Performance Analysis of InterpolatedShrink Method in Image De-Noising

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ABSTRACT
The de-noising of an image corrupted by Gaussian noise is a classical problem in signal or image processing. An image is often corrupted by noise during its acquisition and transmission. Image de-noising is used to reduce the noise while retaining the important features in the image. Always there exists a tradeoff between the removed noise and the blurring in the image. The use of wavelet transform for signal de-noising has emerged as an important technique during the last decade. The wavelet transform is preferred over conventional Fast Fourier Transform(FFT) based image de-noising technique ,because of its capability to give detailed spatial-frequency information. In this paper, we tried to analyze the performance of InterpolatedShrink method in image de-noising using various wavelet family, such as Haar,Doubechies,Symlet and Coiflets, for Gaussian noise.

General Terms
Computer Science- Image Processing.

Keywords
De-noising, Thresholding, Discrete Wavelet Transform, Gaussian noise, IntepolatedShrink

1. INTRODUCTION
Image de-noising is a problem of prime importance in image processing field, ranging from medical imaging to satellite imaging. With rapid growth in digital technology, the Engineers and scientists are gathering data and analyzing it at an ever increasing pace. Generally the data collected by sensors is corrupted by noise, either as a result of data acquisition process or due to interfering natural phenomena or it may be due to transmission errors. Before analyzing the image or data, the noise must be removed. In image de-noising, the aim is to suppress noise as much as possible while retaining the important image features. Image de-noising still remains a challenge for researchers since a tradeoff between the removed noise and the blurring in the image always exists.

In recent years, lot of research has been done on image de-noising[1-3], Basically there are two approaches to image de-noising, spatial filtering method and transform domain filtering method [4]. A conventional way to remove noise from a noisy image is to use the spatial filter. Spatial filters are also classified into linear and non-linear filters. The spatial filters usually smooth the image to reduce the noise, but, in the process also blur the image. To overcome the weakness of the spatial filtering, several new techniques have been developed that improve on spatial filters by removing the noise more effectively while preserving the image features. A different class of techniques exploit the decomposition of image into the wavelet basis and de-noise the image by shrinking the wavelet coefficients[5-7].

This paper is organized as follows. Section II, gives brief review of image de-noising method by Interpolatedshrink. The Experimental results and discussions are presented in section III and finally, we give conclusion in Section IV.

2. REVIEW OF IMAGE DENOISING METHOD BY INTERPOLATED SHRINK

Wavelet based image de-noising methods have been very effective because of their ability to capture the energy of a signal in few energy transform values. The multi-resolution analysis performed by the wavelet transform is one of the powerful tools. In the wavelet domain, thresholding is used to separate the information from noise. Donoho and Johnstone [8,9]have developed several wavelet de-noising methods by thresholding the wavelet coefficients arising from the standard discrete wavelet transform.

An image $B(i,j)$ corrupted by Gaussian noise $Z(i,j)$ can be represented as $B(i,j) = A(i,j) + \sigma Z(i,j)$, where $Z(i,j)$ is noise free image and σ the standard deviation of noise.

The Donoho's wavelet based de-noising scheme can be summarized as follows:

1. Transform the noisy image $B(i,j)$ into an orthogonal domain by 2D Discrete Wavelet Transform (DWT).

2. Apply soft or hard thresholding to the resulting wavelet coefficients by the threshold $\lambda = \sigma\sqrt{2\log n^2}$.

3. Perform inverse 2D discrete wavelet transform to obtain the de-noised image.

The DWT of image is a non redundant image representation with better spatial and spectral localization of image formation, compared to other multi-scale representations such as Gaussian and Laplacian pyramid. An image can be decomposed into a sequence of different spatial resolution images using DWT. The method is also called decimated wavelet transform, since it decimates the signal into sub bands. With DWT an image can be decomposed more than once. Decomposition can be continued until the desired level is reached[10]. The Gaussian noise will nearly average out in low frequency wavelet coefficients and only detail wavelet coefficients need to be thresholded. A different class of methods exploits the decomposition of the image into wavelet basis and shrinks the wavelet coefficients to de-noise the image [11, 12]. The shrinkage rule defines, how we apply the threshold. Here threshold plays important role in de-noising process. Finding an optimum threshold is a tedious process because a small threshold value will retain the noisy coefficients, where as a large threshold value leads to the loss of coefficients that carry image signal details. There are two main approaches, hard thresholding, is the simplest method and is a keep or kill method whereas soft thresholding has nice mathematical properties and it shrinks the coefficients above the threshold in absolute value. It is a shrink or kill method.

The hard threshold signal

X is X if $|X| > thr$

X is 0 if $|X| < thr$

The soft threshold signal

X is $(|X| - thr)$ if $|X| > thr$

X is 0 otherwise

The hard thresholding deletes all coefficients that are smaller than the threshold, and keeps the others unchanged. On the other hand soft thresholding also deletes the coefficients under the threshold, but scales the ones that are left. Hard thresholding creates discontinuities in the reconstructed signal, while soft does not. The BayesShrink, VisuShrink and SureShrink along with NormalShrink[13],are well known methods based on wavelet transform. These approaches have been proposed, by considering the influence of other wavelet coefficients on the current wavelet coefficient, to be thresholded. The motivation of this idea is that a large wavelet coefficient will probably have large wavelet coefficients at its neighbors. This is because even when coefficients are uncorrelated and close, there are still significant higher order neighbor correlations, like a strong positive covariance in amplitude between neighbor coefficients. Cai and Silverman [14] proposed a method that takes the immediate neighbor coefficients into account for 1D signal. Here $d_{j,k}$ is the set of wavelet coefficients of signal corrupted with noise and if

$$S^2{}_{j,k} = d^2{}_{j,k-1} + d^2{}_{j,k} + d^2{}_{j,k+1} \leq \lambda^2, \quad (1)$$

then, wavelet coefficients $d_{j,k}$ are set to zero, otherwise shrink it using

$$d_{j,k} = d_{j,k}\left(1 - \frac{\lambda^2}{S^2{}_{j,k}}\right), \quad (2)$$

where $\lambda = \sigma\sqrt{2log\mathrm{n}^2}$ and n is length of the signal. 2D wavelet transform is performed for image de-noising. At every decomposition level, four frequency sub bands are created. This process continues until the required level is reached. Some more methods have been proposed based on statistical modeling of wavelet coefficients [15,16]. By extending the idea of Cai and Silverman for 2D image case, Chen et al.[17] proposed new method namely NeighShrink which thresholds the wavelet coefficients according to the sum of the squares of all the wavelet coefficients within a neighborhood window.

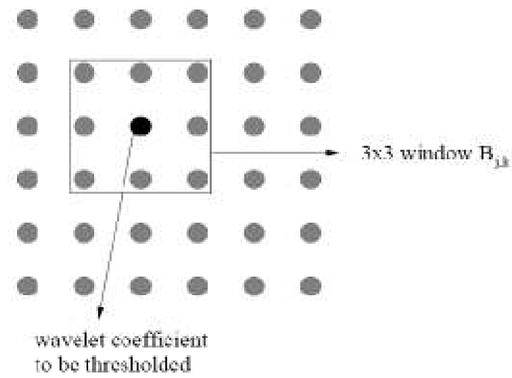

Figure 1. Neighborhood window with size 3×3.

In the new approach called InterpolatedShrink [18], for each noisy wavelet coefficient W_{ij} to be shrinked, it incorporates a square neighboring window B_{ij} centered at it. The neighboring window size can be represented as LxL, where L is a positive odd number. Figure 1, shows a 3×3. neighboring window centered at the wavelet coefficient to be shrinked. The key point is that the interpolated value $M^2{}_{i,l}$ of the neighborhood is determined by sorting the window coefficients. The window coefficients are replaced by the new interpolated value. In this method, there is no abrupt change in the neighboring window coefficients around the current wavelet coefficients, which in turn help in determining the threshold value. Also, even if there is an abrupt change at the edges, the new value will help in preserving the edge information. The wavelet coefficients are then thresholded, according to sum of squares of all the wavelet coefficients $S^2{}_{j,k}$ within a neighborhood window. The interpolated median value is slightly greater than median value of the sorted window coefficients, which is more appropriate in determining the threshold value and preservation of edge information. The sum is calculated using the new coefficients in the formula

$$S^2{}_{j,k} = \sum_{(i,l)\epsilon B_{j,k}} M^2{}_{i,l} \quad (3)$$

and different wavelet coefficient sub bands are thresholded. Then Shrink it according to the following formula:

$$d_{j,k} = d_{j,k}B_{j,k}, \quad (4)$$

with $B_{j,k} = \left(1 - \frac{\lambda^2}{S^2{}_{j,k}}\right)+$. The + sign in the formula means it takes only positive value, and $\lambda = \sigma\sqrt{2\mathrm{logn}^2}$ is the threshold value.

3. RESULTS AND DISCUSSIONS

In this section, performance of InterpolatedShrink method in image de-noising is analyzed experimentally using various wavelet family, such as Haar, Doubechies, Symlet and Coiflets. The experiments are conducted on some natural grayscale test images like Lena and Barbera of size 512*512 at different Gaussian noise levels ,Standard deviation σ is varied from 5 to 50 in steps of 5 and results are tabulated. In Table 1, we present consolidated results of de-noising values for Lena and Barbara Images. Figure 2 and 3, shows some de-noised image results for lena and Barbara with Gaussion noise **(** σ=25 **)**. The peak signal-to-noise ratio (PSNR) in decibels (dB), is defined as as

$$PSNR = 10 \times \log \frac{255^2}{MSE}(dB) \qquad (5)$$

with

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left(I(i,j) - K(i,j)\right)^2 \qquad (6)$$

where I and K being the original image and de-noised image, respectively.

Also from the Table 1, it is observed that for the Lena image, The wavelet Db6 gives better performance at low noise levels (for σ = 5,10 and 15),where as Db2 gives better results for higher noise level (for σ =20,25,30,35,45 and 50) . In case of Barbara image, Coif4 gives the best results at low noise levels(for σ =5,10,15 and 20) and Db6 gives intermediate noise levels (for σ = 25,30,35 and40) and Db2 at low noise levels(for σ = 45 and 50).

Table 1. Image de-noising results for different levels of Gaussian noise added images using various wavelet filters

Noise std. deviation	**Lena Image**							
σ	**Haar**	**Db2**	**Db4**	**Db6**	**Db8**	**Db10**	**Sym2**	**Sym4**
σ=05	36.5359	36.6788	37.7811	**37.8287**	37.3746	37.3736	36.6788	37.1246
σ=10	32.1259	32.0735	34.3561	**34.4222**	33.5091	33.0330	32.0735	33.0080
σ=15	29.4913	29.3892	32.3257	**32.3699**	31.0748	30.3248	29.3892	30.5676
σ=20	27.5953	27.4839	**30.8621**	30.8301	29.2590	28.3059	27.4839	28.6982
σ=25	26.1078	25.8766	**29.6968**	29.6348	27.7989	26.6499	25.8766	27.2006
σ=30	24.8748	24.5254	**28.7272**	28.6404	26.5736	25.2853	24.5354	25.8813
σ=35	23.8094	23.3830	**27.9023**	27.7845	25.5232	24.1099	23.3830	24.7898
σ=40	22.8745	23.7200	**27.1693**	27.0364	24.5965	23.0804	23.7200	23.8335
σ=45	22.0354	22.8711	**26.5270**	26.2762	23.7080	22.1642	22.8711	22.9690
σ=50	21.2765	21.1015	**25.9455**	25.6583	23.0198	21.3716	22.1015	22.1925

Noise std. deviation	**Lena Image**							
σ	**Sym6**	**Sym8**	**Sym10**	**Coif1**	**Coif2**	**Coif3**	**Coif4**	**Coif5**
σ=05	37.6852	37.6483	37.3012	37.4442	37.5556	37.7628	37.6240	37.6999
σ=10	34.0559	34.1390	33.08887	33.9001	33.9352	34.2600	33.9499	33.9707
σ=15	31.8230	32.0160	30.4080	31.8973	31.7083	32.1025	31.6687	31.4166
σ=20	30.1571	30.4020	28.4150	30.4420	30.0410	30.4928	29.9623	29.3772
σ=25	28.8037	29.0947	26.8573	29.2659	28.6987	29.1952	28.5641	27.6984
σ=30	27.6613	28.5701	25.5098	28.3052	27.5657	28.0990	25.7734	26.2632
σ=35	26.6698	27.0239	24.3661	27.4815	26.7882	27.1437	24.6264	24.9793
σ=40	23.7864	26.1708	23.3463	26.7488	25.9292	26.2994	23.9171	23.9624
σ=45	24.9960	25.4067	22.4382	26.0905	25.1750	25.5439	22.7166	22.8959
σ=50	24.2776	24.7128	21.6187	25.4948	24.4715	24.8542	21.9017	22.0895

Noise std. deviation	Barbara Image							
σ	**Haar**	**Db2**	**Db4**	**Db6**	**Db8**	**Db10**	**Sym2**	**Sym4**
σ=05	35.5937	35.9649	36.6817	36.7854	36.1820	36.5012	35.9649	36.0982
σ=10	30.5467	30.9830	32.4390	32.6656	31.7224	31.8556	30.9830	31.5748
σ=15	27.6790	28.7239	30.0730	30.2998	29.0038	29.7587	28.7239	28.7572
σ=20	25.7045	26.8738	28.4213	28.6233	27.1403	26.9776	26.8738	26.6543
σ=25	24.1695	25.4280	27.1128	**27.3660**	25.7838	25.3910	25.4280	25.2193
σ=30	22.8989	24.2785	26.0661	**26.3411**	24.5913	24.0627	24.2785	24.369
σ=35	21.9422	23.2905	25.4368	**25.5096**	23.5770	22.9514	23.2905	23.0463
σ=40	21.0371	22-4327	24.7521	**24.7582**	22.7082	21.9592	22.4327	22.1823
σ=45	20.2179	21.6718	**24.1588**	24.1387	21.9370	21.1028	21.6718	21.4217
σ=50	19.6066	20.9765	**23.6396**	23.5939	21.2453	20.3064	20.9765	20.7464

Noise std. deviation	Barbara Image							
σ	**Sym6**	**Sym8**	**Sym10**	**Coif1**	**Coif2**	**Coif3**	**Coif4**	**Coif5**
σ=05	36.7450	36.8413	36.6206	36.2800	36.7313	**36.8806**	36.2196	36.4271
σ=10	32.1742	32.4702	31.8477	31.7184	32.3376	**32.5538**	31.4448	31.7813
σ=15	29.7038	29.8850	28.9977	29.3952	29.8746	**30.3261**	28.8384	29.1274
σ=20	27.9540	28.0388	26.9610	27.7542	28.1816	**28.4894**	26.9954	27.2612
σ=25	26.6373	26.6071	25.4081	26.5085	26.8592	27.1015	25.8046	25.6446
σ=30	25.6059	25.5006	24.1290	25.8819	25.7209	26.0258	25.9802	24.4432
σ=35	24.8637	24.5868	23.0251	25.1284	24.8860	25.1178	24.8302	23.3843
σ=40	24.2317	23.8104	22.0747	24.4816	24.1559	24.3297	23.9774	22.4240
σ=45	23.3262	23.1420	21.2241	23.7182	23.4942	23.6365	23.2477	21.5757
σ=50	22.6672	22.5629	20.4604	23.1957	22.9007	23.0167	22.5563	20.7849

Coif1-29.2659(db) Coif2-28.6987 (db) Coif3-29.1952(db) Coif4 -28.5641 db) Coif5-27.6984(db)

Db2- 25.8766(db) Db4 - 29.6968 (db) Db6- 29.6348 (db) Db8-27.7989 (db) Db10-26.6499(db)

Sym2- 25.8766(db) Sym4 - 27.2006(db) Sym6-28.8037 (db) Sym8 -29.0947(db) Sym10- 26.8573(db)

Figure 2. Denoising results for Lena image with Gaussian noise (σ=25)

Coif1 -26.5085(db) Coif2-26.8592(db) Coif3-27.1015(db) Coif4-25.8046(db) Coif5-25.6446 (db)

Db2 - 25.4280(db) Db4 - 27.1128 (db) Db6- 27.3660 (db) Db8-25.7838(db) Db10 - 25.3910(db)

Sym2- 25.4280(db) Sym4 - 25.2193 (db) Sym6 - 26.6373(db) Sym8-26.6071 (db) Sym10- 25.4081(db)

Fig 3. Denoising results for Barbara image with Gaussian noise (σ=25)

4. CONCLUSION

In This paper,The performance analysis of InterpolatedShrink method for image de-noising is evaluated. The InterpolatedShrink method is applied for de-noising images with Gaussian Noise,using various wavelet families, such as Haar, Doubechies, Symlet and Coiflets. The experimental results for different wavelets show that, the highest PSNR(dB) varies with varying noise levels. In general, InterpolatedShrink method combined with wavelet filter Db4 produces better results for removing Gaussian noise at higher levels.

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