Fuzzy gb- Continuous Maps in Fuzzy Topological Spaces

S. S. Benchalli
Department of Mathematics
Karnatak University. Dharwad
Karnataka State, India

Jenifer J.Karnel
Department of Mathematics
S.D.M College of Engineering
and Technology, Dharwad
Karnataka State, India

ABSTRACT
The purpose of this paper is to introduce a new form of generalized mapping namely fgb-continuous, fgb-irresolute mappings, fgb-closed maps, fgb-open and fgb*-open maps in fuzzy topological spaces. Some of their properties and characterization have been proved. As an application of these generalized fuzzy sets, fuzzy $gbT_{1/2}$-space , fgb-homeomorphism and fgb*-homeomorphism are introduced and discussed in detail.

Keywords
Fgb-closed sets, fgb-neighbourhood , fgbq-neighbourhood, fgb-continuous, fgb-irresolute mappings, fgb-closed maps, fgb*-open maps, fuzzy $gbT_{1/2}$-space,fgb-homeomorphism and fgb*-homeomorphism.

1. INTRODUCTION
Zadeh in [9] introduced the fundamental concept of fuzzy sets. Fuzzy topology was introduced by Chang [7]. The theory of fuzzy topological spaces was subsequently developed by several authors. The concept of b-open sets in general topology was introduced by Andrijevic [1]. The concept of fuzzy b-open set and fuzzy gb-closed sets are by Benchalli and Jenifer [3].

Here the concept of fuzzy gb-neighbourhood ,fuzzy gbq-neighbourhood, fuzzy gb-continuous, fuzzy gb-irresolute mappings, fuzzy gb-closed maps, fuzzy gb*-closed, fuzzy $gbT_{1/2}$-spaces and fuzzy gb- homeomorphism maps in fuzzy topological spaces are introduced. Their properties and some characterizations are obtained.

2. PRELIMNARIES
Throughout the present paper (X, τ), (Y, σ) and (Z, γ) (or simply X, Y and Z) mean fuzzy topological spaces (abbreviated as fts) .Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be a mapping from a fts X to fts Y. The definition of fuzzy sets, fuzzy topological spaces and other concepts by Chang and Zadeh can be found in [7, 9]. The members of τ are called open fuzzy sets and their compliments are closed fuzzy sets. Let A be a fuzzy set of fts X. We denote the closure and interior of A by cl(A) and int(A) respectively. A fuzzy point [8] x_t in X is a fuzzy set having support $x \in X$ and value $t \in (0,1]$. No separation axioms are assumed unless explicitly stated.

2.1 Definition [3] A fuzzy set A in a fts X is called
(i) fuzzy b-open set iff $A \leq (IntCl(A) \vee ClInt(A))$.
(ii) fuzzy b-closed set iff $(IntCl(A) \wedge ClInt(A)) \leq A$.

2.2 Definition **[3]** Fuzzy b-closure and fuzzy b-interior of a fuzzy set A is and given by
(i) $bCl(A) = \wedge\{B$: B is a fuzzy b-closed set of X and $B \geq A\}$.
(ii) $bInt(A) = \vee\{C$: C is a fuzzy b-open set of X and $A \geq C\}$.

2.3 Definition A fuzzy set A of a fts (X, τ) is called:
(i) a generalized closed (g – closed) fuzzy set [2] if $Cl(A) \leq B$ whenever $A \leq B$ and B is fuzzy open set in (X, τ).
(ii) a fuzzy generalized b-closed (briefly fgb – closed) fuzzy set [3] if $bCl(A) \leq B$ whenever $A \leq B$ and B is fuzzy open in (X, τ).

Complement of fuzzy g – closed (resp. fuzzy gb – closed fuzzy set) set are called fuzzy g–open (resp. fuzzy gb – open) set.

2.4 Definition Let X, Y be two fuzzy topological spaces. A function f: $X \rightarrow Y$ is called
(i) fuzzy continuous (f–continuous) [7] if $f^{-1}(B)$ is fuzzy open set in X, for every fuzzy open set B of Y
(ii) fg–continuous mapping [2] if $f^{-1}(A)$ is fuzzy g – closed set in X, for every fuzzy closed set A of Y.
(iii) fb – continuous mapping [4] if $f^{-1}(A)$ is fuzzy b – closed set in X, for every fuzzy closed set A of Y.
(iv) fb* - continuous mapping [4] if $f^{-1}(A)$ is fuzzy b - closed set in X, for every fuzzy b-closed set A of Y.
(v) fb-closed mapping [4] if f(A) is fuzzy b-closed in Y for every fuzzy closed set A in X.
(vi) fb*-closed mapping [4] if f(A) is fuzzy b-closed in Y for every fuzzy b-closed set A in X.

2.5 Definition A fuzzy topological space (X, τ) is called a
(i) fuzzy $T_{1/2}$- space [2] if every fuzzy g – closed set in X is a fuzzy closed set in X.
(ii) $fbT_{1/2}$- space [4] if every fbg– closed set in X is a fuzzy b-closed set in X.

2.6 Definition A fuzzy set A of a fts (X,τ) is called a fuzzy gb-closed [4] if $bCl(A) \leq B$ whenever $A \leq B$ and B is fuzzy open.

2.7 Remark A fuzzy set A of a fts (X,τ) is called a gb-open [4] (gb-open) fuzzy set if its complements 1-A is fuzzy gb-closed set.

3. FUZZY gb-CONTINUOUS MAPS IN FTS
In this section, we introduce fuzzy gb-continuous maps, fuzzy gb-irresolute maps, fuzzy gb-closed maps, fuzzy gb-open maps and fuzzy gb- homeomorphism in fuzzy topological spaces and study some of their properties.

3.1 Definition A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy generalized b-continuous (briefly fgb-continuous), if $f^{-1}(A)$ is fgb-closed set in X, for every fuzzy-closed set A in Y.

3.2 Theorem f: $(X,\tau) \to (Y,\sigma)$ is fgb-continuous iff the inverse image of each fuzzy open set of Y is fgb-open set of X.

Proof Let B be a fgb-open set of Y. 1-B is fgb-closed in Y. Since f : X→Y is fgb-continuous $f^{-1}(1-B) = 1- f^{-1}(B)$ is fgb-closed set of X. Hence $f^{-1}(B)$ is fgb-open set of X.

Converse, is obvious.

3.3 Definition A map f : $(X,\tau) \to (Y,\sigma)$ is said to be fuzzy gb-continuous (briefly fgb-continuous) if the inverse image of every fuzzy open set in Y is fgb-open set in X.

3.4 Theorem If f: $(X,\tau) \to (Y,\sigma)$ is fgb-continuous then (a) for each fuzzy point x_p of X and each A∈Y such that $f(x_p)$ *q*A, there exists a fgb-open set A of X such that $x_p \in B$ and $f(B) \leq A$.

(b) for each fuzzy point x_p of X and each A∈Y such that $f(x_p)$ *q*A, there exists a fgb-open set B of X such that x_p *q* B and $f(B) \leq A$.

Proof (a)Let x_p be a fuzzy point of X,then $f(x_p)$ is a fuzzy point in Y.Now let A ∈ Y be a fgb-open set such that $f(x_p)$ *q*A. Put B= $f^{-1}(A)$. Since f : X→Y is fgb-continuous B is fgb-open set of X and $x_p \in B$, $f(B)=f(f^{-1}(A)) \leq A$.

(b) Let x_p be a fuzzy point of X, and let A∈ Y such that $f(x_p)$ *q*A. Put B= $f^{-1}(A)$. Since f: X→Y is fgb-continuous B is fgb-open set of X, such that x_p *q* B and $f(B) = f(f^{-1}(A)) \leq A$.

3.5 Theorem Every f-continuous function is fgb-continuous function

Proof Let f: X → Y be a f-continuous function. Let A be an open fuzzy set in Y. Since f is f-continuous, $f^{-1}(A)$ is open in X. And so $f^{-1}(A)$ is fgb-open set in X. Therefore f is fgb-continuous function

The converse of the above theorem need not be true as seen from the following example.

3.6 Example Let X =Y ={a, b} and the fuzzy sets A and B be defined as follows: A = {(a, 1), (b, 0.9)},

B={(a,0.4),(b, 0.5)} Consider τ= {0,1, A} and σ = {0,1, B}.

bO(X)={0,1,A,(a,α),(b,β)},where α> 0 or β>0.1.

bC(X)= {0,1,A,(a, α),(b, β)}, where α = 0 or β < 0.1.

Then (X, τ) and (Y, σ) are fts. Let f: X →Y be the identity map. Then f is fgb-continuous map but not fuzzy-continuous, since for the fuzzy open set B in Y, $f^{-1}(B)$ is not fuzzy closed set in X but it is fgb-closed in X.

3.7 Definition A mapping f :$(X,\tau) \to (Y,\sigma)$ is said to be fuzzy b-generalized irresolute (briefly fgb-irresolute),if $f^{-1}(A)$ is fgb-closed set in X, for every fgb-closed set A in Y.

3.8 Theorem A mapping f :$(X,\tau) \to (Y,\sigma)$ is fgb-irresolute mapping if and only if the inverse image of every gb - open fuzzy set in Y is gb-open fuzzy set in X.

3.9 Theorem Every fgb-irresolute mapping is fgb-continuous .

Proof Let f: X→Y is fgb-irresolute . Let F be a closed fuzzy set in Y, Then F is fgb-closed fuzzy set in Y. Since f is fgb-irresolute, $f^{-1}(V)$ is a gb-closed fuzzy set in X. Hence f is fgb-continuous .

The converse of the above theorem need not be true as seen from the following example.

3.10 Example Let X = Y ={a, b} and the fuzzy sets A, B, C, D and E be defined as follows. A={(a, 0.9), (b, 0.9)}, B = {(a, 0.8), (b, 0.5)}, C = {(a, 0.7), (b, 0.5)}, D={(a, 0.5), (b, 0.2)}, E={(a, 0.5), (b, 0.6)}. Consider T = {0, 1, A, B, C, D} and σ = {0, 1, C}. Then (X, T) and (Y, σ) are fts. Define f: X → Y by f(a)=c, f(b)=a and f(c)=b. Then f is fgb-continuous but not fgb-irresolute as the fuzzy set E is gb-closed fuzzy set in Y, but $f^{-1}(E)$ = C is not gb-closed fuzzy set in X.

3.11 Theorem Let f :$(X,\tau) \to (Y,\sigma)$, g: $(Y,\sigma) \to (Z, \gamma)$ be two functions. Then

(1) g •f: X→ Z is fgb-continuous, if f is fgb-continuous and g are f-continuous.

(2) g•f: X→ Z is fgb- irresolute, if f and g are fgb-irresolute functions.

(3) g•f: X→ Z is fgb – continuous if f is fgb- irresolute and g is fgb-continuous.

Proof (1) Let B be fuzzy closed subset of Z. Since g :Y →Z is fuzzy continuous, by definition $g^{-1}(B)$ is fuzzy closed set of Y.Now f : X→Y is fgb-continuous and $g^{-1}(B)$ is fuzzy closed set of Y, so by definition 3.3, $f^{-1}(g^{-1}(B)) = (g\bullet f)^{-1}(B)$ is fgb-closed in X. Hence g • f : X → Z is fgb-continuous.

(2) Let g :Y→Z fgb-irresolute and let B be fgb-closed subset of Z.Since g is fgb-irresolute by definition 3.7, $g^{-1}(B)$ is fgb-closed set of Y. Also f : X→Y is fgb-irresolute, so $f^{-1}(g^{-1}(B)) = (g\bullet f)^{-1}(B)$ is fgb-closed. Thus g • f : X → Z is fgb-irresolute.

(3) Let B be fuzzy b-closed subset of Z.Since g : Y →Z is fgb-continuous, $g^{-1}(B)$ is fgb-closed subset of Y. Also f : X →Y is fgb-irresolute, so every fgb-closed set of Y is fgb-closed in X.Hence $f^{-1}(g^{-1}(B)) = (g\bullet f)^{-1}(B)$ is fgb-closed set of X. Thus g•f : X →Z is fgb- continuous.

3.12 Definition A fuzzy topological space (X,τ) is fuzzy $gbT_{1/2}$-space (in short $fgbT_{1/2}$-space) if every fgb-closed set in X it is fuzzy b-closed in X.

3.13 Theorem A fuzzy topological space X is $fgbT_{1/2}$-space if and only if every fuzzy set in X is both fuzzy b-open and fgb-open.

Proof Let X be $fgbT_{1/2}$- space and let A be fgb-open set in X. Then 1-A is gb-closed. By hypothesis every fgb-closed set is fuzzy b-closed, 1-A is fuzzy b-closed set and hence A is fuzzy b-open in X.

Conversely, let A be fgb- closed. Then 1-A is fgb-open which implies 1-A is fuzzy b-open. Hence A is fuzzy b-closed. Every fgb-closed set in X is fuzzy b-closed .Therefore X is $fgbT_{1/2}$- space.

3.14 Theorem In a fts X every fuzzy $T_{1/2}$- space is $fgbT_{1/2}$-space.

3.15 Theorem If f : (X, τ)→ (Y, σ) is fb*-continuous and g :(Y, σ)→ (Z,γ) is fgb-continuous then g • f : (X, τ) → (Z, γ) is fb*-continuous if Y is $fgbT_{1/2}$- space.

Proof Suppose A is fuzzy b-closed subset of Z. Since g :Y → Z is fgb-continuous by definition,every inverse image of fuzzy closed set of Z is fgb-closed in Y.Hence $g^{-1}(B)$ is fgb-closed subset of Y.Now Y is $fgbT_{1/2}$- space and by definition , every fgb-closed set is fuzzy b-closed in Y. Hence $g^{-1}(B)$ is fuzzy b-closed subset of Y. Also f :X→Y is fb*-continuous so by definition, inverse image of fuzzy b-closed set in Y is fuzzy b-closed in X. Hence $f^{-1}(g^{-1}(B))=(g\bullet f)^{-1}(B)$ is fuzzy b-closed. Thus $g \bullet f : X \to Z$ is fb*-continuous.

3.16 Theorem Let $f : (X,\tau) \to (Y,\sigma)$ be fgb-continuous. Then f is fb-continuous if X is $fgbT_{1/2}$ -space.
Proof Let B be fuzzy closed set in Y.Since $f : X \to Y$ is fgb-continuous, $f^{-1}(B)$ is fgb-closed subset in X. Since X is $fgbT_{1/2}$-space, by hypothesis , every fgb-closed set is fuzzy b-closed . Hence $f^{-1}(A)$ is fuzzy b-closed subset in X. Therefore $f : X \to Y$ is fb-continuous.

3.17 Theorem Let $f :(X, \tau) \to (Y, \sigma)$ be onto,fgb-irresolute and fb*-closed. If X is $fgbT_{1/2}$ -space, then Y is $fgbT_{1/2}$-space.
Proof Let A be a fgb-closed set in Y. Since $f : X \to Y$ is fgb-irresolute, $f^{-1}(A)$ is fgb-closed set in X. As X is $fgbT_{1/2}$-space, $f^{-1}(A)$ is fuzzy b-closed set in X. Also $f : X \to Y$ is fb*-closed, so $f(f^{-1}(A))$ is fuzzy b-closed in Y. Since f is onto, $f(f^{-1}(A))=A$. Thus A is fuzzy b-closed in Y. Hence Y is also $fgbT_{1/2}$-space.

3.18 Theorem If the bijective map $f : (X,\tau) \to (Y,\sigma)$ is f-open and fgb-irresolute, then f is fgb-irresolute.
Proof Let A be a fgb-closed set in Y and let $f^{-1}(A) \le B$ where B is a fuzzy open set in X . Clearly, $A \le f(B)$. Since $f : X\to Y$ is f-open map,by definition f (B) is fuzzy open in Y and A is fgb-closed set in Y. Then $bCl(A) \le f(B)$, and hence $f^{-1}(bCl(A)) \le B$. Also f is fgb-irresolute and bCl(A) is a fuzzy b-closed set in Y, then $f^{-1}(bCl(A))$ is b-closed set in X.Thus $bCl(f^{-1}(A)) \le bCl(f^{-1}(bCl(A))) \le B$. So $f^{-1}(A)$ is fgb -closed set in X. Hence $f : X\to Y$ is fgb-irresolute map.

3.19 Theorem Let f: $(X,\tau)\to(Y,\sigma)$ be fgb-continuous and g: $Y\to Z$ be fg-continuous. Then $g\bullet f$ fgb-continuous if Y is fuzzy $T_{1/2}$-space.
Proof Let A be fuzzy closed set in Z. Since g is fg-continuous, $g^{-1}(A)$ is fg-closed in Y. But Y is fuzzy $T_{1/2}$-space and so $g^{-1}(A)$ is fuzzy closed in Y.Since f is fgb-continuous $f^{-1}(g^{-1}(A)) = (g\bullet f)^{-1}(A)$ is fgb-closed in X. Hence $g\bullet f$ fgb-continuous.

3.20 Definition A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be fuzzy gb-open (briefly fgb-open) map if the image of every fuzzy open set in X, is fgb-open set in Y.

3.21Definition A mapping $f:(X, \tau)\to(Y, \sigma)$ is said to be fuzzy gb-closed (briefly fgb-closed) map if the image of every fuzzy closed set in X is fgb-closed set in Y.

3.22 Definition A mapping $f:(X,\tau) \to(Y,\sigma)$ is said to be fuzzy gb*-open (briefly fgb*-open) map if the image of every fgb-open set in X, is fgb-open set in Y.

3.23 Definition A mapping $f:(X,\tau) \to (Y,\sigma)$ is said to be fuzzy gb*-closed (briefly fgb*-closed) map if the image of every fgb-closed set in X is fgb-closed set in Y.

3.24 Remark Every fgb*-open (fgb*-closed) mapping is fgb-open(fgb-closed)

The converse of all of the above statements are not true **.**

3.25 Example Let X={a,b},Y={x,y}, A={(a,0.8),(b,0.6)} , B={(a,0.4),(b,0.3)} .Let τ={0,1,A}, σ ={0,1,B}. Then the mapping $f:(X, \tau)\to (Y, \sigma)$ defined by f(a)=x and f(b)=y is fb-open but not fb*-open.

3.26 Theorem If $f : (X,\tau) \to(Y,\sigma)$ is f-closed and $g:(Y,\sigma) \to(Z,\gamma)$ is fgb-closed, then g o f is fgb-closed.

Proof For a fuzzy closed set in X, f (A) is fuzzy closed in Y. Since $g:Y\to Z$ is fgb-closed g(f(A)) is fgb-closed in Z. g(f(A)) =(gof)(A) is fgb-closed in Z. Therefore gof is fgb-closed.

3.27 Theorem If $f : (X,\tau) \to(Y,\sigma)$ is a fgb- open map and Y is $fgbT_{1/2}$ -space, then f is a f- open map.
Proof Let A be an fuzzy open set in X. Then f(A) is $fgbT_{1/2}$ -space fgb-open set in Y since f is fgb- open map. Again since Y is $fgbT_{1/2}$ -space, f(A) is fuzzy open set in Y. Hence f: $X\to Y$ be a fuzzy open map.

3.28 Theorem If $f : (X,\tau) \to(Y,\sigma)$ be a fgb- closed map and Y is $fgbT_{1/2}$ - space, then f is a f-closed map.

3.29 Theorem A map $f : (X,\tau) \to(Y,\sigma)$ is fgb- closed if and only if for each fuzzy set A of Y and for each fuzzy open set B such that $f^{-1}(A) \le B$, there is a fgb- open set C of Y such that $A \le C$ and $f^{-1}(C) \le B$.
Proof Suppose f is fgb- closed map. Let A be a fuzzy set of Y, and B be a fuzzy open set of X, such that $f^{-1}(A) \le B$. Then C = 1-f(1-B) is a fgb - open in Y such that $A \le C$ and $f^{-1}(C) \le B$.

Conversely, suppose that F is a fuzzy closed set of X. Then $f^{-1}(1-f(F))\le 1-F$, and 1-F is fuzzy open set. By hypothesis, there is a fgb - open set C of Y such that $1-f(1-B) \le C$ and $f^{-1}(C) \le 1-F$. Therefore $F\le 1-f^{-1}(C)$. Hence $1-C \le f(C) \le f(1- f^{-1}(C)) \le 1-C$, which implies f(F) = 1 - C. Since 1 - C is fgb - closed set, f(F) is fgb - closed set and thus f is a fgb - closed map.

3.30 Theorem If $f : (X,\tau) \to(Y,\sigma)$ and g: $Y\to Z$ are fgb-closed maps and Y is $fgbT_{1/2}$ -space, then gof: $X \to Z$ is fgb-closed map.

3.31 Theorem Let f: $X\to Y$, $g:Y\to Z$ be two maps such that $g\bullet f : X \to Z$ is fgb - closed map.

i) If f is fuzzy continuous and surjective, then g is fgb - closed map.
ii) If g is fgb-irresolute and injective, then f is fgb-closed map.
Proof i) Let F be a fuzzy closed set of Y. Then $f^{-1}(F)$ is fuzzy closed set in X as f is fuzzy continuous. Since $g \bullet f$ is fgb-closed map, $(g\bullet f)(f^{-1}(F)) = g(F)$ is fgb-closed in Z. Hence g: $Y\to Z$ fgb - closed map.
ii) Let F be a fuzzy closed set in X. Then $(g\bullet f)(F)$ is fgb-closed in Z, and so $g^{-1}(g\bullet f)(F) = f(F)$ is fgb-closed in Y. Since g is fgb-irresolute and injective. Hence f is a fgb- closed map.

3.32 Theorem If A is fgb-closed fuzzy set in X and f: $X\to Y$ is bijective, f-continuous and fgb-closed, then f(A) is fgb-closed fuzzy set in Y.
Proof Let $f(A) \le B$ where B is an fuzzy open set in Y. Since f is f-continuous, $f^{-1}(B)$ is an fuzzy open set containing A. Hence $bCl(A) \le f^{-1}(B)$ as A is fgb-closed set. Since f is fgb-closed, f(bCl(A)) is fgb-closed set contained in the fuzzy open

set B, which implies $bCl(f(bCl(A))) \leq B$ and hence $bCl(f(A)) \leq B$. So f(A) is fgb-closed set in Y.

3.33 Theorem If $f:(X,\tau)\rightarrow(Y,\sigma)$ is fgb-closed and $g:(Y,\sigma)\rightarrow(Z,\gamma)$ is fgb*-closed, then g•f is fgb*-closed. *Proof* For a fuzzy closed set in X, f (A) is fgb-closed in Y. Since $g:Y\rightarrow Z$ is fgb*-closed g(f(A)) is fgb-closed in Z. g(f(A)) =(g•f)(A) is fgb-closed in Z. Therefore g•f is fgb-closed.

3.34Theorem If f: $X\rightarrow Y$ and g: $Y\rightarrow Z$ are fgb*- closed maps, then g•f: $X \rightarrow Z$ is fgb*-closed map.

3.35 Theorem Let f: $X\rightarrow Y$, g:$Y\rightarrow Z$ be two maps such that g•f:$X \rightarrow Z$ is fgb* - closed map.

i) If f is fgb-continuous and surjective, then g is fgb - closed map.
ii) If g is fgb-irresolute and injective, then f is fgb*-closed map.
Proof i) Let F be a fuzzy closed set of Y. Then $f^{-1}(F)$ is fgb-closed in X as f is fgb-continuous. Since g•f is fgb*-closed map, (g•f) $(f^{-1}(F))$ = g(F) is fgb-closed set in Z. Hence g: $Y\rightarrow Z$ fgb - closed map.
ii) Let F be a fgb-closed set in X. Then (g•f) (F) is fgb-closed fuzzy set in Z. Since g is fgb-irresolute and injective g^{-1}(g•f)(F) =f(F) is fgb-closed in Y. Hence f is a fgb*- closed map.

3.36 Definition A function f : $(X,\tau) \rightarrow(Y,\sigma)$ is called fuzzy gb - homeomorphism (briefly fgb- homeomorphism) if f and f^{-1} are fgb- continuous.

3.37 Theorem Every f-homeomorphism is fgb-homeomorphism.
Proof Let f: $X \rightarrow Y$ be fuzzy homeomorphism. Then f and f^{-1} are f-continuous. By theorem 3.9 f and f^{-1} are fgb – continuous. Hence f is fgb- homeomorphism.

The converse of the above theorem need not be true as seen from the following example.

3.38 Example Let X = Y ={a, b, c} and the fuzzy sets A, B and C be defined as follows. A = {(a, 1), (b, 0.8), (c, 0.8)}, B = {(a, 0.3), (b, 0.6), (c, 0.8)},C ={(a, 0.4), (b, 0.6), (c, 0.8)}. Consider τ = {0, 1, A} and σ = {0, 1, B}. Then (X, τ) and (Y,σ) are fts. Define f: $X\rightarrow Y$ by f(a)=a, f(b)=c and f(c)=b. Then f is f fgb-homeomorphism but not f - homeomorphism as A is open in X f(A) = A is not open in Y. f^{-1}: $Y\rightarrow X$ is not f-continuous.

3.39 Theorem Let f : $(X,\tau) \rightarrow(Y,\sigma)$be a bijective function. Then the following are equivalent:
a) f is fgb - homeomorphism.
b) f is fgb - continuous and fgb- open maps.
c) f is fgb-continuous and fgb-closed maps.
Proof (a) $\Rightarrow$ (b): Let f be fgb - homeomorphism. Then f and f^{-1} are fgb - continuous. To prove that f is fgb- open map. Let A be an fuzzy open set in X. Since f^{-1}:$Y\rightarrow X$ is fgb-continuous, $(f^{-1})^{-1}(A)$ =f (A) is fgb - open in Y. Therefore f(A) is fgb - open in Y. Hence fgb- open map.
(b)$\Rightarrow$(a) Let f be fgb- open and fgb- continuous map. To prove that f^{-1}:$Y\rightarrow X$ is fgb - continuous. Let A be an fuzzy open set in X. Then f (A) is fgb - open set in Y since f is fgb - open map. Now $(f^{-1})^{-1}(A)$ = f (A) is fgb - open set in Y. Therefore f^{-1}: $Y\rightarrow X$ is fgb - continuous. Hence f is fgb - homeomorphism.
(b) $\Rightarrow$(c) Let f be fgb - continuous and fgb - open map. To prove that f is fgb - closed map. Let B be a closed fuzzy set in X. Then 1 - B is fuzzy open set in X. Since f is fgb - open map, f(1-B) is fgb - open fuzzy set in Y. Now f(1-B)=1 - f(B). Therefore f(B) is fgb-closed in Y. Hence f is a fgb - closed map.
(c) $\Rightarrow$ (b) Let f be fgb - continuous and fgb-closed map. To prove that f is fgb-open map. Let A be an fuzzy open set in X. Then 1-A is a fuzzy closed set in X. Since f is fgb - closed map, f(1–A) is fgb-closed in Y. Now f(1-A)=1-f(A). Therefore f(A) is fgb - open in Y. Hence f is fgb - open map.

3.40 Theorem If f : $(X,\tau) \rightarrow(Y,\sigma)$ fgb-homeomorphism and g: $Y\rightarrow Z$ is fgb- homeomorphism and Y is $fgbT_{1/2}$ -space, then gof: $X \rightarrow Z$ is fgb- homeomorphism.
Proof To show that g•f and $(g\bullet f)^{-1}$ are fgb- continuous. Let A be an open fuzzy set in Z. Since g: $Y\rightarrow Z$ is fgb - continuous, $g^{-1}(A)$ is fgb -open in Y. Then $g^{-1}(A)$ is open fuzzy set in Y as Y is $fgbT_{1/2}$-space. Also since f : $X\rightarrow Y$ is fgb- continuous, $f^{-1}(g^{-1}(A))=(g\bullet f)^{-1}(A)$ is fgb - open in X. Therefore g•f is fgb - continuous.

Again, let A be an fuzzy open set in X. Since f^{-1}: $Y\rightarrow X$ is fgb - continuous, $(f^{-1})^{-1}(A)$) = f(A) is fgb-open set in Y. And so f(A) is fuzzy open set in Y as Y is $fgbT_{1/2}$ -space. Also since g^{-1}:$Z\rightarrow Y$ is fgb - continuous, $(g^{-1})^{-1}(f(A))$ = g(f(A)) = (g•f)(A) is fgb-open set in Z. Therefore $((g\bullet f)^{-1})^{-1}(A)$ = (g•f)(A) is fgb-open fuzzy set in Z.Hence $(g\bullet f)^{-1}$ is fgb - continuous. Thus g •f is fgb - homeomorphism.

3.41 Definition A function f : $(X,\tau) \rightarrow(Y,\sigma)$ is called fuzzy gb* - homeomorphism (briefly fgb*- homeomorphism) if f and f^{-1} are fgb- irresolute.

3.42 Theorem Every fgb*- homeomorphism is fgb-homeomorphism.

Proof Let f : $(X,\tau) \rightarrow(Y,\sigma)$ be fgb*- homeomorphism. Then f and f^{-1} are fgb- irresolute mappings. By theorem 3.9 f and f^{-1} are fgb- continuous. Hence f : $X \rightarrow Y$ is fgb-homeomorphism.

3.43 Theorem If f : $(X,\tau) \rightarrow(Y,\sigma)$, g: $Y\rightarrow Z$ be fgb*-homeomorphism then their composition g•f: $X \rightarrow Z$ is fgb*-homeomorphism.

Proof Let A be a fgb-open set in Z. Then since g: $Y\rightarrow Z$ is f:$X\rightarrow Y$, $g^{-1}(A)$ is fgb-open in Y. Also since f: $X\rightarrow Y$ is fgb-irresolute, $(f^{-1}(g^{-1}(A))$ = $(g\bullet f)^{-1}$ (A) is fgb-open in X. Therefore g•f : $X \rightarrow Z$ is fgb- irresolute. Again, let A be a fgb-open set in X. Then since f^{-1} : $Y \rightarrow X$ is fgb-irresolute, $(f^{-1})^{-1}(A)$ = f(A) is fgb-open in Y. Also g^{-1}: $Z \rightarrow Y$ is fgb-irresolute, $(g^{-1})^{-1}$(f(A) = g(f(A) = (g•f) (A) is fgb-open in Z. Therefore $(g\bullet f)^{-1}$:$Z \rightarrow X$ is fgb-irresolute. Hence g•f : $X \rightarrow Z$ is fgb*- homeomorphism.

3.44 Theorem Let f : $(X,\tau) \rightarrow(Y,\sigma)$be a bijective function. Then the following are equivalent:
a) f is fgb* - homeomorphism.
b) f is fgb - irresolute and fgb*- open maps.
c) f is fgb-irresolute and fgb*-closed maps.
Proof (a) $\Rightarrow$ (b): Let f be fgb* - homeomorphism. Then f and f^{-1} are fgb - irresolute. To prove that f is fgb*- open map. Let A be fgb-open set in X. Since f^{-1} : $Y \rightarrow X$ is

fgb-irresolute, $(f^{-1})^{-1}(A)$ =f (A) is fgb - open in Y. Therefore f(A) is fgb* - open in Y. Hence fgb*- open map.

(b)⇒(a) Let f be fgb*- open and fgb- irresolute map. To prove that f^{-1}:Y→X is fgb -irresolute. Let A be fgb-open fuzzy set in X. Then f (A) is fgb - open set in Y since f is fgb* - open map. Now $(f^{-1})^{-1}(A)$ = f (A) is fgb - open set in Y. Therefore f^{-1}: Y→ X is fgb - irresolute. Hence f is fgb* - homeomorphism.

(b) ⇒(c) Let f be fgb - irresolute and fgb*- open map. To prove that f is fgb* - closed map. Let B be a closed fuzzy set in X. Then 1 - B is fuzzy open set in X. Since f is fgb - open map, f(1-B) is fgb - open in Y. Now f(1-B)=1 - f(B).

Therefore f(B) is fgb-closed in Y. Hence f is a fgb*- closed map.

(c) ⇒ (b) Let f be fgb - irresolute and fgb*-closed map. To prove that f is fgb*-open map. Let A be an fgb-open set in X. Then 1-A is a fgb-closed in X. Since f is fgb* - closed map, f(1–A) is fgb-closed in Y. Now f(1-A)=1-f(A). Therefore f(A) is fgb - open in Y. Hence f is fgb* - open map.

Definition 3.45 Let A be a fuzzy set in fts X and x_p is a fuzzy point of X, then A is called fuzzy generalized b-neighborhood (briefly fgb-neighbourhood) of x_p if and only if there exists a fgb-open set B of X such that $x_p \in B \leq A$.

Definition 3.46 Let A be a fuzzy set in fts X and x_p is a fuzzy point of X, then A is called fuzzy generalized b-q-neighbourhood (briefly fgbq-neighbourhood) of x_p if and only if there exist a fb-open set B such that $x_p\, q\, B \leq A$.

Theorem 3.47 A is fgb-open set in X if and only if for each fuzzy point $x_p \in A$, A is a fgb-neighbourhood of x_p.

Proof Let A be fgb-open set X. For each $x_p \in A$, A ≤ A. Therefore A is a fgb-neighborhood of x_p..

Conversely**,** let A be a fgb-neighbourhood of x_p. That implies, there exist a fgb-open set B such that $x_p \in B \leq A$. Therefore A is fgb-open set in X.

Theorem 3.48 If A and B are fgb-neighborhood of x_p then A Λ B is also a fgb-neighbourhood of x_p.

Theorem 3.49 Let A be a fuzzy set of a fts X. Then a fuzzy point $x_p \in$ bCl (A) if and only if every fgbq-neighbourhood of x_p is quasi-coincident with A.

Theorem 3.51 Let f: (X,τ) → (Y,σ). Then the following statements are equivalent.

(a) f is fgb-irresolute.

(b) for every fgb-closed set A in Y,$f^{-1}(A)$ is fgb-closed in X.

(c) for every fuzzy point x_p of X and every fgb-open set A of Y such that $f(x_p) \in A$, there exist a fgb-open set such that $x_p \in B$ and f(B) ≤ A.

(d) for every fuzzy point x_p of X and every fgb-neighbourhood A of f(x), $f^{-1}(A)$ is a fgb-neighbourhood of x_p.

(e) for every fuzzy point x_p of X and every fgb-neighbourhood A of $f(x_p)$, there is a fgb-neighbourhood B of x_p such that f(B) ≤ A.

(f) for every fuzzy point x_p of X and every fgb-open set A of Y such that $f(x_p)$ q A, there exists a fgb-open set B of X such that x_p q B and f(B) ≤ A.

(g) for every fuzzy point x_p of X and every fgbq-neighbourhood A of $f(x_p)$, $f^{-1}(A)$ is a fgbq-neighbourhood of x_p.

(h) for every fuzzy point x_p of X and every fgbq-neighbourhood A of $f(x_p)$, there exists a fgbq-neighbourhood B of x_p such that f(B) ≤ A.

Proof (a)⇒ (b) Obvious.

(b) ⇒ (a) A is a fgb-closed set in Y implies 1–A is fgb-open in Y.f^{-1}(1-A) is fgb-open in X implies $f^{-1}(A)$ is fgb-closed in X. Hence f is fb-irresolute.

(a) ⇒ (c) Obvious.

(c) ⇒ (a) Let A be a fgb-open set in Y and $x_p \in f^{-1}(A)$ implies $f(x_p) \in A$. Then there exist a fgb-open set B in X such that $x_p \in B$ and f(B) ≤ A. Hence $x_p \in B \leq f^{-1}(A)$. $f^{-1}(A)$ is fgb-open in X. Hence f is fgb-irresolute.

(a) ⇒ (d) Obvious.

(d) ⇒ (a) Obvious.

(d) ⇒ (e) Let x_p be a fuzzy point of X and A be a fgb-neighbourhood of $f(x_p)$. Then $B = f^{-1}(A)$ is a fgb-neighbourhood of x_p and $f(B) = f(f^{-1}(A)) \leq A$.

(e) ⇒ (c) Let x_p be a fuzzy point of X and A be a fgb-open set such that $f(x_p) \in A$. Then A is a fgb -neighbourhood of $f(x_p)$. Hence there is fgb-neighbourhood B of x_p in X such that $x_p \in B$ and f(B) ≤ A. Hence there is fgb-open set C in X such that $x_p \in C \leq B$ and f (C) ≤ f(B) ≤ A.

(a) ⇒ (f) Let x_p be a fuzzy point of X and A be a fgb-open set in Y such that $f(x_p)$ *q* A. Let $B = f^{-1}(A)$. B is a fgb-open set in X, such that $x_p\, q\, B$ and $f(B)= f(f^{-1}(A)) \leq A$.

(f) ⇒ (a) Let A be a fuzzy open set in Y and $x_p \in f^{-1}(A)$. Clearly $f(x_p) \in A$.$[x_p(x)] = 1 - x_p(x)$.Then $f(1-x_p)$ *q* A .Hence there exists a fgb-open set B of X such that $(1-x_p)$ *q* B and f(B) ≤A. Now $(1- x_p)\, q\, B \Rightarrow (1-x_p)(x) + B(x) = 1-p + B(x) > 1 \Rightarrow B(x) > p \Rightarrow x_p \in B$. Thus $x_p \in B \leq f^{-1}(A)$. Hence $f^{-1}(A)$ is fgb-open in X.

(f) ⇒ (g) Let x_p be a fuzzy point of X and A be fgbq-neighbourhood of $f(x_p)$. Then there is fgb-open set C in Y such that x_p *q* C ≤A. By hypothesis there is a fgb-open set B of X such that x_pq B and f(B) ≤ C. Thus $x_p\, q\, B \leq f^{-1}(C) \leq f^{-1}(A)$. Hence $f^{-1}(A)$ is a fgbq-neighbourhood of x_p.

(h) ⇒ (f)) Let x_p be a fuzzy point of X and A be fgb-open in Y such that $f(x_p)$ *q* A. Then A is fgbq-neighbourhood of $f(x_p)$.So there is a fgbq-neighbourhood C of x_p such that f(C) ≤ A. Since C is a fgbq-neighbourhood of x_p there exists a fgb-open set B of X such that $x_p\, q\, B \leq C$. Hence x_p q B and f(B) ≤ A.

4. ACKNOWLEDGMENTS

The authors are grateful to the University Grants Commission, New Delhi, India for its financial support under UGC SAP II to the Department of Mathematics, Karnatak University, Dharwad, India.

5. CONCLUSION

It is interesting to work on the compositions of weaker and stronger forms of mappings and various properties of fgb-closed sets. Compositions of mappings can be tried with other forms of generalized closed fuzzy sets.

5. REFERENCES

[1] D.Andrijevic,On b-open sets, Mat.vesnik 48 (1996), 59-64.

[2] G. Balasubramanian and P.Sundaram, On some generalizations of fuzzy continuous, Fuzzy Sets and Systems, 86 (1997), 93-100.

[3] S.S.Benchalli and Jenifer Karnel, On Fuzzy b-open sets in Fuzzy Topological Space J.Computer and Mathematical Sciences **1**(2010),127-134.

[4] S.S.Benchalli and Jenifer Karnel**,** On Fuzzy b-neighbourhoods and Fuzzy b-mappings in Fuzzy Topological Space, J.Computer and Mathematical Sciences 1(6),(2010),696-701.

[5] S.S.Benchalli and Jenifer Karnel**,** Weaker and stronger forms of fuzzy b-irresolute maps**,** Int. J. Math., Anal.,(communicated).

[6] S.S.Benchalli and Jenifer Karnel, On fbg-closed sets and fb-seperation Axioms in Fuzzy Topological Spaces, (communicated).

[7] C.L.Chang, Fuzzy topological spaces, J. Math. Anal. Appl. (1968), 182-190. [9] L.A. Zadeh, Fuzzy sets, Inform. and Control, 8 (1965), 338-353.

[8] Pao-Ming Pu and Ying- Ming Liu, Fuzzy Topology-I.Neighborhood structure of fuzzy point and Moore-smith convergence, *J. Math.Anal.Appl.* **6** (1980) 571-599.

[9] L.A.Zadeh, Fuzzy sets, *Inform.Contr*. **8**(1965) 338-353.