Common Fixed Point Theorems in Fuzzy Metric Space using Implicit Relation

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ABSTRACT
In this paper, we prove common fixed point theorems in fuzzy metric spaces for weakly compatible mappings along with property (E.A.) satisfying implicit relation. Property (E.A.) buys containment of ranges without any continuity requirement besides minimizing the commutativity conditions of the maps to commutativity at their point of coincidence. Moreover, property (E.A.) allows replacing the completeness requirement of the space with a more natural condition of closeness of the range.

KEYWORDS
Common fixed point; Weakly compatible maps; property (E.A.)..

1. INTRODUCTION
It proved a turning point in the development of mathematics when the notion of fuzzy set was introduced by Zadeh [22] which laid the foundation of fuzzy mathematics. Fuzzy set theory has applications in applied sciences such as neural network theory, stability theory, mathematical programming, modeling theory, engineering sciences, medical sciences (medical genetics, nervous system), image processing, control theory, communication etc.

Kramosil and Michalek[9] introduced the notion of a fuzzy metric space by generalizing the concept of the probabilistic metric space to the fuzzy situation. George and Veeramani[5] modified the concept of fuzzy metric spaces introduced by Kramosil and Michalek[9]. There are many view points of the notion of the metric space in fuzzy topology for instance one can refer to Kaleva and Seikkala [8], Kramosil and Michalek [9], George and Veeramani [5].

Regan and Abbas[2] obtained some necessary and sufficient conditions for the existence of common fixed point in fuzzy metric spaces .

Popa ([14]- [15]) introduced the idea of implicit function to prove a common fixed point theorem in metric spaces . Singh and Jain[7] further extended the result of Popa ([14]- [15]) in fuzzy metric spaces. For the reader convenience, we recall some terminology from the theory of fuzzy metric spaces.

2. PRELIMINARIES
Definition 2.1. ([22]) Let X be any non empty set. A fuzzy set M in X is a function with domain X and values in [0, 1].

Definition 2.2. ([17]) A mapping $*$: $[0, 1]\times[0, 1] \to [0, 1]$ is called a continuous t-norm if ([0, 1], $*$) is an abelian topological monoid with unit 1such that, $a * b \leq c * d$, for $a \leq c$, $b \leq d$.

Basic examples of t-norm are the Lukasiewicz t-norm T_L, $T_L(a,b) = Max(a+b-1,0)$, t-norm T_P, $T_P(a,b) = ab$, and t-norm T_M, $T_M(a,b) = Min\{a,b\}$.

Definition 2.3. ([9]) The 3 − tuple (X,M, $*$) is called a fuzzy metric space in the sense of Kramosil and Michalek if X is an arbitrary set, $*$ is a continuous t − norm and M is a fuzzy set in $X^2\times[0,\infty)$ satisfying the following conditions:
(a) $M(x, y, t) > 0$,
(b) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(c) $M(x, y, t) = M(y, x, t)$,
(d) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(e) $M(x, y, .) : [0,\infty) \to [0, 1]$ is a continuous function,
for all $x, y, z \in X$ and $t, s > 0$.

Note that, M(x, y, t) can be thought as degree of nearness between x and y with respect to t. It is known that M(x, y, .) is nondecreasing for all $x, y \in X$([5]).
A sequence $\{x_n\}$ in X converges to x if and only if for each $t > 0$ there exists $n_0 \in N$, such that,
$$M(x_n, x, t) = 1 , \quad \text{for all } n \geq n_0.$$
The sequence $(x_n)_{n\in N}$ is called Cauchy sequence if $\lim_{n\to\infty}M(x_n,x_{n+p},t) = 1$,for all $t > 0$ and $p \in N$. A fuzzy metric space X is called complete if every Cauchy sequence is convergent in X.

Lemma 2.4. ([5]) Let (X,M, $*$) be a fuzzy metric space. Then M is a continuous function on $X^2 \times (0,\infty)$.
In 1999, Vasuki [21] introduced the notion of weakly commuting.

Definition 2.5. Two self-mappings *f* and *g* of a fuzzy metric space (X,M,$*$) are said to be weakly commuting if $M(fgx, gfx, t) \geq M(fx, gx, t)$, for each $x \in X$ and for each $t > 0$.

In 1994, Mishra [12] generalized the notion of weakly commuting to compatible mappings in fuzzy metric spaces akin to the concept of compatible mapping in metric spaces, see [7].

Definition 2.6. Let f and g be mappings from a fuzzy metric space (X,M,$*$) into itself. A pair of map {f,g} is said to be compatible if $lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$, whenever $\{x_n\}$ is a sequence in X such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = u$ for some $u \in X$ and for all $t > 0$.

In 1999, Vasuki [21] initiated the concept of non compatible of mapping in fuzzy metric spaces .

Definition 2.7. Let *f* and *g* be self mappings on a fuzzy metric space(X,M,$*$). The mappings *f* and *g* are said to be non compatible if $lim_{n\to\infty}$ M(fgx_n, gfx_n, t) $\neq$ 1, whenever {x_n} is a sequence in X such that $lim_{n\to\infty}$ fx_n = $lim_{n\to\infty}$gx_n = u, for some u $\in$ X and for all t > 0.

Definition 2.8.[2] A pair of mappings f and g from a fuzzy metric space (X,M, $*$) into itself are weakly compatible if they commute at their coincidence points,i.e., fx = gx implies that fgx = gfx.

In 2002, Aamri and Moutawakil[1] introduced the concept of property (E.A.).

Definition 2.9.[1] Two self maps A and S of a metric space (X, d) are said to satisfy property (E.A.) if there exists a sequence {x_n} in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ = t, for some t $\in$ X.

In a similar mode, it is said that two self-maps A and S of a fuzzy metric space (X, M,$*$) satisfy property (E.A.), if there exists a sequence {x_n} in X such that $\lim_{n\to\infty}$M(Ax_n, Sx_n, t) = 1.

Property (E.A.) buys containment of ranges without any continuity requirements, besides minimize the commutativity conditions of the maps to the commutativity at their points of coincidence. Moreover , property (E.A.) allows replacing the completeness requirement of the space with a more natural condition of closeness of the range space.

Example 2.10.[2] Let (X,M, $*$) be a fuzzy metric space, where X = [0, 2] with minimum t − norm, and M(x, y, t) = $\frac{t}{t+d(x,y)}$ for all t > 0 and for all x, y ∈ X. Define the self maps f and g as follows:

$$f\,x = \begin{cases} 2, & when\ x \in [0,1] \\ \frac{x}{2}, & when\ 1 < x \leq 2 \end{cases};$$

$$g\,x = \begin{cases} 0, & when\ x = 1 \\ \frac{x+3}{5}, & otherwise \end{cases};$$

Let {$x_n = 2 - \frac{1}{n}$} is a sequence in X such that $\lim_{n\to\infty}$ f x_n = $\lim_{n\to\infty}$ g x_n = z. By definition of f and g, we have z ∈ {1}. Thus {f, g} satisfies property (E.A.).

We note that weakly compatible and property (E.A.) are independent to each other.

Example 2.11.[11] Let X = [0, 1] with the usual metric space d, i.e., d(x, y) = |x – y|.

Define (X,M, $*$) = $\left(\frac{t}{t+d(x,y)}\right)$ for all x, y in X and for all t > 0 and also define,

$$f\,x = \begin{cases} 1-x & if\ x \in [0,\frac{1}{2}] \\ 0 & if\ x \in (\frac{1}{2},1] \end{cases}; \quad g\,x = \begin{cases} \frac{1}{2} & if\ x \in [0,\frac{1}{2}] \\ \frac{3}{4} & if\ x \in (\frac{1}{2},1] \end{cases}.$$

Consider the sequence {x_n} = $\left\{\frac{1}{2} - \frac{1}{n}\right\}$, n $\geq$ 2, we have $lim_{n\to\infty}$f $\left(\frac{1}{2} - \frac{1}{n}\right)$ = $\frac{1}{2}$ = $lim_{n\to\infty}$g $\left(\frac{1}{2} - \frac{1}{n}\right)$. Thus , the pair (f, g) satisfies property (E.A.). Further, f and g are weakly compatible since x = $\frac{1}{2}$ is their unique coincidence point and fg($\frac{1}{2}$) = f($\frac{1}{2}$) = g($\frac{1}{2}$) = gf($\frac{1}{2}$). We further observe that $lim_{n\to\infty}$(fg $\left(\frac{1}{2} - \frac{1}{n}\right)$, fg $\left(\frac{1}{2} - \frac{1}{n}\right)$) $\neq$ 1, therefore, the pair (f, g) is non-compatible.

Example 2.12.[11] Let X = R^+ and d be the usual metric on X. Define (X,M, $*$) = $\left(\frac{t}{t+d(x,y)}\right)$ for all x, y in X and for all t > 0 and also define f, g : X → X by fx = 0, if 0 < x $\leq$ 1 and fx = 1, if x > 1 or x = 0, and gx = [x], the greatest integer that is less than or equal to x, for all x $\in$ X. Consider a sequence {x_n} = $\left\{1 + \frac{1}{n}\right\}$, n $\geq$ 2 in (1, 2), then we have $\lim_{n\to\infty}$ f x_n = 1 = $\lim_{n\to\infty}$ g x_n. Similarly for the sequence {y_n} = $\left\{1 - \frac{1}{n}\right\}$, n $\geq$ 2 in (0, 1), we have $\lim_{n\to\infty}$ f x_n = 0 = $\lim_{n\to\infty}$ g x_n . Thus the pair (f, g) satisfies property (E.A.). However, f and g are not weakly compatible as each u_1 $\in$ (0, 1) and u_2 $\in$ (1, 2) are coincidence points of f and g, where they do not commute. Moreover, they commute at x = 0, 1, 2,…. But none of these points are coincidence points of f and g. Further, we note that pair (f, g) is noncompatible. Thus we can conclude that property E.A. does not imply weak compatibility.

Here , we note that two noncompatible self-mappings of a fuzzy metric space (X,M, $*$) satisfy property E.A.

Definition 2.13[2] The mappings A, B, S and T from a fuzzy metric space (X,M, $*$) into itself are said to satisfy common property (E.A.) if there exists sequences {x_n} and {y_n} in X such that, $\lim_{n\to\infty}$ Ax_n = $\lim_{n\to\infty}$ Sx_n = $\lim_{n\to\infty}$ By_n = $\lim_{n\to\infty}$ Ty_n = u for some u ∈ X.

3. IMPLICIT RELATIONS

Let **(Φ)** be the set of all real continuous functions $\phi : (R^+)^6 \to R^+$ satisfying the following condition:

(M_1) : ϕ(u, v, u, v, v, u) $\geq$ 0 imply u $\geq$ v, for all u, v ∈ [0,1].

(M_2) : ϕ(u, v, v, u, u, v) $\geq$ 0 imply u $\geq$ v, for all u, v ∈ [0,1].

(M_3) : ϕ(u, u, v, v, u, u) $\geq$ 0 imply u $\geq$ v, for all u, v ∈ [0,1].

Example 3.1.

(i) $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \min\{t_2,t_3, t_4, t_5, t_6\}$.

(ii) $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1^2 - \min\{t_i t_j : i, j \in \{2, 3, 4, 5, 6\}\}$.

(iii) $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1^3 - \min\{t_i t_j t_k : i, j,k \in \{2, 3, 4, 5, 6\}\}$.

4. MAIN RESULTS

Theorem 4.1.: Let (X, M, $*$) be a fuzzy metric space with $*$ continuous t-norm. Let A, B, S, T be self mappings of X satisfying:

(4.1) A(X) $\subset$ T(X) and B(X) $\subset$ S(X),

(4.2) pair (A,S) or (B,T) satisfies the property (E.A.),

(4.3) For some $\phi \in (\Phi)$ and for all x,y $\in$ X and every t > 0,

ϕ\{M(Ax, By, t), M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t)\} ≥ 0,

(4.4) pairs (A,S) and (B,T) are weakly compatible,

(4.5) One of A(X), B(X), S(X) or T(X) is a closed subset of X.

Then A, B, S, T have a unique common fixed point in X.

Proof : Suppose that (B,T) satisfy the property (E.A.), then there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$. Since $B(X) \subset S(X)$, there exists a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sy_n = z$. Now we show that $\lim_{n\to\infty} Ay_n = z$.

On putting $x = y_n$ and $y = x_n$ in (4.3) ,we have

$\phi\{M(Ay_n, Bx_n, t), M(Sy_n, Tx_n, t), M(Sy_n, Ay_n, t), M(Tx_n, Bx_n, t), M(Sy_n, Bx_n, t), M(Tx_n, Ay_n, t)\} \geq 0.$

Proceeding limit as $n\to\infty$, in view of (Φ), we have $\lim_{n\to\infty} Ay_n = z$.

Since S(X) is a closed subset of X, therefore z = Su for some $u \in X$, subsequently, we have ,

$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Ay_n = Su = z.$

From (4.3), putting $x = u$, $y = x_n$, we have,

$\phi\{M(Au, Bx_n, t), M(Su, Tx_n, t), M(Su, Au, t), M(Tx_n, Bx_n, t), M(Su, Bx_n, t), M(Tx_n, Au, t)\} \geq 0$.

Letting $n\to\infty$, in view of (Φ), we have Au = Su.

The weak compatibility of A and S implies that ASu = SAu and then Az = ASu = Sz = SSu = Sz . Since $A(X) \subset T(X)$, therefore there exists a point $v \in X$ such that Au = Tv. We claim that Tv = Bv, put x = u and y = v in (4.3) , we have

ϕ\{M(Au, Bv, t), M(Su, Tv, t), M(Su, Au, t), M(Tv, Bv, t), M(Su, Bv, t), M(Tv, Au, t)\} ≥ 0,

i.e., ϕ\{M(Au, Bv, t), 1, 1, M(Au, Bv, t), M(Au, Bv, t), 1\} ≥ 0,

in view of (Φ), we get Au = Bv = Tv.

Therefore , Au = Su = Tv = Bv = z. The weak compatibility of B and T implies that

BTv = TBv and TTv = TBv = BTv = BBv, i.e., Tz = Bz.

Now we prove Au(= z) is a common fixed point of A, B, S and T, from (4.3) , it follows that

ϕ\{M(Az, Bv, t), M(Sz, Tv, t), M(Sz, Az, t), M(Tv, Bv, t), M(Sz, Bv, t), M(Tv, Az, t)\} ≥ 0.

i.e., ϕ\{M(Az, z, t), M(Az, z, t), M(Az, Az, t), M(z, z, t), M(Az, z, t), M(z, Az, t)\} ≥ 0.

i.e., ϕ\{M(Az, z, t), M(Az, z, t), 1, 1, M(Az, z, t), M(z, Az, t)\} ≥ 0,

Now in view of (Φ), we get, Az = z.

Hence, z = Az = Sz and z is a common fixed point of A and S.Similarly, one can prove that Bv = z is also a common fixed point of B and T. Therefore we conclude that z is a common fixed point of A, B, S and T.

The proof is similar when T(X) is assumed to be a closed subset of X. The cases in which A(X) or B(X) is a closed subset of X are similar to the cases in which T(X) or S(X) respectively is closed.

Uniqueness, suppose $z \neq w$ be another fixed point, then from (4.3),

ϕ\{M(Az, Bw,t), M(Sz, Tw, t), M(Sz, Az, t), M(Tw, Bw, t), M(Sz, Bw, t), M(Tw, Az, t)\} ≥ 0.

i.e., ϕ\{M(z, w, t), M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t), M(w, z, t)\} ≥ 0.

i.e., ϕ\{M(z, w, t), M(z, w, t), 1, 1, M(z, w, t), M(w, z, t)\} ≥ 0,

in view of (Φ), we get z = w.

Remark: Since two noncompatible self-mappings of a fuzzy metric space (X,M, $*$) satisfy property E.A. , we obtain the following corollary.

Corollary 4.2. Let A, B, S and T be self-mappings of a fuzzy metric space (X,M, $*$) satisfying (4.1) and (4.2). Suppose that (A, S) or (B, T) are noncompatible and the pairs (A, S) and (B,T) are weakly compatible. If the range of one A, B, S and T is a closed subset of X, then A, B, S and T have a unique common fixed point in X.

Example 4.4. Let (X, M, $*$) be a fuzzy metric space, where X = [0, 1] with continuous t – norm defined by $a * b = \min\{a, b\}$, $M(x, y, t) = \frac{t}{t+|x-y|}$, for all $x, y \in X$ and $t > 0$.

Define Ax = Bx = 1 and

$$Sx = Tx = \begin{cases} \frac{1+2x}{5} & if\ 0 \leq x < 1, \\ 1 & if\ x = 1, \end{cases} \quad \text{for all } x \in X.$$

Then all the conditions of theorem 4.1 are satisfied with

$$\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \min\{t_2, t_3, t_4, t_5, t_6\}.$$

Clearly 1 is the unique common fixed point of A, B, S and T.

Example 4.5. Let (X, M, $*$) be a fuzzy metric space, where X = [0, 2) with continuous t – norm defined by $a * b = \min\{a, b\}$ and $M(x, y, t) = \frac{t}{t+|x-y|}$, for all $x, y \in X$ and $t > 0$.

Define Ax = Bx = 1 and

$$Sx = \begin{cases} 1, & if\ x\ is\ rational \\ \frac{2}{3}, & if\ x\ is\ irrational \end{cases} ; \quad Tx = \begin{cases} 1, & if\ x\ is\ rational \\ \frac{1}{3}, & if\ x\ is\ irrational \end{cases}$$

;

Let $\phi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \min\{t_2, t_3, t_4, t_5, t_6\}$.

Then all the conditions of theorem 4.1 are satisfied with $A(X) = \{1\} \subset \{1, \frac{2}{3}\} = S(X)$. Clearly 1 is the unique common fixed point of A, B, S and T.

In our next result, we prove a common fixed point theorem for mappings satisfying common property (E.A.).

Theorem 4.3. : Let (X,M,$*$) be a fuzzy metric space with $*$ continuous t-norm. Let A, B, S, T be self mappings of X satisfying (4.3) .Then A, B, S, T have a unique common fixed point in X, provided the pairs (A,S) and (B,T) satisfy common property (E.A.). T(X) and S(X) are closed subsets of X and the pairs (A,S) and (B,T) are weakly compatible.

Proof: Suppose that (A,S) and (B,T) satisfy a common property (E.A.),then there exists two sequences $\{x_n\}$ and $\{y_n\}$,such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Ay_n = z$ for some z in X.

Since T(X) and S(X) are closed subsets of X, therefore z = Su = Tv for some u,v ∈ X. we claim that Au = z. To prove this, replace x by u and y by x_n in (4.3) , we have

$\phi\{M(Au, Bx_n, t), M(Su, Tx_n, t), M(Su, Au, t), M(Tx_n, Bx_n, t), M(Su, Bx_n, t), M(Tx_n, Au, t)\} \geq 0$.

Letting $n\to\infty$ and in view of (Φ), we have Au = z = Su. Now we prove that Bv = Tv, for this put x = u and y = v in (4.3) , we have

$\phi\{M(Au, Bv, t), M(Su, Tv, t), M(Su, Au, t), M(Tv, Bv, t), M(Su, Bv, t), M(Tv, Au, t)\} \geq 0$.

i.e., $\phi\{M(Tv, Bv, t), M(Tv, Tv, t), M(Tv, Tv, t), M(Tv, Bv, t), M(Tv, Bv, t), M(Tv, Tv, t)\} \geq 0$.

i.e., $\phi\{M(Tv, Bv, t), 1, 1, M(Tv, Bv, t), M(Tv, Bv, t), 1)\} \geq 0$.

In view of (Φ), we have Tv = Bv and hence Au =z =Su = Bv = Tv. The rest of the proof follows from theorem 4.1.

5. CONCLUSION

The aim of this paper is to strengthen the results and to emphasize the role of property E.A. in the existence of common fixed points and prove our main result for a pair of weakly compatible mappings along with property E.A.

Our improvement in this paper is four fold:

(i) to relax the continuity requirement of maps completely,

(ii) to minimize the commutativity requirement of the maps to the point of coincidence,

(iii) to weaken the completeness requirement of the space,

(iv) property E.A. buys containment of ranges without any continuity requirement to the points of coincidence.

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