Type-2 TSK Fuzzy Logic System and its Type-1 Counterpart

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ABSTRACT
An interval type-2 TSK fuzzy logic system can be obtained by considering the membership functions of its existed type-1 counterpart as primary membership functions and assigning uncertainty to cluster centers, standard deviation of Gaussian membership functions and consequence parameters. In many cases it has been difficult to determine the spread percentages for these parameters to obtain an optimal model. In order to develop robust and reliable solutions for the problems, this paper distinguishes the differences between type-2 TSK system and its counterpart, analyzes the sensibility of the outputs of a type-2 TSK fuzzy system, and discusses the approximation capacities of type-2 TSK FLS and its type-1 counterpart as well.

General Terms
Research article

Keywords
fuzzy logic system, membership functions, uncertainty, sensibility, capability

1. INTRODUCTION
Takagi-Sugeno-Kang (TSK) qualitative modeling based on fuzzy logic [1, 2], as known as TSK modeling, was proposed in an effort to develop a systematic approach to generating fuzzy rules from a given input-output data set. TSK fuzzy logic systems (FLSs) are widely used for model-based control and model-based fault diagnosis. This is due to the system's properties of, on one hand being a general nonlinear approximator that can approximate every continuous mapping, and on the other hand being a piecewise linear model that is relatively easy to interpret [3] and whose linear submodels can be exploited for control and fault detection.

Based on the extension principle [4, 5], Mendel and his co-authors extended previous studies and established a complete type-2 fuzzy logic theory with the handling of uncertainties [6]. Type-2 TSK FLS was presented in 1999 [7]. Because the universal approximation property and the capability of handling rule uncertainties in a more complete way, interval type-2 FLSs are gaining more and more in popularity. More and more fuzzy experts see the shortcomings of type-1 FLS, and apply type-2 FLS to situations where uncertainties abound.

One of design methods for a type-2 TSK FLS is by considering the membership functions of an existed type-1TSK FLS as primary membership functions (MFs) and assigning uncertainty to cluster centers, standard deviation of Gaussian MF and consequence parameters.. There is no theory that guarantees that the type-2 TSK FLS have the potential to outperform its type-1 counterpart. In many cases it has been difficult to determine the spread percentages for these parameters to obtain an optimal model.

The aim of this paper is to distinguish the differences between type-2 TSK system and its counterpart, ascertain how the root-means-square-error (RMSE) of a type-2 TSK model depend upon spread percentage of cluster centers and consequent parameters. The approximation capacities of type-2 TSK FLS and its type-1 counterpart are discussed.

In this paper, Section 1 contains TSK fuzzy modeling development and some introductory remarks. Section 2 recalls the initial theoretical foundation: type-1 and type-2 TSK fuzzy model. In Section 3, the algorithm of interval type-2 TSK FLS is presented. Type-2 TSK fuzzy logic system is obtained directly from its type-1 counterpart by considering the membership functions of its existed type-1 counterpart as primary membership functions and assigning uncertainty to cluster centers, standard deviation of Gaussian membership functions and consequence parameters. Section 4 is a function approximation example to analyze the influence of spread percentages of cluster centers and consequent parameters to the outputs of a type-2 model and Section 5 is the discussion of approximation capacities of type-2 model and its type-1 counterpart. The results show that spread percentages have great influence on different factors of performance of a type-2 TSK model, and a type-2 model has greater capability comparing with its type-1 counterpart. Section 6 contains concluding remarks and future research recommendations.

2. THEORETICAL FOUNDATION
The proposed linguistic approach by Zadeh [8, 9] is effective and versatile in modeling ill-defined systems with fuzziness or fully-defined systems with realistic approximations. Fuzzy qualitative modeling has the capability to model complex system behavior in such a qualitative way that the model is more

effective and versatile in capturing the behavior of ill-defined systems with fuzziness or fully defined system with realistic approximation. In the literature, different modeling techniques can be found, and TSK FLS has attracted much attention. TSK FLS consists of rules with fuzzy antecedents and mathematical function in the consequent part. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviors of the system in those regions. The main difference with more traditional [10] (Mamdani FL) fuzzy rules is that the consequents of the rules are a function of the values of the input variables, instead of fuzzy sets.

2.1 Type-1 TSK fuzzy system

A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of a system. For a multi-input-single-output (MISO) first–order type-1 TSK model, its *k*th rule can be expressed as:

$$IF\ \ x_1\ is\ Q_1^k\ and\ x_2\ is\ Q_2^k\ and\ ...\ and\ x_n\ is\ Q_n^k,$$

$$THEN\ \ Z\ is\ w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + ... + p_n^k x_n$$

where x_1, x_2 …, x_n and *Z* are linguistic variables; Q_1^k, Q_2^k, …, and Q_n^k are the fuzzy sets on universe of discourses *U*, *V*, …, and *W*, and $p_0^k, p_1^k, p_2^k, \ldots, p_n^k$ are regression parameters.

2.2 Type-2 TSK fuzzy system

Mendel in his book [6] presented the architecture of interval type-2 TSK model and proposed a complete computation method for it. Detailed type-2 fuzzy sets and interval type-2 FLS background material can be found in [11].

A generalized *k*th rule in a type-2 TSK fuzzy model can be expressed as

$$IF\ \ x_1\ is\ \tilde{Q}_1^k\ and\ x_2\ is\ \tilde{Q}_2^k and\ ...\ and\ x_n\ is\ \tilde{Q}_n^k,$$

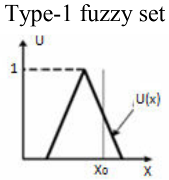
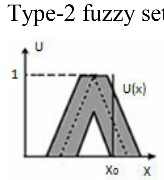
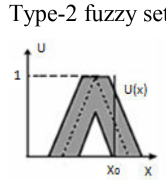
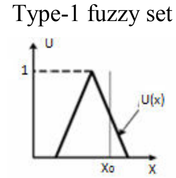
$$THEN\ Z\ is\ \tilde{w}^k = \tilde{p}_0^k + \tilde{p}_1^k x_1 + \tilde{p}_2^k x_2 + ... + \tilde{p}_n^k x_n$$

where $\tilde{p}_0^k$, $\tilde{p}_1^k$, …, $\tilde{p}_n^k$ are consequent parameters, $\tilde{w}^k$ is the output from the *k*th IF-THEN rule in a total of *M* rules fuzzy model, $\tilde{Q}_1^k$, $\tilde{Q}_2^k$, …, and $\tilde{Q}_n^k$ are fuzzy sets on universe of discourses.

2.3 Comparison between type-1 and type-2 TSK fuzzy system

Type-1 and type-2 TSK FLSs are characterized by IF-THEN rules and no defuzzification is needed in the inference engine, but they have different antecedent and consequent structures. Assuming FLSs with m rules and n antecedents in each rule, a type-1 TSK FLS is compared with a type-2 TSK FLS in Table 1. From Table 1, a type-2 TSK FLS has more design degrees of freedom than does a type-1 TSK FLS because its type-2 fuzzy sets are described by more parameters than type-1 fuzzy sets [6]. This suggests that a type-2 TSK FLS has the potential to outperform a type-1 TSK FLS because of its larger number of design degrees of freedom.

Table 1. Comparison between type-1 and type-2 TSK Fuzzy Logic System

TSK FLS		Type-1	Type-2		
			A2-C1	A2-C0	A1-C1
Structure	Antecedents	Type-1 fuzzy set 	Type-2 fuzzy set 	Type-2 fuzzy set 	Type-1 fuzzy set 
	Consequent parameters	Crisp number	Fuzzy number	Crisp number	Fuzzy number
Output		A crisp point	An interval set of output A point output		
Number of design parameters		(3p+1)M*	(5p+2)M*	(4p+1)M*	(4p+2)M*

* There are M rules and each rule has p antecedents in the fuzzy system

3. OBTAINING A TYPE-2 TSK SYSTEM FROM ITS TYPE-1 COUNTERPART

An interval type-2 TSK FLS can be obtained by considering the membership functions (MFs) of its existed type-1 counterpart as primary MFs and assigning uncertainty to cluster centers, standard deviation of Gaussian MF and consequence parameters. In the type-2 TSK fuzzy algorithm as shown in Fig. 1 [12], a width a_j^k of cluster center x_v^{k*} is extended to both two directions of cluster center x_v^{k*}, as shown in Fig. 2. By doing so, cluster centers are expanded from a certain point to a fuzzy number:

$$\tilde{x}_v^{k*} = [x_v^{k*}(1-a_j^k), x_v^{k*}(1+a_j^k)] \quad (1)$$

where a_j^k is the spread percentage of cluster centre x_v^{k*} in Fig. 2. The cluster center x_v^{k*} becomes a constant width interval valued fuzzy set $\tilde{x}_v^{k*}$.

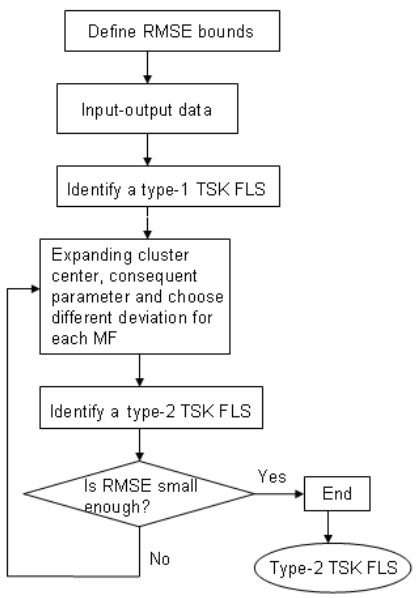


Fig 1: Diagram of type-2 TSK FLS

Hence, the premise membership is a type-2 fuzzy set, *i.e.*

$$\tilde{Q}_v^k = \exp\left[-\frac{1}{2}\left(\frac{x_v - x_v^{k*}(1 \pm a_v^k)}{\sigma_v^k}\right)^2\right] \quad (2)$$

where the standard deviation of Gaussian MF σ_v^k is with different values for each rule.

Consequent parameters are obtained by expanding consequent parameters from its type-1 counterpart to fuzzy numbers by eq.(5) where b_j^k is the spread percentage of fuzzy numbers $\tilde{p}_j^k$.

$$\tilde{p}_j^k = p_j^k(1 \pm b_j^k) \quad (3)$$

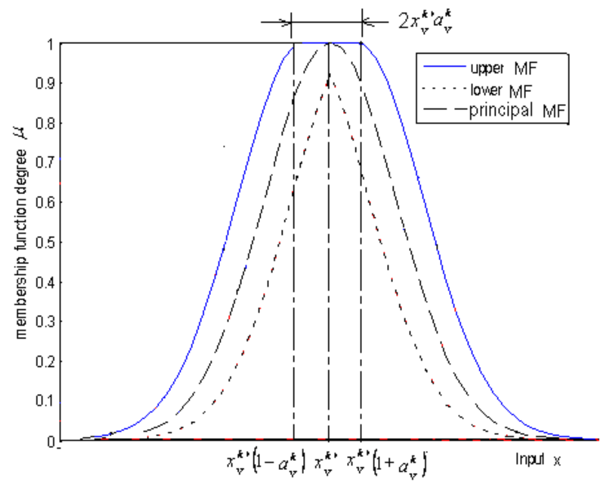


Fig. 2 Spread of cluster center

Because that the starting point for the least-squares method to design a type-1 TSK FLS is a type-1 fuzzy basis function expansion [13], the performance of a type-2 TSK FLS is evaluated using the following RMSE:

$$RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n}(W_{js} - W_{jm})^2} \quad (4)$$

4. INFLUENCE OF SPREAD PERCENTAGE OF CLUSTER CENTERS AND CONSQUANT PARAMETERS TO THE OUTPUT OF A TYPE-2 SYSTEM

In this paper, a function approximation example is used for analysis of Influence of spread percentages of cluster centers and consequent parameters to the outputs of a type-2 model.

Table 2 is a six-rule type-1 fuzzy model for approximating the following function

$$y = -(x-2.5)^3 + 3.25 \quad (5)$$

This tyep-1 fuzzy model is obtained by using subtractive clustering based type-1 TSK FLS identification algorithm described in [14, 15].

In order to extend a type-1 TSK FLS to its type-2 counterpart with emphasis on interval set, antecedent MFs have to be changed from type-1 fuzzy sets to type-2 fuzzy sets. Consequent parameters have also to be changed from a certain number to a fuzzy number. Based on the type-1 TSK model rules in Table 1, a six- rule type-2 TSK model can be expended by assigning uncertainty a_j^k, b_j^k and σ_j^k to cluster centers, standard deviation of Gaussian MF and consequence parameters.

In order to estimate how RMSE depends upon spread percentage of cluster centers and consequent parameters, The value of a_j^k and b_j^k are chosen from [0, 0.4] and that of σ_j^k is chosen from [0.2, 0.6]. The step sizes are selected as 0.01, 0.01 and 0.001.

Table 2. Six-rule type-1 fuzzy model for $y = -(x-2.5)^3 + 3.25$

Rule	If x, then $z = p_1 \times x + p_0$
1	If $x = \exp(-\frac{1}{2}\left(\frac{x-2.5*(1\pm 22.057\%)}{0.26133}\right)^2$, then $z = 2.1948 \times x(1\pm 26.361\%) - 1.9308(1\pm 10.524\%)$
2	If $x = \exp(-\frac{1}{2}\left(\frac{x-1.5*(1\pm 10.67\%)}{0.26539}\right)^2$, then $z = 3.8481 \times x(1\pm 26.361\%) - 4.3995(1\pm 10.524\%)$
3	If $x = \exp(-\frac{1}{2}\left(\frac{x-3.5*(1\pm 2.8392\%)}{0.39298}\right)^2$, then $z = 4.6866 \times x(1\pm 26.361\%) - 10.734(1\pm 10.524\%)$
4	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.8125*(1\pm 5.087\%)}{0.26071}\right)^2$, then $z = 8.9872 \times x(1\pm 26.361\%) - 11.736(1\pm 10.524\%)$
5	If $x = \exp(-\frac{1}{2}\left(\frac{x-0.3125*(1\pm 5.8798\%)}{0.28858}\right)^2$, then $z = 31.698 \times x(1\pm 26.361\%) - 32.089(1\pm 10.524\%)$
6	If $x = \exp(-\frac{1}{2}\left(\frac{x}{0.39063}\right)^2$, then $z = 41.433 \times x(1\pm 26.361\%) - 3.0343(1\pm 10.524\%)$

Figures 3 to 5 depict influence of them on RMSE of the type-2 fuzzy model.

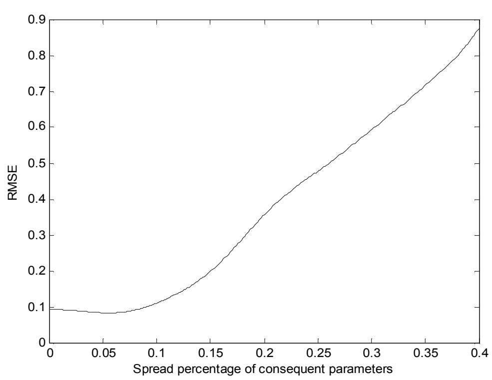


Fig. 3 Influence of a_j^k to RMSE of a type-2 fuzzy model

It is observed that a_j^k and σ_j^k, for which RMSE of type-2 model is relatively sensitive, would require future characterization, as opposed to b_j^k for which RMSE of the model is relatively insensitive. The value of a_j^k and σ_j^k decide the size of bounded regions of the union of all antecedent primary memberships – the footprint of uncertainty (FOU) which is the area between upper MF and lower MF of type-2 MF in Fig. 6.

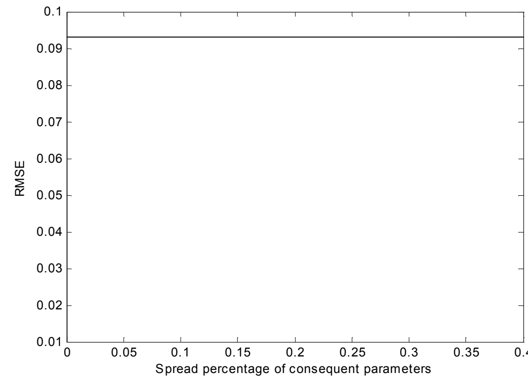


Fig. 4 Influence of b_j^k to RMSE of a type-2 fuzzy model

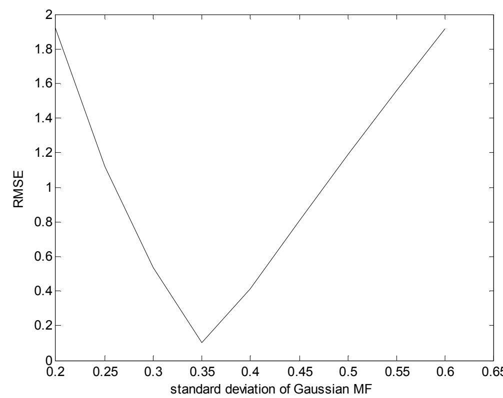


Fig. 5 Influence of σ_j^k to RMSE of a type-2 fuzzy model

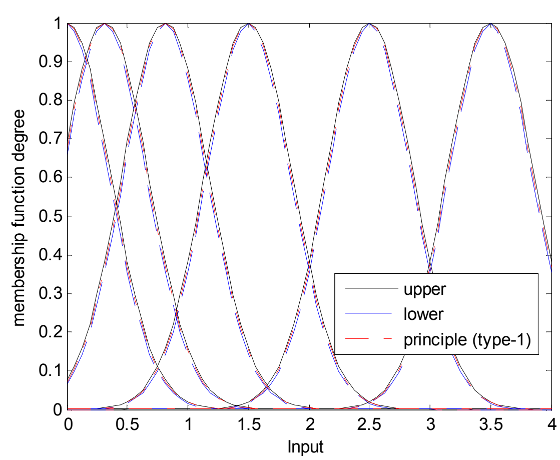


Fig. 6 MFs for type-2 TSK FLS and its type-1 counterpart

The type-2 TSK model provides more information, not only crisp output as that of type-1 TSK model, but also the interval set of the output. This interval set of the output has the information about the uncertainties that are associated with the crisp output. Outputs of a type-2 fuzzy model is very sensitive for uncertainty of consequent parameters b_j^k, especially, the interval set of the output. Table 3 summarizes influence of a_j^k, b_j^k and σ_j^k on type-2 model's RMSE, model output and Gaussian MFs.

Table 3. Summary of influence of a_j^k, b_j^k and σ_j^k to the outputs of a type-2 fuzzy model

Influence	a_j^k	b_j^k	σ_j^k
RMSE	Yes	No	Yes
Model output	Yes	Yes, Significant	Yes
Gaussian MFs	Yes, Significant	No	Yes
Model error	Yes	No	Yes

5. CAPABILITY COMPARISON

As known, there is no mathematical approve that type-2 fuzzy model always performs better than its type-1 counterpart. There is no theory that guarantees that a type-2 TSK FLS have the potential to outperform its type-1 counterpart.

To check out if the type-2 model really has the greater approximation capacity than that of its type-1 counterpart, the original data sets from the mathematical function in eq. (5) are added random noises to evaluate generalization capability of the two type fuzzy models. Random noises are chosen from different intervals, added to both input and output. RMSE of both models are calculated by comparing the model crisp outputs to the original outputs (without noise). The behaviors of the type-2 fuzzy model and its type-1 counterpart under those data sets are listed in Table 4.

It is observed that better performance is always obtained by using the type-2 model. When noise is much smaller than input date, RMSE of type-1 system becomes stable. It means that type-1 system cannot model it. These results prove that type-2 system is able to model more complex input-output relationship to achieve the universal approximation property. Specially for problems with high precision requirement, type-2 FLS has the capability to develop robust and reliable solutions.

The proposed algorithm of interval type-2 TSK FLS has been used in fuzzy modeling and uncertainty prediction in high precision manufacturing [16-18].

Table 4. RMSE of type-2 fuzzy model and its type-1 counterpart

	Noise Interval	RMSE	
		Type-2	Type-1
1	[-0.05, 0.05]	0.203240	0.204300
2	[-0.005, 0.005]	0.086282	0.094973
3	[-0.0005, 0.0005]	0.083890	0.093465
4	[-0.00005, 0.00005]	0.084833	0.093013
5	[-0.000005, 0.000005]	0.084240	0.093038
6	[-0.0000005, 0.0000005]	0. 083267	0.093037
7	[-0.00000005, 0.00000005]	0.082853	0.093037
8	[-0.000000005, 0.000000005]	0. 081515	0.093037

6. CONCLUSION

This paper distinguishes the differences between type-2 TSK system and its counterpart, analyzes the sensibility of the outputs of a type-2 TSK fuzzy system, and discusses the approximation capacities of type-2 TSK FLS and its type-1 counterpart as well. Spread percentage of cluster canter has great influence on RMSE of a type-2 FLS. The interval output of a type-2 TSK FLS is only influenced by spread percentage of consequent parameters. Type-2 FLS has greater approximation capacity than that of its type-1 counterpart and it has the advantage to develop robust and reliable solutions for the problems with high precision requirement.

7. ACKNOWLEDGMENTS

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