## **Dynamic version of Traveling Salesman Problem**

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## ABSTRACT

In the classical version of Traveling salesman problem, the targets which have to be visited are stationary but in real life there are large numbers of instances where the targets are in motion. In this paper, Moving target TSP with resupply is being studied and new algorithm is designed for moving target TSP with resupply when all targets are moving away from the origin with positive constant velocity and the goal is to minimize the total intercepting time taken by the salesman. An algorithm is also designed When all targets are moving towards the origin with the positive constant velocity in a straight line and a single salesman (moving with the constant velocity) has to intercept these targets in a particular way with the constraint that after intercepting every target, salesman must come back to the origin for resupply and the goal is to minimize the total intercepting time taken by the salesman.

### **General Terms**

Algorithms, Theory.

## Keywords

Algorithm, Traveling salesman problem, Moving target traveling salesman problem with resupply.

## 1. INTRODUCTION

The step by step solution of a particular problem is known as algorithm[3,4,5,6,9,11,12]. The rate of growth of function with respect to the input size is known as time complexity [7,8] of algorithm. It is well known fact that the Travelling Salesman Problem [10] is NP complete problem [3, 4] in field of combinatorial optimization [10] studied in operations research.

Any problem in class P is also in NP, since we would be able to solve it in polynomial time, we can also verify it in polynomial time. Problem X polynomial transforms to problem Y if given any input x to X, we can construct in polynomial time an input y to Y such that x is a yes instance of X iff y is a yes instance of Y  $X \leq_P Y$ .

Problem Y is NP-Hard if

For every problem X in NP,  $X \leq_P Y$  (the problem X is

polynomial time reducible to problem Y)

Problem Y is NP-complete if

Y is in NP and

For every problem X in NP,  $X \leq_P Y$ .( the problem X is polynomial time reducible to problem Y)

No polynomial-time algorithm has been discovered which give the optimal solution for an NP-Complete problem and no super polynomial lower bound has been proved for any NP-Complete problem From the above definition it is clear that traveling salesman problem is NP complete problem

In the classical traveling salesman problem, there is salesman and he has to traverse n number of cities exactly once in such a way that the total distance traveled by salesman is minimized. Moving target traveling salesman problem[1] can be defined as follows:

A set  $G=\{g_1,g_2,g_3,\_\_\_,g_n\}$  of targets each  $g_i$  is moving at constant velocity  $v_i$  from an initial position  $p_i$  and a salesman starts at the origin and having speed  $v>v_i$ , the goal is to find the fastest tour which starts and end at the origin, which intercepts all targets.

## 2. RELATED WORK

In moving target traveling salesman problem with resupply, the salesman must come back to the origin for resupply after intercepting each target. Suppose  $d_i$  be the initial distance between the target  $g_i$  and the origin i.e.  $d_i = |p_i|$  where  $p_i$  represents the position of target  $g_i$  in the given plane. As velocity is vector quantity So we have assumed that when the targets are moving away from the origin then the velocity is positive and when the targets are moving towards the origin then the velocity is negative. In Moving-Target TSP with Resupply when all targets are moving away from the origin with the positive constant velocities where all targets move directly away from the origin, an optimal tour (time has to be minimized) intercepts the targets in increasing order of their respective ratios  $d_i/v_i$  [1].

## 3. **RESULTS AND DISCUSSION**

In section 4, an algorithm is being designed. When all targets are moving away from the origin with the positive constant velocity in a straight line and a single salesman (moving with the constant velocity) has to intercept these targets in a particular way with the constraint that after intercepting every target, salesman must come back to the origin for resupply and the goal is to minimize the total intercepting time taken by the salesman. In section 5, an algorithm is being designed. When all targets are moving towards the origin with the positive constant velocity in a straight line and a single salesman (moving with the constant velocity) has to intercept these targets in a particular way with the constraint that after intercepting every target, salesman must come back to the origin for resupply and the goal is to minimize the total intercepting time taken by the salesman.

## 4. MOVING TARGET TRAVELING SALESMAN PROBLEM WITH RESUPPLY WHEN ALL TARGETS ARE MOVING AWAY FROM THE ORIGIN

In this section an algorithm is being designed When all targets are moving away from the origin with the positive constant velocity in a straight line and a single salesman (moving with the constant velocity) has to intercept these targets in a particular way with the constraint that after intercepting every target, salesman must come back to the origin for resupply and the goal is to minimize the total intercepting time taken by the salesman.

#### **Proposed algorithm**

#### Input:-

If G={  $g_1, g_2, g_3, \_$  \_ \_ ,  $g_n$  } be the n number of targets

di be the initial distance of target gi.

 $v_i$  be the initial velocity of the target  $g_i$ .

 $t_{\rm i}$  be the time taken by salesman to intercept the target  $g_{\rm i}$ 

v be the velocity of salesman.

T be the total time taken by salesman to intercepts all targets

 $s_i$  be the additional distance traveled by  $i^{th} \, \text{target}$ 

#### **Output:-**

The intercepting order in which the salesman must intercept the targets so that the total time can be minimized

#### **Pseudo code:**

1.for i=1 to n

 $if(v_i > v)$ 

print " the algorithm cannot determine the optimal tour because in that case salesman will not be able to intercept the all targets"

2. for i=1 to n

 $z_i = d_i / v_i$ 

}

3 Sort the list  $z_i$  by using merge sort.

4 for i=1 to n print d<sub>i</sub>

```
5 for i=1 to n
```

```
print v<sub>i</sub>
```

```
6 t_1 = d_1/(v-v_1)
```

```
7 s_1=0
8 T=2*t_1
```

```
9 for i=2 to n
```

```
{
b<sub>1</sub>=0
```

```
for j=1 to i-1
```

{

```
if(j==1)b_i = b_i + 2^* t_i
```

```
else
```

 $b_{j} {=} \, b_{j{\text{-}}1} + 2 {}^{*} t_{j};$  }

```
j- -
t_i=(d_i + ((b_j)*v_i))/(v-v_i)
s_i=(b_j)*(v_i)
```

```
T = 2 * t_i + T
```

}
10 for i=1 to n

print(t<sub>i</sub>)

```
11 for i=1 to n
```

print s<sub>i</sub>

12 print T

#### Time complexity of the proposed algorithm

If  $t_1$ = time complexity of step  $1 = \theta(n)$ .  $t_2$ = time complexity from step  $2 = \theta(n)$ .  $t_3$ = time complexity from step  $3 = \theta(nlgn)$ .  $t_4$ = time complexity from step  $4 = \theta(n)$ .  $t_5$ = time complexity from step  $5 = \theta(n)$ .  $t_6$ = time complexity from step 6 to  $8 = \theta(1)$ .  $t_7$ = time complexity from step  $9 = \theta(n^2)$ .  $t_8$ =time complexity from step 10 to  $12 = \theta(n)$ . T(n)=Resultant time complexity of algorithm T(n)= $t_1$ +  $t_2$ +  $t_3$ +  $t_4$ +  $t_5$ +  $t_6$ +  $t_7$  +  $t_8 = \theta(n) + \theta(n) + \theta(nlg(n))$ +  $\theta(n) + \theta(n) + \theta(1) + \theta(n^2) + \theta(n) = \theta(n^2)$ 

#### **Experimental Results**

When there are two numbers of targets which are initially at the distance 5 and 20 from the origin, moving with the velocities 6 and 100. The salesman is moving with the constant velocity 200. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{2,1\}$ . It means the salesman will intercept the target  $g_1$  And the total

intercepting time comes out to be 0.476289 and if the salesman firstly intercepts the target  $g_1$  and after then the salesman intercept the target  $g_2$  then total intercepting time comes out to be 0.554639 Which is greater than the total intercepted time determined from the proposed algorithm. The relative error is the intercepting time of the targets determined from the proposed algorithm with the compared with the intercepting time of the other order. The relative error in this particular case comes out to be 16.393408 as shown in the table 1.

Table 1 Moving target tsp with	resunnly for two targ	ets when all targets are	moving away from the origin
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Target	di	vi	V	Intercepting order	Additional Distance	Time taken to intercept individual target	Total intercepting time	Relative Error
g1	5	6	200	2,1	0	0.200000	0.476289	0
-					2.400000	0.038144		
<b>g</b> <sub>2</sub>	20	100	-	1,2	0	0.025773	0.554639	16.393408

When there are three numbers of targets which are initially at the distance 5, 7 and 6 from the origin, moving with the velocities 6, 4 and 3. The salesman is moving with the constant velocity of 20. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{1,2,3\}$ . It means the salesman will intercept the target  $g_1$  and after then salesman will intercept the target  $g_2$  and  $g_3$  And the total intercepting time comes out to be 3.339286. And If the salesman tends to intercept the target in  $\{1,3,2\}$  order then the intercepting time comes out to be 3.383403 and the relative error comes out to be 1.321150. If the salesman tends to intercept the target in  $\{2,3,1\}$  order then the intercepting time comes out to be 4.223740 and the relative error comes out to be 26.48632. If the salesman tends to intercept the target in  $\{2,1,3\}$  order then the intercepting time comes out to be 3.870799 and the relative error comes out to be 15.916965. If the salesman tends to intercept the target in  $\{3,1,2\}$  order then the intercepting time comes out to be 3.912815 and the relative error comes out to be 17.175198. If the salesman tends to intercept the target in  $\{3,2,1\}$  order then the intercepting time comes out to be 4.305673 and the relative error comes out to be 28.939929. The intercepting time of all orders along with the relative error is shown in the table 2.

When there are three numbers of targets which are initially at the distance 5, 20 and 6 from the origin, moving with the velocities 6, 4 and 9. The salesman is moving with the constant velocity of 40. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{3,1,2\}$ . It means the salesman will intercept the target  $g_3$  and after then salesman will intercept the target  $g_1$  and  $g_2$  And the total intercepting time comes out to be 2.110689. If the salesman tends to intercept the target in  $\{1,2,3\}$  order then the intercepting time comes out to be 2.711575 and the relative error comes out to be 28.468713. If the salesman tends to

intercept the target in {1,3,2} order then the intercepting time

comes out to be 2.152435 and the relative error comes out to be 1.97783756. If the salesman tends to intercept the target in  $\{2,1,3\}$  order then the intercepting time comes out to be 3.228126 and the relative error comes out to be 52.941811. If the salesman tends to intercept the target in  $\{2,3,1\}$  order then the intercepting time comes out to be 3.193970 and the relative error comes out to be 51.323572. If the salesman tends to intercept the target in  $\{3,2,1\}$  order then the intercepting time comes out to be 51.323572. If the salesman tends to intercept the target in  $\{3,2,1\}$  order then the intercepting time comes out to be 15.483001. The intercepting time for orders are given in table3.

When there are four numbers of targets which are initially at the distance 7, 8, 9 and 15 from the origin, moving with the velocities 5,6, 5 and 6. The salesman is moving with the constant velocity of 20. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{2,1,3,4\}$ . It means the salesman will intercept the target  $g_1$  and  $g_4$  And the total intercepting time comes out to be 13.156009.

## Table 2 Moving target tsp with resupply for three targetswhen all targets are moving away from the origin

Target	di	vi	V	Intercepting orde r	Additional Distance	Time taken to intercept the individual target	Total time	Relative Error
g <sub>1</sub>	5	6	20	1,2,3	0 2.857143	0.357143		
					2.837143	0.010071	3.339286	0
					5.839286	0.696429	-	
				1,3,2	0	0.357143		
					2.142857	0.478992	3.383403	1.321150
					6.689075	0.855567		
<b>g</b> <sub>2</sub>	7	4		2,3,1	0	0.437500		
					2.625000	0.507353	4.223740	26.48632
					11.338236	1.167017		
				2,1,3	0	0.437500		
					5.250000	0.732143	3.870799	15.916965
					7.017858	0.765756	-	
<b>g</b> <sub>3</sub>	6	3	-	3,1,2	0	0.352941		
					4.235294	0.659664	3.912815	17.175198
					8.100841	0.943803	-	
				3,2,1	0	0.352941		
					2.823529	0.613971	4.305673	28.939929
					11.602942	1.185924	1	

# Table 3 Moving target tsp with resupply for three targets when all targets are moving away from the origin

Target	di	vi	V	Intercepting order	Additional Distance	Time taken to intercept individual target	Total time	Relative Error	
g1	5	6	40	1,2,3	0	0.147059			
					1.176471	0.588235	2.711575	28.468713	
					13.235295	0.620493			
				1,3,2	0	0.147059			
					2.647059	0.278937	2.152435	1.97783756	
					3.407970	0.650221			
g <sub>2</sub>	20	4		2,3,1	0	0.555556	3.193970	51.323572	
					10.000000	0.516129			
					12.860215	0.525300			
				2,1,3	0	0.555556			
					6.666667	0.343137	3.228126	52.941811	
					16.176472	0.715370			
<b>g</b> <sub>3</sub>	6	9	-	3,1,2	0	0.193548			
					2.322581	0.215370	2.110689	0	
					3.271347	0.646426			
				3,2,1	0	0.193548			
					1.548387	0.598566	2.437487	15.483001	
					9.505376	0.426629			

When there are eight numbers of targets which are initially at the distance 2,4,3,2,8,9,7 and 5 from the origin, moving with the velocities 1,5,4,3,3,6,6 and 4. The salesman is moving with the constant velocity of 50. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{4,3,2,7,8,6,1,5\}$  and the total intercepting time comes out to be 2.820404. But if the salesman intercepts the targets in the intercepting order {8,7,6,5,1,2,3,4} then the intercepting time comes out to be 3.387424 and the relative error corresponding to this particular order is 20.104212. If the salesman intercepts the targets in the intercepting order  $\{1,2,3,4,5,6,7,8\}$  then the intercepting time comes out to be 3.135190 and the relative error corresponding to this particular order is 11.1610515. If the salesman intercepts the targets in the intercepting order  $\{8,7,6,5,4,3,2,1\}$  then the intercepting time comes out to be 3.347504 and the relative error corresponding to this particular order is 18.6888119.

**4** In this section, an algorithm is being designed When all targets are moving towards the origin with the positive constant velocity in a straight line and a single salesman (moving with the constant velocity) has to intercept these targets in a particular way with the constraint that after intercepting every target, salesman must come back to the origin for resupply and the goal is to minimize the total intercepting time taken by the salesman

## **Proposed algorithm**

**Input:-**If G={  $g_1, g_2, g_3, \dots, g_n$  } be the n number of targets

d<sub>i</sub> be the initial distance of target g<sub>i</sub>.

v<sub>i</sub> be the initial velocity of the target g<sub>i</sub>.

ti be the time taken by salesman to intercept the target gi

v be the velocity of salesman.

T be the total time taken by salesman to intercepts all targets

s<sub>i</sub> be the additional distance traveled by i<sup>th</sup> target

#### **Output:-**

The intercepting order in which the salesman must intercept the targets so that the total time can be minimized.

#### Pseudo code:

```
1. for i=0 to n-1

\begin{cases} z_i = d_i/v_i \\ z_i = d_i/v_i \end{cases}
2 Sort the list z_i in the deceasing order by using merge sort.

3 for i=0 to n-1

print d_i

4 for i=0 to n-1

print v_i

5 t_0=d_0/(V+v_0)

6 s_0=0
```

```
7 T=2*t<sub>0</sub>
8 for(i=0 to n-2)
     {
       b_0 \!\!=\!\! 0
        for(j=0 to i-1)
              if(j==0)
              b_i = b_i + 2 t_i
              else
              b_{j} = b_{j-1} + 2*t_{j}
              }
              i---
              t_i = (d_i - ((b_i) * v_i))/(V + v_i)
              s_i = (b_i)^* (v_i)
              T=2 \star t_i+T
9 for(i=0 to n-1)
              print(t<sub>i</sub>)
10 for(i=0 to n-1)
              Print "the additional distance si"
```

11 print "the total time taken by salesman is T" **Time complexity of the proposed algorithm** 

If  $t_1$  = time complexity of step  $1 = \theta(n)$ .

 $t_2$  = time complexity of step 2 =  $\theta(nlgn)$ .

 $t_3$ = time complexity of step 3 =  $\theta(n)$ .

 $t_4$ = time complexity of step 4=  $\theta(n)$ .

 $t_5$  = time complexity of step 5 to 7 =  $\theta(1)$ .

 $t_6$  = time complexity of step 8 =  $\theta(n^2)$ ...

 $t_7$  = time complexity of step 9 =  $\theta(n)$ .

 $t_8$ =time complexity from step 10 to 11= $\theta(n)$ .

T(n)=Resultant time complexity of algorithm

 $T(n) = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 = = \theta(n^2)$ 

Experimental Results When there are two numbers of targets which are initially at the distance 41 and 20 from the origin, moving with the velocities 1 and 5. The salesman is moving with the constant velocity 40. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{1,2\}$ . It means the salesman will intercept the target  $g_1$  and after then salesman will intercept the target g<sub>2</sub> And the total intercepting time comes out to be 2.444444 and if the salesman firstly intercepts the target  $g_2$ and after then the salesman intercept the target g1 then total intercepting time comes out to be 2.845528 Which is greater than the total intercepted time determined from the proposed algorithm. The relative error is the intercepting time of the targets determined from the proposed algorithm with the compared with the intercepting time of the other order. The relative error in this particular case comes out to be 16.4100801 as shown in the table 4.

Target	di	v <sub>i</sub>	V	Intercepting order	Time taken to intercept individual target	Total intercepting time	Relative Error
g1	41	1	40	2,1	0.44444 0.978320	2.845528	16.410080 1
<b>g</b> <sub>2</sub>	20	5		1,2	1	2.44444	0

 Table 4 Moving target tsp with resupply for two targets

 when all targets are moving towards the origin.

When there are three numbers of targets which are initially at the distance 5, 7 and 3 from the origin, moving with the velocities 1, 5 and 3. The salesman is moving with the constant velocity of 20. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{1,2,3\}$ . It means the salesman will intercept the target  $g_1$  and after then salesman will intercept the target  $g_2$  and  $g_3$  And the total intercepting time comes out to be 0.885963. And If the salesman tends to intercept the target in  $\{1,3,2\}$  order then the intercepting time comes out to be 0.927702 and the relative error comes out to be 4.711144. If the salesman tends to intercept the target in  $\{2,3,1\}$  order then the intercepting time comes out to be 1.086708 and the relative error comes out to be 22.65839. If the salesman tends to intercept the target in  $\{2,1,3\}$  order then the intercepting time comes out to be 0.987329 and the relative error comes out to be 11.441335. If the salesman tends to intercept the target in  $\{3,1,2\}$  order then the intercepting time comes out to be 0.987329 and the relative error comes out to be 11.441335. If the salesman tends to intercept the target in  $\{3,2,1\}$  order then the intercepting time comes out to be 1.124472and the relative error comes out to be 26.920875. The intercepting time of all orders along with the relative error is shown in the table 5.

When there are three numbers of targets which are initially at the distance 10, 12 and 15 from the origin, moving with the velocities 5, 6 and 20. The salesman is moving with the constant velocity of 80. When the proposed algorithm is applied to this particular input then the intercepting order of input targets comes out to be  $\{2,1,3\}$ . It means the salesman will intercept the target  $g_2$  and after then salesman will intercept the target  $g_1$  and  $g_3$  And the total intercepting time comes out to be 0.588919. If the salesman intercepts the targets in any other order then the different intercepting times for the different intercepting order is given in the table 6.

# Table 5 Moving target tsp with resupply for three targetswhen all targets are moving towards the origin.

Target	di	vi	V	Intercepting order	Time taken to intercept individual target	Total time	Relative Error
g <sub>1</sub>	5	1	20	1,2,3	0.238095 0.184762 0.020124	0.885963	0
				1,3,2	0.238095 0.068323 0.157433	0.927702	4.711144
g <sub>2</sub>	7	5		2,3,1	0.280000 0.057391 0.205963	1.086708	22.65839
				2,1,3	0.280000 0.211429 0.002236	0.987329	11.441335
g <sub>3</sub>	3	3		3,1,2	0.130435 0.225673 0.137557	0.987329	11.441335
				3,2,1	0.130435 0.227826 0.203975	1.124472	26.920875

# Table 6 Moving target tsp with resupply for three targetswhen all targets are moving towards the origin.

Target	di	vi	V	Intercepting order	Time taken to intercept individual target	Total time	Relative Error
g <sub>1</sub>	10	5	80	1,2,3	0.117647 0.123119 0.053694	0.588919	0
				1,3,2	0.117647 0.102941 0.108755	0.658687	11.846790
g <sub>2</sub>	12	6		2,3,1	0.139535 0.094186 0.090150	0.647743	9.98847040
				2,1,3	0.139535 0.101231 0.053694	0.588919	0
g <sub>3</sub>	15	20		3,1,2	0.150000 0.100000 0.104651	0.709302	20.441351
				3,2,1	0.150000 0.118605 0.086047	0.709302	20.441351

## **5. CONCLUSION**

In this paper, Moving target TSP with resupply is being studied and new algorithm is designed for moving target TSP with resupply when all targets are moving away from the origin with positive constant velocity and a single salesman (moving with the constant velocity) has to intercept these targets in a particular way with the constraint that after intercepting every target, salesman must come back to the origin for resupply and the goal is to minimize the total intercepting time taken by the salesman. An algorithm is also designed When all targets are moving towards the origin with the positive constant velocity in a straight line and a single salesman (moving with the constant velocity) has to intercept these targets in a particular way with the constraint that after intercepting every target, salesman must come back to the origin for resupply and the goal is to minimize the total intercepting time taken by the salesman. Topics for future research include when the salesman is also moving with the positive acceleration and when all targets move towards the origin or away from the origin with the positive constant accelerations.

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