})\} \leq f_a(xx^{-1}) = f_a(e)$ implies (i). To verify (ii), it follows that x ε G_{fa} and $G_{fa} \neq \phi$.

Now let x,y ε Gf_a.

$f_a(xy^{-1}) \geq \min \{f_a(x), f_a(y^{-1})\}= \min \{f_a(x), f_a(y)\} = \min \{ f_a(e), f_a(e)\} = f_a(e)$

but from (i) $f_a(xy^{-1}) \leq f_a(e)$ for all x, yε G and so $f_a(xy^{-1}) = f_a(e)$ which then $xy^{-1} \ \varepsilon \ G_{fa}$ is fuzzy soft group of G.

Corollary 3.2: Let 'G' be a finite group and f_a be a fuzzy soft group of G. Consider the subset H of G given by H = {x ε G / $f_a(x) = f_a(e)$}, then H is a Crisp subgroup of G.

Proposition 3.3: Let f_a and f_b be two fuzzy soft groups of G, then $f_a \cap f_b$ is fuzzy soft group of G.

Proof: Let x,y ε G.

(FSG1) $(f_a \cap f_b)(xy) = \min \{ f_a(xy), f_b(xy)\}$

$\geq \min \{ \min \{f_a(x), f_a(y)\}, \min(f_b(x), f_b(y)\}$

$= \min \{\min\{ f_a(x), f_b(x)\}, \min \{ f_ay), f_b(y)\}$

$= \min \{ (f_a \cap f_b)(x), (f_a \cap f_b)(y)\}$

$= \min \{ f_a(x^{-1}), f_b(x^{-1})\}$

$= \min \{ f_a(x), f_b(x)\}$

$= (f_a \cap f_b)(x)$

Proposition 3.4: If 'f_a' is fuzzy soft group, then the set ∪(f_a ; t) is fuzzy soft group $\forall$ t ε Im(f_a).

Proof: Let t ε Im(A) ⊆ [0,1], x,y ε ∪(f_a;t), then $f_a(x) \geq t$, $f_a(y) \geq t$. since f_a is fuzzy soft group of G.

$f_a(xy) \geq \min \{f_a(x), f_a(y)\} \geq t$ implies $xy \, \varepsilon \cup (f_a;t)$. Let $x \, \varepsilon \cup (f_a \, ; \, t)$ implies $f_a(x^{-1}) = f_a(x) \geq t$ which gives $x^{-1} \, \varepsilon \cup (f_a;t)$. Thus $\cup(f_a;t)$ is fuzzy soft group of G.

Proposition 3.5: If 'f_a' is fuzzy soft group such that all non-empty level subset $\cup(f_a;t)$ is fuzzy soft group of G, then 'f_a' is fuzzy soft group of G.

Proof: Assume that the non-empty level set $\cup(f_a;t)$ is fuzzy soft group of G. If $t_0 = \min \{f_a(x), f_a(y)\}$ and for x, y ε G, then x, y $\varepsilon \cup(f_a;t)$ so $f_a(xy) \geq t_0 = \min\{f_a(x), f_a(y)\}$ which implies that the condition (FSG1) is valid. For $x^{-1} \varepsilon$ G, then $x^{-1} \varepsilon \cup(f_a;t_0)$ thus $f_a(x^{-1}) = t_0 = f_a(x)$ which gives the condition (FSG2) is valid and therefore 'f_a' is fuzzy soft group of G.

Proposition 3.6: A set of necessary and sufficient condition for a fuzzy soft set of a group G to be a fuzzy soft group of G is that $f_a(xy) \geq \min \{f_a(x), f_a(y)\} \, \forall$ x, y in G.

Proof: Let 'f_a' be a fuzzy soft group of G, then

$f_a(xy^{-1}) \geq \min \{f_a(x), f_a(y)\}$

$= \min \{f_a(x), f_a(y)\}$ for x, y ε G.

For the Converse part,

Suppose that 'f_a' be fuzzy soft set of the group G of which e is the identity element.

Now $f_a(yy^{-1}) \geq \min \{f_a(y), f_a(y)\}$ or

$f_a(e) \geq f_a(y)$.

Now $f_a(ey^{-1}) \geq \min \{f_a(e), f_a(y)\}$ or

$f_a(y^{-1}) \geq f_a(y,e)$

also,

$f_a(xy) \geq \min \{f_a(x), f_a(y)\} \geq \min\{f_a(x), f_a(y)\}$.

Proposition 3.7: Let G and G^1 be two groups and $\theta : G \rightarrow G^1$ be a soft homomorphism. If f_b is fuzzy soft group of G then the pre-image $\theta^{-1}(f_b)$ is fuzzy soft group of G.

Proof: Assume that f_b is fuzzy soft group of G^1.

Let x, y ε G.

(FSG1) $\mu_{\theta^{-1}(f_b)}(xy)$

$\quad = \mu_{f_b} (\theta\theta(xy))$

$\quad = \mu_{f_b} (\theta(x).\theta(y))$

$\quad \geq \min \{ \mu_{f_b} (\theta(x), \mu_{f_b} (\theta(y)\}$

$\quad = \min \{ \mu_{\theta^{-1}f_b} (x), \mu_{\theta^{-1}f_b} (y)\}$

(FSG2) $\mu_{\theta^{-1}f_b} (x^{-1})$

$\quad = \mu_{f_b} (\theta(x^{-1}))$

$\quad = \mu_{f_b} (\theta^{-1}x) = \mu_{f_b} (\theta x)$

$\quad = \mu_{\theta^{-1}f_b} (x)$

$\therefore \quad \theta^{-1}(f_b)$ is fuzzy soft group of G.

Proposition 3.8: Let $\theta: G \rightarrow G^1$ be an epimorphism and f_b be fuzzy soft set in G^1. If $\theta^{-1}(f_b)$ is fuzzy soft group of G, f_b is fuzzy soft group of G^1.

Proof: Let x, y ε G^1, then there exists a, b ε G such that $\theta(a) = x$, $\theta(b) = y$.

It follows that,

(FSG1) $\mu_{f_b} (xy) = \mu_{f_b} (\theta(a)\theta(b))$

$\quad = \mu_{f_b} (\theta(ab)$

$\quad = \mu_{\theta^{-1}f_b} (ab)$

$\quad \geq \min \{ \mu_{\theta^{-1}f_b} (a), \mu_{\theta^{-1}f_b} (b)\}$

$\quad \geq \min \{ \mu_{f_b} (\theta(a), \mu_{f_b} (\theta(b)\}$

$\quad \geq \min\{ \mu_{f_b} (x), \mu_{f_b} (y)\}$

(FSG2) $\mu_{f_b} (x^{-1}) = \mu (\theta(a^{-1})) = \mu_{f_b} (\theta(a^{-1}) = \mu_{\theta^{-1}f_b} (a^{-1})$

Definition 2.10: Let f_a be a fuzzy soft set in G. Let $\theta: G \rightarrow$ G be a map and define $f_A^{\theta} = G \rightarrow [0, 1]$ by $f_A^{\theta}(x) = f_A (\theta(x))$.

Proposition 3.9: If 'f_a' is fuzzy soft group of G and $\theta : G \rightarrow G^1$ be a soft homomorphism of G, then the fuzzy soft set $f_a^\theta = \{ x, f_a^\theta(x) / x \, \varepsilon \, G\}$ is fuzzy soft group of G.

Proof: Let x, y ε G, then

(FSG1) $f_a^\theta(xy)$ $= f_a(\theta(xy)$

$= f_a(\theta(x).\theta(y))$

$\geq \min \{ f_a(\theta(x), f_a(\theta(y)\}$

$= \min \{ f_a^\theta(x), f_a^\theta(y)\}$

(FSG2) $f_a(x^{-1})$ $= f_a(\theta(x^{-1}))$

$= f_a(\theta^{-1}(x))$

$= f_a^\theta(x)$

Thus f_a^θ is fuzzy soft group of G.

Proposition 3.10: If 'f_a' is fuzzy soft group of G. Let f_a^+ be a fuzzy soft set in G defined by $f_a^+(x) = f_a(x) + 1 - f_a(e) \, \forall \, x \, \varepsilon$ G, then f_a^+ is normal fuzzy soft group of G which contains f_a.

Proof: For any x, y ε G, it follows that

$f_a^+(x)$ $= f_a(x) + 1 - f_a(e) = 1.$

(FSG1) $f_a^+(xy)$ $= f_a(xy) + 1 - f_a(e)$

$\geq \min \{ f_a(x), f_a(y)\} + 1 - f_a(e)$

$= \min \{ f_a(x) + 1 - f_a(e), f_a(y) + 1 - f_a(e)\}$

$= \min \{f_a^+(x) f_a^+(y)\}$

(FSG1) $f_a^+(x^{-1})$ $= f_a(x^{-1}) + 1 - f_a(e)$

$= f_a(x) + 1 - f_a(e)$

$= f_a^+(x)$

Hence $f_a^+(x)$ is normal fuzzy soft group of G. Clearly $f_a \subseteq f_a^+$.

Proposition 3.11: If $\{ f_{a_i} \}_{i \, \varepsilon \, fa}$ is a family of fuzzy soft groups of G, then $\cap f_{a_i}$ is fuzzy soft group G whose $\cap f_{a_i} = \{ (x, \wedge f_{a_i} (x) / x \varepsilon G\}$, where i ε f_a.

Proof: Let x, y ε G, then for i ε f_a, it follows that

(FSG1) $\cap f_{a_i} (xy) = \wedge f_{a_i} (xy)$

$\geq \wedge \min \{ f_{a_i} (x), f_{a_i} (y)\}$

$= \min \{ (\wedge f_{a_i} (x)), (\wedge f_{a_i} (y)\}$

$= \min \{ (\cap f_{a_i})(x), (\cap f_{a_i})(y) \}$

(FSG2) Let x ε G. For i ε f_a, it gives that

$\cap f_{a_i} (x^{-1}) = \wedge f_{a_i} (x^{-1})$

$\geq \wedge f_{a_i} (x)$

$= (\cap f_{a_i})(x)$

Hence $(\cap f_{a_i})$ is fuzzy soft group of G.

Proposition 3.12: If 'f_a' is fuzzy soft group of G, then f_a^c is fuzzy soft group of G.

Proof: Let x, y ε G.

(FSG1) $f_a^c(xy) =$ $1 - f_a(xy)$

$\leq 1 - \min \{f_a(x), f_a(y)\}$

$= \max \{1 - f_a(x), 1 - f_a(y)\}$

$= \max \{ f_a^c(x), f_a^c(y)\}$

(FSG2) $f_a^c(x^{-1}) = 1 - f_a(x^{-1})$

$= 1 - f_a(x) = f_a^c(x)$

(FSG3) $f_a^c(e)$ $= 1 - f_a(e) = 1 - 1 = 0$

Proposition 3.13: Let f_a and f_b be two fuzzy soft groups of G then $f_a \cup f_b$ is fuzzy soft group of G.

Proof: Let x, y ε G.

(FSG1) $(f_a \cup f_b)(xy) = \max \{f_a(xy), f_b(xy)\}$

$\geq \max \{\min \{ f_a(x), f_a(y)\}, \min \{f_b(x), f_b(y)\}$

$\geq \min \{\max \{f_a(x), f_a(y)\}, \max \{f_b(x), f_b(y)\}$

$\geq \min \{\max \{f_a(x), f_b(x)\}, \max \{f_a(y), f_b(y)\}$

$\geq \min \{ (f_a \cup f_b)(x), (f_a \cup f_b)(y)\}$

(FSG1) $(f_a \cup f_b)(x^{-1}) = \max\{f_a(x^{-1}), f_b(x^{-1})\}$

$= \max \{ f_a(x), f_b(x)\}$

$= (f_a \cup f_b)(x)$

(FSG3) $(f_a \cup f_b)(e) = \max\{ f_a(e), f_b(e)\}$

$= \max \{1, 1\}$

$= 1$

Proposition 3.14: Let f_a and f_b be two fuzzy soft groups of G, then $f_a \times f_b$ is fuzzy soft group of G.

Proof: Let x, y ε G.

(FSG1) $(f_a \times f_b)(xy) = \min \{f_a(xy), f_b(xy)\}$

$\geq \min\{\min\{f_a(x), f_a(y)\}, \min\{f_b(x), f_b(y)\} \}$

$\geq \min\{\min \{f_a(x), f_b(x)\}, \min(f_a(y), f_b(y)\}\}$

$\geq \min \{ (f_a \times f_b)(x), (f_a \times f_b)(y)\}$

(FSG2) $(f_a \times f_b)(x^{-1}) = \min\{ f_a(x^{-1}), f_b(x^{-1})\}$

$\geq \min \{ f_a(x), f_b(x)\}$

$= (f_a \times f_b)(x)$

(FSG3) $(f_a \times f_b)(e) = \min \{ f_a(e), f_b(e)\}$

$= \min \{ 1, 1\}$

$= 1$

Proposition 3.15: Let 'f_a' be a fuzzy soft group over X, (ϕ, χ) be a fuzzy soft homomorphism from X to Y, then $(\phi,\chi) f_a$ is a fuzzy soft group over Y.

Proof: Let k ε χ_a and y_1, y_2 ε Y. If $\phi^{-1}(y_1) = 0$, $\phi^{-1}(y_2) = 0$ the proof is straight forward.

Let us assume that there exist x_1, x_2 ε X such that $\phi(x_1) = y_1$, $\phi(x_2) = y_2$.

$\phi(f_k)(y_1 y_2^{-1}) = $

$$\bigvee_{(\varphi(f) = y_1 y_2^{-1})} \bigvee_{(\chi(a) = k)} f_a(t)$$

$$\geq \bigvee_{(\chi(a) = k)} f_a(x_1, x_2^{-1})$$

$$\geq$$

$$\bigvee_{(\chi(a) = k)} \min\{f_a(x_1), f_a(x_2)\}$$

This is equality is satisfied for each

x_1, x_2 ε X, which satisfy $\phi(x_1) = y_1$,

$\phi(x_2) = y_2$. We have,

$\phi(f_k)(y_1 y_2^{-1})$

$$= \min\{ \bigvee_{(\varphi(f_1)=y_1)} \bigvee_{(\chi(a)=k)} f_a(t_1), \bigvee_{(\varphi(f_2)=y_2)} \bigvee_{(\chi(a)=k)} f_a(t2)\}$$

$= \min \{ \phi(f)_k(y_1). \phi(f)_k(y_2)\}$

Proposition 3.16: Let g_b be a fuzzy soft group over Y and (ϕ, χ) be a fuzzy soft homomorphism from X to Y, then $(\phi,\chi)^{-1} g_b$ is a fuzzy soft group over X.

Proof: Let a ε χ^{-1}_b and x_1, x_2 ε X.

$\phi^{-1}(g)_a(x_1.x_2^{-1}) = g_{\chi(a)}(\phi x_1 x_2^{-1})$

$= g_{\chi(a)} \{ \phi(x_1).\phi(x_2)\}$

$\geq \min \{ g_{\chi(a)}(\phi(x_1)) . g_{\chi(a)}((x_1))\}$

$= \min \{ \phi^{-1}(g)_a(x_1), \phi^{-1}(g)_a(x_2)\}$

Proposition 3.17: Let 'f_a' be a fuzzy soft set over X, then 'f_a' is a fuzzy soft group over X iff $\forall$ a ε A and for arbitrary α ε [0, 1] with $(fa)_\alpha \neq 0$, the α- level soft set $(f_a)_\alpha$ is a fuzzy soft group over X.

Proof: Let 'f_a' be a fuzzy soft group over X, then for each a ε A, f_a is a fuzzy subgroup of X. For arbitrary α ε [0, 1] with $(fa)_\alpha \neq 0$.

Let x, y ε $(f_a)_\alpha$, then $f_a(x) \geq \alpha$ and $f_a(y) \geq \alpha$.

$\therefore$(FSG1) $_a(xy) \geq \min \{f_a(x), f_a(y)\}$

$\geq \min \{\alpha, \alpha\} = \alpha.$

Hence xy ε $(f_a)\alpha$.

Furthermore,

(FSG2) $f_a(x^{-1}) \geq f_a(x) \geq \alpha$, then $x^{-1} \, \varepsilon \, (f_a)_\alpha$.

We obtain that $(f_a)_\alpha$ is a subgroup of X, $\forall$ a ε A, there exists a ε A such that f_a is not a fuzzy sub group of X, then there exist x_0, y_0 ε X such that, $f_a(x_0 y_0^{-1}) \leq \min\{f_a(x_0), f_a(y_0^{-1})\}$.

Let $f_a(x_0 y_0^{-1}) = \lambda$, $f_a(x_0) = \beta$ and $f_a(y_0) = \delta$.

We have, $\lambda < \min\{\beta, \delta\}$.

Let $\alpha = \dfrac{\lambda + \min\{\beta, \delta\}}{2}$, $\lambda < \alpha < \min\{\beta, \delta\}$.

Now we have, $f_a(x_0 y_0^{-1}) = \alpha < \alpha$. Hence x_0, $y_0^{-1} \notin (f_a)_\alpha$. But since $\beta > \min\{\beta, \delta\}$ and $\delta > \min\{\beta, \delta\} > \alpha$. We obtain $f_a(x_0) > \alpha$ and $f_a(y_0) > \alpha$. (i.e) x_0, y_0 ε $(f_a)_\alpha$. This contradicts with the fact that $(f_a)_\alpha$ is fuzzy soft group over X.

Definition 2.11: Let $\phi : X \to Y$ and $\chi : A \to B$ be two functions, where A and B are parameter sets for the crisp sets X and Y respectively, then the pair (ϕ, χ) is called a fuzzy soft function from X to Y.

Proposition 3.18: If $f_{a_1}, f_{a_2}, \ldots, f_{a_n}$ be fuzzy soft groups of the groups χ_1, χ_2, .. χ_n respectively, then $f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi$ is fuzzy soft groups of $\chi_1 \times \chi_2 \times \ldots \times \chi_n$.

Proof: For all elements $(x_1 x_2 \ldots x_n)$, $(y_1 y_2 \ldots y_n)$ ε $\chi_1 \times \chi_2 \times \ldots \times \chi_n$.

(FSG1) $(f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)((x_1 x_2 \ldots x_n), (y_1 y_2 \ldots y_n))$

$= (f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)(x_1 y_1, x_2 y_2, \ldots x_n y_n)$

$\geq \min\{f_{a_1}(x_1 y_1), f_{a_2}(x_2 y_2), \ldots f_{a_n}(x_n y_n)\}$

$\geq \min\{\min\{$

$f_{a_1}(x_1) f_{a_1}(y_1)\}, \min\{f_{a_2}(x_2) f_{a_2}(y_2)\}.. \min\{f_{a_n}(x_n), f_{a_n}(y_n)\}$

$\geq \min\{\min\{$

$f_{a_1}(x_1) f_{a_2}(x_2) \cdots f_{a_n}(x_n)\}, \min\{f_{a_1}(y_1)\} f_{a_2}(y_2).. f_{a_n}(y_n)\}\}$

$\geq \min\{ ((f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)((x_1 x_2 \ldots x_n), (f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)(y_1 y_2 \ldots y_n)\}$

(FSG2) $(f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)(x_1, x_2, \ldots x_n)^{-1}$

$= (f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)((x_1^{-1}, x_2^{-1}, \ldots x_n^{-1})$

$= \min\{ f_{a_1}(x_1^{-1}) f_{a_2}(x_2^{-1}) \ldots f_{a_n}(x_n^{-1})\}$

$\geq \min\{ f_{a_1}(x_1) f_{a_2}(x_2) \ldots f_{a_n}(x_n)\}$

$\geq (f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)(x_1 x_2 \ldots x_n)$

(FSG3) $(f_{a_1} \chi f_{a_2} \chi \ldots f_{a_n} \chi)(e_1, e_2, \ldots e_n)$

$= \min\{ f_{a_1}(e_1) f_{a_2}(e_2) \ldots f_{a_n}(e_n)\}$

$= \min\{1, 1, \ldots\ldots 1\}$

$= 1$

4. CONCLUSION

In this paper we studied the algebraic properties of fuzzy soft sets in group structures. This work focused on fuzzy soft groups, homomorphism of fuzzy soft groups and pre-image of fuzzy soft groups. To extend this work one could study the properties of fuzzy soft sets in other algebraic structures such as rings, nor-rings, field and ideals.

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