Performance Analysis of M*N Equalizer based Minimum Mean Square Error (MMSE) Receiver for MIMO Wireless Channel

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ABSTRACT

The effect of fading and interference effects can be combated with equalizer. This paper analyses the performance of MMSE equalizer based receiver for MIMO wireless channel .The BER characteristics for the various transmitting and receiving antenna is simulated in mat lab tool box and many advantages and disadvantages the system is described. The simulation carried out signal processing lab show that the MMSE equalizer based receiver is a good choice for removing some ISI and minimizes the total noise power. The results show that the BER decreases as the m x n antenna configurations is increased.

General Terms

Equalizer, Bit error rate, Signal to noise ratio (eb/N0),transmitting antenna, receiving antenna

Keywords

MIMO (Multiple Input Multiple output), MMSE(Minimum Mearn Square Error) ,ISI(Inter Symbol Interference), ,SNR (Signal to Noise Ratio)

1. INTRODUCTION

Wireless communication technology has shown that when multiple antennas at both transmitter and receiver are employed it provides the possibility of higher data rates compared to single antenna systems [1] [2]. The system with multiple antennas at the transmitter and receiver is commonly referred is to as multiple input multiple output (MIMO) systems. The multiple antennas are thus used to increase data rates through multiplexing or to improve performance through diversity. This method offers higher capacity to wireless systems and the capacity increases linearly with the number of antennas and link range with out additional bandwidth and power requirements. MIMO channel model [4][5] is depicted in Figure 1 with M transmitter and N receiver antennas. It can be achieved by higher spectral efficiency and link reliability or diversity (reduced fading).

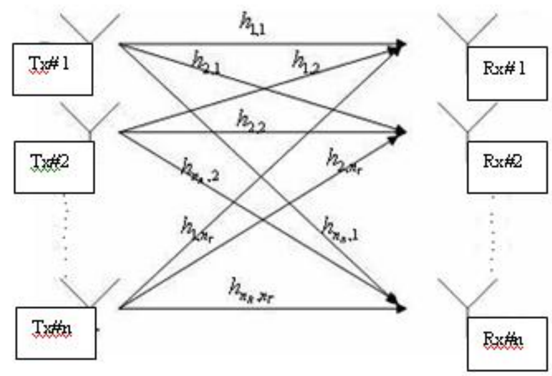

Figure 1 MIMO Model.

2. MMSE EQUALIZER

In MIMO wireless communication, an equalizer is employed which is a network that makes an attempt to recover a signal that has suffers with an Inter symbol Interference(ISI) and proves the BER characteristics and maintains a good SNR.

A Minimum Mean Square Error (MMSE) estimator is a method in which it minimizes the mean square error (MSE), which is a common measure of estimator quality. Minimum mean-square error equalizer, which does not usually eliminate ISI completely but instead, minimizes the total power of the noise and ISI components in the output.

2.1 Definition

Let X be an unknown random variable, and let Y be a known random variable. An estimator $\hat{X}(y)$ (X cap) is any function of the measurement Y, and its Mean square error is given mathematically given by equation 1

$$MSE = E\left\{(\hat{X} - X^2)\right\} \qquad 1$$

where the expectation is taken over both X and Y.

The MMSE estimator is then defined as the estimator achieving minimal MSE. Generally, it very difficult to determine a closed form for the MMSE estimator. In these cases, one possibility is to seek the technique minimizing the MSE within a particular class, such as the class of linear estimators. The linear MMSE estimator is the estimator achieving minimum MSE among all estimators of the form *AY* + *b*. If the measurement *Y* is a random vector, *A* is a matrix and *b* is a vector.

2.2 Minimum Mean Square Error (MMSE) equalizer for 2×2 MIMO channel:

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = [h_{1,1} \quad h_{1,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \tag{2}$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = [h_{2,1} \quad h_{2,2}]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2 \tag{3}$$

Equivalently the above equation is rewritten as

$$y = Hx + n \tag{4}$$

Where ,

Y= received symbol is the channel matrix, x is the input symbol,

'n' is the noise getting added.

The equation can be represented in matrix notation as follows:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{5}$$

The Minimum Mean Square Error **(**MMSE**)** approach tries to find a coefficient W which minimizes the criterion, ,

$$E\left\{\left[W_{y-x}\right]\left[W_{y-x}\right]^H\right\}$$

.To solve for x, we need to find a matrix W which satisfies $WH = I$.The Minimum Mean Square Error (MMSE) detector for meeting this constraint is given by,

$$W = [H^H H + N_0 I]^{-1} H^H \tag{6}$$

Where W - Equalization Matrix and

H - Channel Matrix

This matrix is known as the pseudo inverse for a general m x n matrix

Where

$$H^H H = \begin{bmatrix} h_{1,1}^* & h_{2,1}^* \\ h_{1,2}^* & h_{2,2}^* \end{bmatrix}\begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} = \begin{bmatrix} |h_{1,1}|^2 + |h_{2,1}|^2 & h_{1,1}^* h_{1,2} + h_{2,1}^* h_{2,2} \\ h_{1,2}^* h_{1,1} + h_{2,2}^* h_{2,1} & |h_{1,2}|^2 + |h_{2,2}|^2 \end{bmatrix} \tag{7}$$

In fact, when the noise term is zero, the MMSE equalizer reduces to Zero Forcing equalizer. this model can be extended to m x n antenna configuration

2.3 Estimation of minimum mean square error:

An example can be shown by using a linear combination of random variable estimates X_1,X_2, X_3 to estimate another random variable X_4 using X_4ˆ If the random variables *X* = [X_1,X_2,X_3,X_4] are real Gaussian random variables with zero mean and covariance matrix given by

$$\text{Cov}(x) = E\,[XX^T] \tag{8}$$

we will estimate the vector X_4 and find coefficients a_i such that the estimate

$$\hat{X}_4 = \sum_{i=1}^{3} a_i X_i \tag{9}$$

is an optimal estimate of X_4. We will use the autocorrelation matrix, R, and the cross correlation matrix, C, to find vector A, which consists of the coefficient values that will minimize the estimate. The autocorrelation matrix *R* is easily determined

3. SIMULATION RESULTS AND DISCUSSIONS

The simulations were carried out at signal processing lab .Now let us consider the simulation analysis of MMSE equalizer for 2 x n antenna configurations which means keeping the transmitter antenna as two and vary the number of antennas in the receiver side. From the mat lab figure window shown in figure 2 its evident that the BER decreases as the a receiver antenna increases for the zero forcing equalizer. For a better clarity the data from the figure window is taken and plotted in the form of bar chart as shown in figure no 3.

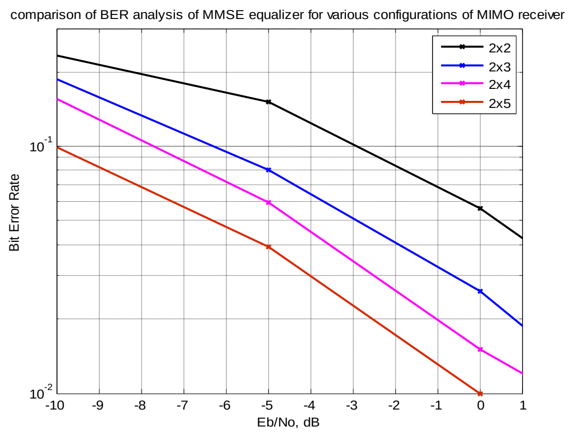

Figure no :2:BER characteristics for a 2xn MIMO system

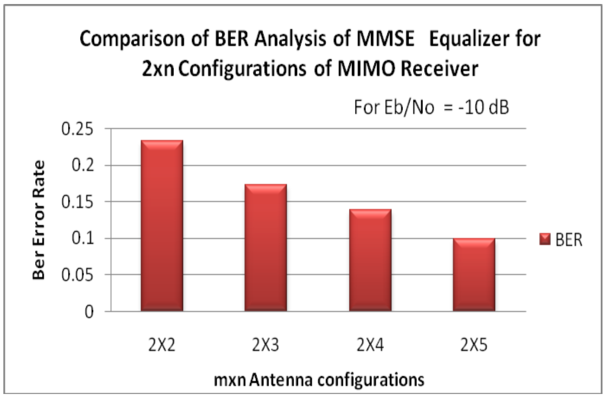

Figure No 3. Bar chat showing the comparison of BER in 2 x n MIMO

Similarly now the antenna configuration is varied namely 3x n Figure 4 and 5 show that in this configuration as the no of receivers (n) is increased keeping the no of transmitters (m=3) as constant, the Bit Error Rate (BER) decreases in MMSE Equalizer with increase in receivers.

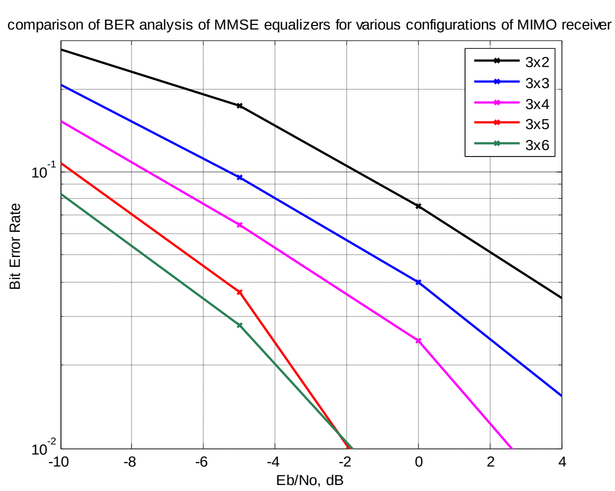

Figure no :4 :BER characteristics for a 2xn MIMO system

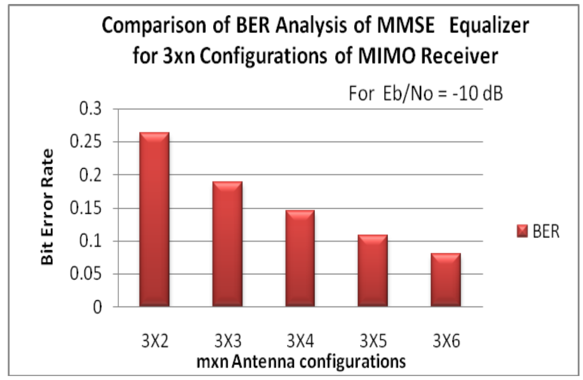

Figure no: 4: BER characteristics for a 3 x n MIMO system

Similarly now the antenna configuration is varied namely 4x n Figure 6 and 7 show that in this configuration as the no of receivers (n) is increased keeping the no of transmitters (m=4) as constant ,the Bit Error Rate (BER) decreases in MMSE Equalizer.

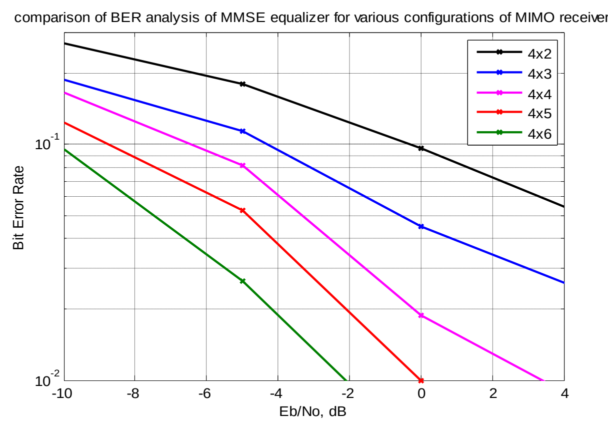

Figure no: 6: BER characteristics for a 4 x n MIMO system

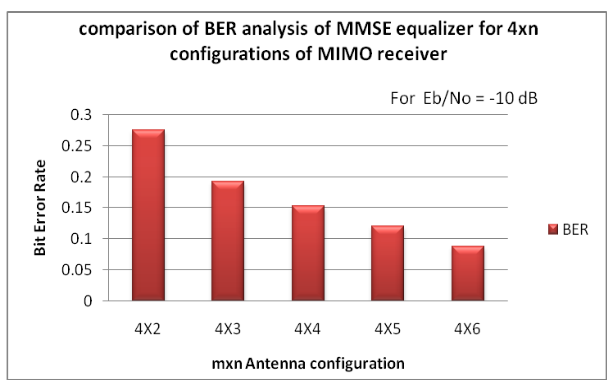

Figure No 7. Bar chat showing the comparison of BER in 4 x n MIMO.

It is observed that when the number of transmitter is kept constants, and the receivers are varied the BER decreases with increase in the numbers of receivers. Figure 8 shows the consolidated Bar Graph, in which the no of transmitters (m=2, 3 and 4) is kept constant and receiver configuration is increased .

It is evident that the Bit Error Rate (BER) decreases in MMSE Equalizer**.**

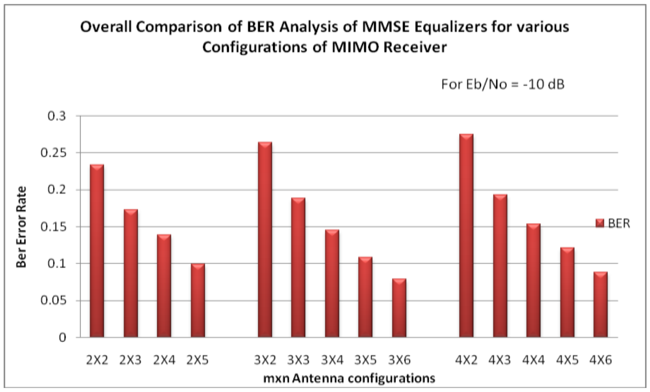

Figure No 8. Comparison of BER characteristics with fixed number of transmitter and variable receiver for MIMO wireless communication-MMSE equalizer

4. ACKNOWLEDGMENTS

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5. CONCLUSION

This paper is a simulation study on the performance analysis of M x N Equalizer based MMSE receiver for MIMO wireless channel. Hence a more balanced linear equalizer is the Minimum mean square error equalizer, which however is not eliminate ISI completely but instead minimizes the total power of the noise and ISI components in the output. From the simulation results it is evident that the BER decreases as the number of receiving antenna increases with respect to number of transmitting antenna in MMSE equalizer based MIMO receiver.

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