

Fuzzy b- Compact and Fuzzy b-closed Spaces

S. S. Benchalli
Professor

Department of Mathematics
Karnatak University, Dharwad
Karnataka State, India

Jenifer J.Karnel
Senior Lecturer

Department of Mathematics
S.D.M College of Engineering
and Technology, Dharwad
Karnataka State, India

ABSTRACT

A new form of fuzzy compact spaces namely fuzzy b-compact spaces, b-closed spaces and fbg-compact spaces with the concept of fuzzy b-open set are introduced. Some characterization on their properties are obtained. The invariance of fuzzy b-compact spaces, b-closed spaces and fbg-compact spaces under fuzzy mapping and their hereditary property are also investigated.

Keywords

Fuzzy b-Compactness, fuzzy b-closed spaces, fbg-compact spaces and fuzzy topological space.

1. INTRODUCTION

After Zadeh [7] introduced the concept of a fuzzy subset, Chang [4] used it to define fuzzy topological space. There after, several concepts of general topology have been extended to fuzzy topology and compactness is one such concept. Compactness for fuzzy topological spaces was first introduced by Chang [4]. The concept of b-open sets in fuzzy settings was introduced by S.S.Benchalli and Jenifer [1]. The purpose of this paper is to introduce fuzzy b-compact, fuzzy b-closed spaces and fbg-compact spaces using fuzzy filterbases. Some characterization, hereditary property, invariance under mapping for these spaces are investigated.

2. PRELIMINARIES

Throughout this paper X and Y mean fuzzy topological spaces (fts, for short). The notations $Cl(A)$, $Int(A)$ and \bar{A} will stand respectively for the fuzzy closure, fuzzy interior and complement of a fuzzy set A in a fts X . The support of a fuzzy set A in X will be denoted by $S(A)$ (i.e $S(A) = \{x \in X : A(x) \neq 0\}$). A fuzzy point x_t [6] in X is a fuzzy set having support $x \in X$ and value $t \in (0, 1]$. The fuzzy sets in X taking on respectively the constant value 0 and 1 are denoted by 0_X and 1_X respectively. For two fuzzy sets A and B in X , we write $A \leq B$ if $A(x) \leq B(x)$ for each $x \in X$.

2.1 Definition A fuzzy set A in a fts X is said to be fuzzy b-open (b-closed) [3] set iff $A \leq (IntCl(A) \vee (ClInt(A)))$, ($(IntCl(A) \wedge (ClInt(A))) \leq A$).

2.2 Definition A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be (a) fuzzy b*-continuous [2] if $f^{-1}(A)$ is fuzzy b-open set in X , for each fuzzy b-open set A in Y .

(b) fuzzy b*-open [2] if for every fuzzy b-open set A in X , $f(A)$ is fuzzy b-open set in Y .

2.3 Definition [1] Let A be a fuzzy set in a fts X . Then its b-closure and b-interior are denoted and defined by

- (i) $bCl(A) = \bigwedge \{B : B \text{ is a fuzzy b-closed set of } X \text{ and } B \geq A\}$.
- (ii) $bInt(A) = \bigvee \{C : C \text{ is a fuzzy b-open set of } X \text{ and } A \geq C\}$.

2.4 Definition [3] A fuzzy set A in a fts X is called fuzzy generalized b-closed (briefly fbg – closed) fuzzy set if $bCl(A) \leq B$ whenever $A \leq B$ and B is fuzzy b-open in (X, τ) .

2.5 Definition A fuzzy set A is quasi-coincident [6], with a fuzzy set B denoted by $A q B$ iff there exist $x \in X$ such that $A(x) + B(x) > 1$. If A and B are not quasi-coincident then we write $A \bar{q} B$. $A \leq B \Leftrightarrow A \bar{q} (1-B)$.

2.6 Definition A collection of fuzzy subsets \mathcal{D} of a fts X is said to form a fuzzy filterbase [5], iff for every finite collection

$$\{A_\lambda : \lambda = 1, \dots, n\}, \bigwedge_{\lambda=1}^n A_\lambda \neq 0_X.$$

2.7 Definition A collection μ of fuzzy sets in a fts X is said to be a cover [5] of a fuzzy set u of X iff $(\bigvee_{A \in \mu} A)(x) = 1_X$,

for every $x \in S(u)$. A fuzzy cover of a fuzzy set u in a fts X is said to have a finite subcover iff there exists a finite subcollection $\eta = \{A_1, \dots, A_n\}$ of μ such that $(\bigvee_{\lambda=1}^n A_\lambda)(x) \geq u(x)$, for every $x \in S(u)$.

3. FUZZY b-COMPACT SPACES

3.1 Definition A fts X is said to be fuzzy b-compact iff for every family μ of fuzzy b-open fuzzy sets such that $(\bigvee_{A \in \mu} A) = 1_X$ there is a finite subfamily $\delta \subseteq \mu$ such that $(\bigvee_{A \in \delta} A) = 1_X$ for every $x \in S(u)$.

3.2 Definition A fuzzy set U in a fts X is said to be fuzzy b-compact relative to X iff for every family μ of fuzzy b-open sets such that $(\bigvee_{A \in \mu} A) \geq U(x)$ there is a finite subfamily $\delta \subseteq \mu$ such that $(\bigvee_{A \in \delta} A) \geq U(x)$ for every $x \in S(U)$.

3.3 Theorem A fts X is b-compact iff for every collection $\{A_\lambda : \lambda \in \Lambda\}$ of fuzzy b-closed sets of X having the finite intersection property, $\bigwedge_{\lambda \in \Lambda} A_\lambda \neq 0_X$.

Proof Let $\{A_\lambda : \lambda \in \Lambda\}$ be a collection of fuzzy b-closed sets with the finite intersection property. Suppose that $\bigwedge_{\lambda \in \Lambda} A_\lambda = 0_X$. Then $\bigvee_{\lambda \in \Lambda} (\overline{A_\lambda}) = 1_X$. Since $\{\overline{A_\lambda} : \lambda \in \Lambda\}$ is a collection of fuzzy b-open sets covering X , then from the definition of b-compactness of X it follows that there exists a finite subset $\Delta \subseteq \Lambda$ such that $\bigvee_{\lambda \in \Delta} \overline{A_\lambda} = 1_X$. Then $\bigwedge_{\lambda \in \Delta} A_\lambda = 0_X$, which gives a contradictions. Therefore $\bigwedge_{\lambda \in \Lambda} A_\lambda \neq 0_X$.

Conversely, let $\{A_\lambda : \lambda \in \Lambda\}$ be a collection of fuzzy b-open sets covering X . Suppose that for every finite subset $\Delta \subseteq \Lambda$, we have $\bigvee_{\lambda \in \Delta} A_\lambda \neq 1_X$. Then $\bigwedge_{\lambda \in \Delta} (\overline{A_\lambda}) \neq 0_X$. Hence $\{\overline{A_\lambda} : \lambda \in \Lambda\}$ satisfies the finite interesection property. Then by definition we have $\bigwedge_{\lambda \in \Lambda} (\overline{A_\lambda}) \neq 0_X$ which implies $\bigvee_{\lambda \in \Delta} A_\lambda \neq 1_X$ and this contradicts that $\{A_\lambda : \lambda \in \Lambda\}$ is a fuzzy b-cover of X . Thus X is fuzzy b-compact.

The following characterization on b-compactness is in terms of fuzzy filterbases.

3.4 Theorem A fts is fuzzy b-compact if and only if every filterbases Γ in X , $\bigwedge_{G \in \Gamma} bCl(G) \neq 0_X$.

Proof Let μ be a fuzzy b-open cover which has no finite sub-cover in X . Then for every finite subcollection of $\{A_1, \dots, A_n\}$ of μ , there exists $x \in X$ such that $A_\lambda(x) < 1$ for every $\lambda = 1, \dots, n$. Then $\overline{A_\lambda} > 0$, so that $\bigwedge_{\lambda=1}^n \overline{A_\lambda}(x) \neq 0_X$. Thus $\{\overline{A_\lambda}(x) : A_\lambda \in \mu\}$ forms a filterbases in X . Since μ is fuzzy b-open set cover of X , then $\left(\bigvee_{A_\lambda \in \mu} A_\lambda\right)(x) = 1_X$ for every $x \in X$ and hence $\bigwedge_{A_\lambda \in \mu} bClA_\lambda(x) = 0_X$, which is a contradiction. Then every fuzzy b-open set cover of X has a finite subcover and hence X is fuzzy b-compact.

Conversely, suppose there exists a filterbases Γ in X , $\bigwedge_{G \in \Gamma} bCl(G) = 0_X$, so that $\left(\bigvee_{G \in \Gamma} (bCl(G))\right)(x) = 1_X$ for every $x \in X$ and hence $\mu = \{bCl(G) : G \in \Gamma\}$ is a fuzzy b-open cover of X . Since X is fuzzy b-compact, by definition Γ has a finite subcover such that $\left(\bigvee_{\lambda=1}^n (bCl(G_\lambda))\right)(x) = 1_X$ and hence $\left(\bigwedge_{\lambda=1}^n (G_\lambda)\right)(x) = 1_X$, so that $\bigwedge_{\lambda=1}^n (G_\lambda) = 0_X$, which is a contradiction. Therefore $\bigwedge_{G \in \Gamma} bCl(G) \neq 0_X$ for every filterbases Γ .

3.5 Theorem A fuzzy set u in a fts X is fuzzy b-compact relative to X if and only if for every filterbase Γ such that every finite members of Γ is quasi coincident with u , $\left(\bigwedge_{G \in \Gamma} bCl(G)\right) \wedge u \neq 0_X$.

Proof Suppose U is not fuzzy b-compact relative to X , then there exists a fuzzy b-open set μ covering of U with no finite

subcover v . Then $\left(\bigvee_{A_\lambda \in v} A_\lambda\right)(x) < U(x)$ for some $x \in S(U)$, so that $\left(\bigwedge_{A_\lambda \in v} \overline{A_\lambda}\right)(x) > \overline{U}(x) \geq 0$ and hence $\Gamma = \{\overline{A_\lambda}(x) : A_\lambda \in \mu\}$ forms a filterbases and $\bigwedge_{A_\lambda \in v} \overline{A_\lambda} \not\leq U$. By hypothesis $\left(\bigwedge_{A_\lambda \in v} bClA_\lambda\right) \wedge U \neq 0_X$ and hence $\left(\bigwedge_{A_\lambda \in v} \overline{A_\lambda}\right) \wedge U \neq 0_X$. Then for some $x \in S(U)$, $\left(\bigwedge_{A_\lambda \in \mu} \overline{A_\lambda}\right)(x) > 0_X$, that is $\left(\bigvee_{A_\lambda \in \mu} A_\lambda\right)(x) < 1_X$ which is a contradiction. Hence U is a fuzzy b-compact relative to X .

Conversely, suppose that there exists a filterbases Γ such that every finite members of Γ is quasi-coincident with U and $\left(\bigwedge_{G \in \Gamma} bCl(G)\right) \wedge U \neq 0_X$. Then for every $x \in S(U)$, $\left(\bigwedge_{G \in \Gamma} bCl(G)\right)(x) = 0_X$ and hence $\bigvee_{G \in \Gamma} \overline{bCl(G)}(x) = 1_X$ for every $x \in S(U)$. Thus $\mu = \{\overline{bCl(G)} : G \in \Gamma\}$ is a fuzzy b-open set cover of U . Since U is fuzzy b-compact relative to X , there exists a finite subcover, say $\{bCl(G_\lambda) : \lambda = 1, 2, \dots, n\}$ such that $\left(\bigvee_{\lambda=1}^n \overline{bCl(G_\lambda)}\right)(x) \geq U(x)$ for every $x \in S(U)$. Hence $\left(\bigwedge_{\lambda=1}^n bCl(G_\lambda)\right)(x) \leq \overline{U}(x)$ for every $x \in S(U)$, so that $\bigwedge_{\lambda=1}^n bCl(G_\lambda) \not\leq U$, which is a contradiction. Therefore for every filterbases Γ , every finite member of Γ is quasi-coincident with U . Hence $\left(\bigwedge_{G \in \Gamma} bCl(G)\right) \wedge u \neq 0_X$.

The following theorem proves the hereditary property for fuzzy b-compact spaces.

3.6 Theorem Every fuzzy b-closed subset of a fuzzy b-compact spaces is fuzzy b-compact relative to X .

Proof Suppose Γ be a fuzzy filterbases, Δ be its finite subcollection in X and for a fuzzy b-closed set U let $U \leq \bigwedge_{G \in \Gamma} G$. Let $\Gamma^* = \{U\} \cup \Gamma$. For any finite subcollection Δ^* of Γ^* , if $U \notin \Delta^*$, then $\bigwedge_{G \in \Delta^*} G \neq 0_X$. If $U \in \Delta^*$ and $U \leq \bigwedge_{G \in \Delta^*} G$, then $\bigwedge_{G \in \Delta^*} G \neq 0_X$. Hence Δ^* is a fuzzy filterbases in X . Since X is fuzzy b-compact, then $\left(\bigwedge_{G \in \Gamma} bCl(G)\right) \neq 0_X$, such that $\left(\bigwedge_{G \in \Gamma} bCl(G)\right) \wedge U = \left(\bigwedge_{G \in \Gamma} bCl(G)\right) \wedge bCl(U) \neq 0_X$. Hence by theorem 3.5, U is fuzzy b-compact relative to X .

In the following theorem it is showmn that image of a fuzzy b-compact space under a fuzzy b*-continuous mapping is fuzzy b-compact.

3.7 Theorem If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy b*-continuous and U is fuzzy b-compact relative to X , then $f(U)$ is fuzzy b-compact.

Proof Let $\{A_\lambda : \lambda \in \Lambda\}$ be a fuzzy b-open cover of $S(f(U))$ in Y . For $x \in S(U)$, $f(x) \in f(S(U))$. Since f is fuzzy b*-continuous, $\{f^{-1}(A_\lambda) : \lambda \in \Lambda\}$ is fuzzy b-open cover of $S(U)$ in X . Since U is fuzzy b-compact relative to X , there is a finite subfamily $\{f^{-1}(A_\lambda) : \lambda = 1, \dots, n\}$ such that $S(U) \leq \bigvee_{\lambda=1}^n f^{-1}(A_\lambda)$

$= f^{-1}(\bigvee_{\lambda=1}^n A_{\lambda})$. Hence $S(f(U)) = f(S(U)) \leq f(f^{-1}(\bigvee_{\lambda=1}^n A_{\lambda})) \leq \bigvee_{\lambda=1}^n A_{\lambda}$.
 Therefore $f(U)$ is λ fuzzy b-compact relative to Y .

The pre image of a fuzzy b-compact space under fuzzy b*-open bijective mapping is fuzzy b-compact.

3.8 Theorem If a function $f : (X, \tau) \rightarrow (Y, \sigma)$ is fuzzy b*-open bijective mapping and Y be fuzzy b-compact, then X fuzzy b-compact.

Proof Let $\{A_{\lambda} : \lambda \in \Lambda\}$ be a family of fuzzy b-open covering of X . Then let $\{f(A_{\lambda}) : \lambda \in \Lambda\}$ be a fuzzy b-open cover is a family of fuzzy b-open sets covering Y . Since Y is fuzzy b-compact, by definition there exist a finite family $\Delta \subseteq \Lambda$ such that $\{f(A_{\lambda}) : \lambda \in \Delta\}$ covers Y . Also since f is bijective we have $1_x = f^{-1}(1_y) = f^{-1}(f(\bigvee_{\lambda \in \Delta} A_{\lambda})) = \bigvee_{\lambda \in \Delta} A_{\lambda}$. Thus X is fuzzy b-compact.

3.9 Definition A fts (X, τ) is said to be locally b-compact iff for every fuzzy point p in X there exist $A \in \tau$ such that (i) $p \in A$ (ii) A is fuzzy b-compact.

3.10 Theorem Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy b*-continuous function from a locally b-compact fts (X, τ) onto a fts (Y, σ) . If f is also fuzzy b*-open then Y is locally b-compact.

Proof Let q be a fuzzy point in Y with support y_0 and value y . Then p is a fuzzy point in X with support x_0 and value y . Now $x_0 \in f^{-1}(y_0)$. Then $f(p) = q$. Since p is a fuzzy point in X and X is locally b-compact, by definition there exists a member $A \in \tau$ such that $p \in A$ and A is fuzzy b-compact. Now $A \in \tau$ and f is b*-open, therefore $f(A) \in \sigma$ and $q \in f(A)$, also $f(A)$ is fuzzy b-compact in Y . Thus for a fuzzy point $q \in Y$ there exist a member $f(A) \in \sigma$ such that $q \in f(A)$ and $f(A)$ is fuzzy b-compact. Therefore (Y, σ) is locally b-compact.

4. FUZZY b-CLOSED SPACES

4.1 Definition A fts X is said to be fuzzy b-closed iff for every family λ of fuzzy b-open set such that

$\bigvee_{A \in \lambda} A = 1_x$ there is a finite subfamily $\delta \subseteq \lambda$ such that

$$\left(\bigvee_{A \in \delta} bCl(A) \right)(x) = 1_x, \text{ for every } x \in X.$$

4.2 Definition A fuzzy set U in a fts X is said to be fuzzy b-closed relative to X iff for every family λ of fuzzy b-open set such that $\bigvee_{A \in \lambda} A = U$ there is a finite subfamily $\delta \subseteq \lambda$ such

that $\bigvee_{A \in \delta} bCl(A)(x) = U(x)$, every for $x \in S(U)$.

4.3 Remark Every fuzzy b-compact space is fuzzy b-closed, but the converse is not true.

4.4 Theorem A fts X is fuzzy b-closed iff for every fuzzy filterbases Γ in X , $(\bigwedge_{G \in \Gamma} bCl(G)) \neq 0_x$.

Proof Let μ be a fuzzy b-open set cover of X and let for every finite family of μ , $\bigvee_{A \in \delta} bCl(A)(x) < 1_x$ for some $x \in X$. Then $(\bigwedge_{A \in \delta} \overline{bCl(A)})(x) > 0_x$ for some $x \in X$. Thus

$\{\overline{bCl(A)} : A \in \mu\} = \Gamma$ forms a fuzzy b-open filterbases in X . Since μ is a fuzzy b-open set cover of X , then $(\bigwedge_{A \in \mu} A) = 0_x$

which implies $(\bigwedge_{A \in \Gamma} \overline{bCl(A)})(x) = 0_x$, which is a contradiction. Then every fuzzy b-open μ of X has a finite subfamily δ such that $(\bigvee_{A \in \delta} bCl(A)(x)) = 1_x$ for every $x \in X$.

Hence X is fuzzy b-closed.

Conversely, suppose there exists a fuzzy b-open filterbases Γ in X such that $(\bigwedge_{G \in \Gamma} bCl(G)) = 0_x$. That implies

$$\left(\bigvee_{G \in \Gamma} \overline{bCl(G)} \right)(x) = 1_x \text{ for } x \in X \text{ and hence}$$

$\mu = \{\overline{bCl(G)} : G \in \Gamma\}$ is a fuzzy b-open set cover of X . Since X is fuzzy b-closed, by definition μ has a finite subfamily δ such that $(\bigvee_{G \in \delta} \overline{bCl(G)})(x) = 1_x$ for every $x \in X$, and hence

$$\bigwedge_{G \in \delta} \overline{bCl(G)} = 0_x. \text{ Thus } \bigwedge_{G \in \delta} G = 0_x \text{ is a contradiction. Hence } \bigwedge_{G \in \Gamma} bCl(G) \neq 0_x.$$

4.5 Theorem A fuzzy subset λ in a fts X is fuzzy b-closed relative to X iff for every fuzzy b-open filterbases Γ in X , $(\bigwedge_{G \in \Gamma} bCl(G)) \wedge U \neq 0_x$, there exist a finite subfamily Δ of Γ such that $(\bigwedge_{G \in \Delta} bCl(G)) \overline{q} \cdot$

Proof Let U be a fuzzy b-closed set relative to X . Suppose Γ is a fuzzy b-open filterbases in X such that for every finite subfamily Δ of Γ , $(\bigwedge_{G \in \Delta} bCl(G)) \overline{q} \cdot U$. But $(\bigwedge_{G \in \Gamma} bCl(G)) \wedge U = 0_x$. Then

for every $x \in S(U)$, $(\bigwedge_{G \in \Gamma} bCl(G))(x) = 0_x$ and so

$$\left(\bigvee_{G \in \Gamma} \overline{bCl(G)} \right)(x) = 1_x \text{ for every } x \in S(U). \text{ Then}$$

$\mu = \{\overline{bCl(G)} : G \in \Gamma\}$ is a fuzzy b-open set cover of U and hence there exists a family $\Delta \subseteq \Gamma$ such that $\bigvee_{G \in \Delta} \overline{bCl(G)}(x) \geq U$, so that $(\bigwedge_{G \in \Delta} \overline{bCl(G)}) = \bigwedge_{G \in \Delta} \overline{bCl(G)} \leq \overline{U}$.

Thus $\bigwedge_{G \in \Delta} G \leq \overline{U}$. Therefore $\bigwedge_{G \in \Delta} G \overline{q} \cdot U$, this is a contradiction.

Conversely, Suppose U is not fuzzy b-closed relative to X , then there exists a fuzzy b-open set μ that covers U such that for every finite subfamily $\eta \subseteq \mu$ we have

$$\left(\bigvee_{A \in \eta} bCl(A) \right)(x) \leq U(x) \text{ for some } x \in S(U) \text{ and hence}$$

$$\bigwedge_{A \in \eta} \overline{bCl(A)}(x) \geq (\overline{U(x)}) \geq 0 \text{ for some } x \in S(U). \text{ Thus}$$

$\Gamma = \{\overline{bCl(A)} : A \in \mu\}$ forms a fuzzy b-open filterbases in X . Let $\{\overline{bCl(A)} : A \in \eta\}$ be a finite subfamily such that

$$\bigwedge_{A \in \eta} \overline{bCl(A)} \overline{q} \cdot U. \text{ Then } U \leq \bigvee_{A \in \eta} bCl(A). \text{ So there exist a finite}$$

subfamily $\eta \subseteq \mu$ such that $U \leq \bigvee_{A \in \eta} bCl(A)$ which is a contradiction. Then for each finite family $\Delta = \{\overline{bCl(A)} : A \in \eta\}$

of Γ , we have $\bigwedge_{A \in \mu} (\overline{bCl(A)})_q \cup U$. Hence by definition

$(\bigwedge_{A \in \mu} bCl(\overline{bCl(A)})) \wedge U \neq 0_X$. Hence there exists $x \in S(U)$ such

that $(\bigwedge_{A \in \mu} bCl(\overline{bCl(A)})) \wedge U \neq 0_X$. Then $(\bigvee_{A \in \mu} (\overline{bCl(\overline{bCl(A)})})(x) =$

$(\bigvee_{A \in \mu} (bInt(bCl(A))))(x) < 1_X$ and hence $\bigvee_{A \in \mu} A(x) < 1_X$

which contradicts the fact that μ is a fuzzy b-open set cover of U . Therefore U is a fuzzy b-closed relative to X .

The pre-image of a fuzzy b-closed space under fuzzy b*-open bijective mapping is fuzzy b-closed.

4.6 Theorem Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy b*-continuous surjection. If X is fuzzy b-closed space, then Y is fuzzy b-closed space.

Proof Let $\{A_\lambda : \lambda \in \Lambda\}$ be a fuzzy b-open cover of Y . Since f is fuzzy b*-continuous, $\{f^{-1}(A_\lambda) : \lambda \in \Lambda\}$ is fuzzy b-open cover of X . By hypothesis, there exists a finite subset Δ of Γ such that $\bigvee_{\lambda \in \Delta} bCl(f^{-1}(A_\lambda)) = 1_X$. Since f is surjective and by theorem $1_Y = f(1_X) = f(\bigvee_{\lambda \in \Delta} bCl(f^{-1}(A_\lambda))) \leq \bigvee_{\lambda \in \Delta} bCl(f(f^{-1}(A_\lambda) = \bigvee_{\lambda \in \Delta} bCl(A_\lambda))$. Hence Y is fuzzy b-closed space.

5. FUZZY GENERALIZED b-COMPACT SPACES

Firstly we shall define the concept of fuzzy generalized b-compact spaces.

5.1 Definition A collection $\{A_\lambda\}_{\lambda \in \Gamma}$ of fbg-open sets in X is called fbg-open cover of a fuzzy set B in X if $B \leq \bigvee_{\lambda \in \Gamma} A_\lambda$.

5.2 Definition A topological space X is called fbg-compact if every fbg-open cover of X has a finite subcover.

5.3 Definition A fuzzy set A in X is said to be fbg-compact relative to X if for every collection $\{A_\lambda\}_{\lambda \in \Gamma}$ of fbg-open sets of X such that $A \leq \bigvee_{\lambda \in \Gamma} A_\lambda$, there exists a finite subset Δ of Γ such that $A \leq \bigvee_{\lambda \in \Delta} A_\lambda$.

5.4 Definition A fuzzy set A of X is said to be fbg-compact if A is fbg-compact relative to X .

The following results can be easily proved.

5.5 Theorem Let X is said to be fbg-compact and A be fbg-closed set in X . Then A is fbg-compact.

5.6 Theorem A fbg-continuous image of a fbg-compact space is fuzzy b-compact.

5.7 Theorem If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is fbg-irresolute and if A is fbg-compact relative to X , then $f(A)$ is fbg-compact relative to Y .

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7. CONCLUSION

It is an interesting exercise to work on b-compact spaces and b-closed spaces. Similarly other forms of generalized open set can be applied to define different forms of compact spaces and closed spaces.

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