Impedance Characteristics of Log Periodic Antenna

B. I. Neelgar
GMR Institute of Technology
GMR Nagar,
Rajam-532127, Srikakulum (Dist)
A. P. (India)

Prof. GSN Raju
Principal
A.U. College of Engineering
Andhra University,
Visakhapatnam-530003 A.P. (India)

ABSTRACT

Log Periodic array Antenna is one of the most important and commercially used antennas for T.V. reception. It is used in VHF and UHF bands. Although the analysis of this antenna is reported in literature, the data of self impedance & mutual impedance is not fully available. But, this data is useful for the optimal design of the antenna. In view of this the array above is considered and the analysis is carried out in the present work. The computed data for Self-Impedance as a function of lengths of elements and frequency and the Mutual Impedance as a function of spacings of elements is presented.

Keywords
Log-Periodic, Self Impedance, Mutual Impedance, Hertzian Dipole

1. INTRODUCTION

Log Periodic antenna introduced by DuHamel and Isbell [1] is frequency independent antenna which has a structural geometry such that its impedance is periodic with logarithm of the frequency. Fig. 1 shows a schematic structure of a log periodic antenna.
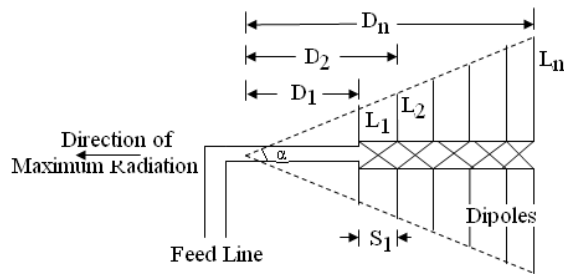

Figure. 1 Log Periodic Antenna Array

The antenna was designed and introduced by DuHamel. The antenna consists of a metal strip whose edges are specified by the angle α/2. The length from the origin to any point on the structure is specified including a distance characteristic as [2]

$$\theta = \theta_0 \sin\left[b \ln\left(\frac{r}{r_0} \right) \right] \tag{1}$$

From equation (1) it can be seen that the values of θ are separated whenever the logarithm of the radial frequency ln(ω) = ln(2πf) differs by 2π/b. The performance the antenna is

periodic as function of logarithm of frequency and hence the name is ***log-periodic***.

The array of log-periodic antenna consists of a sequence of side-by-side parallel linear dipoles forming a coplanar array as shown in Figure 1. The lengths, spacings and diameters increase logarithmically as defined by the geometric ratio τ. That is, [3]

$$\tau = \frac{D_n}{D_{n-1}} = \frac{L_n}{L_{n-1}}, \qquad n = 1,2,3,... \tag{2}$$

The spacing factor σ is defined as

$$\sigma = \frac{S_n}{2L_n} = \frac{S_n}{S_{n-1}}$$

$$S_n = D_{n-1} - D_n \tag{3}$$

$$where\, \alpha = Wedge\ angle = 2\tan^{-1}\left(\frac{1-\tau}{4\sigma} \right)$$

In Yagi_Uda antenna only one element of the array is directly energized by the feed line, while others operate in parasitic mode whereas in log periodic antenna array all the elements are connected to the feed line. Two methods are used to connect feed line wherein in the first type the currents in the elements have same phase relationship as the terminal phases. This produces an endfire beam in the direction of longer elements. In the second method the feed is crisscrossed or transposed between the adjacent elements wherein a 180° phase is added to the terminal of each element. Since the phase between the adjacent closely placed short elements is almost in opposition, very little energy is radiated by them and their interference effects are negligible. But at the same time the longer and larger spaced elements radiate. The mechanical phase reversal between these elements produces a phase progression so that energy is radiated endfire in the direction of shorter elements. The lowest cutoff frequency occurs approximately when the longest element is λ/2. Once the length of longest element is known then the lengths of other elements can be calculated by the relation shown in (2) with the knowledge of τ. The values of τ and σ are obtained from Isbell curves. [4]. With σ the spacings and diameters are obtained with a given frequency band.

2. ANALYSIS OF ARRAY FOR SELF IMPEDANCE

The radiation characteristics of an antenna in the presence of a lossy ground depend substantially on the infinite ground conductivity and in homogeneity [5-7]. The problem was conventionally simplified on the basis of a Hertzian dipole with specified current moment for very low frequency range . But, for higher frequency a finite length antenna should be considered.

In the present analysis, the array is considered to be symmetric and the element half length is not greater than the limit of 5λ/8. The analysis is carried out by King and Wu's three term assumption for the current [8-10]. Thus, for a single centre-fed dipole in free space [11]

$$I(h) = \frac{jV}{60\Gamma_{dA}\cos\beta l}\left[\sin\beta(l - |h|) + A + B\right] \quad for \quad \beta l \neq \frac{\pi}{2} \quad (4)$$

where
$$A = X_X(\cos\beta h - \cos\beta l)$$
$$B = X_Y\left(\cos\frac{\beta h}{2} - \cos\frac{\beta l}{2}\right)$$

or

$$I(h) = \frac{-jV}{60\Gamma_{dA}}\left[\sin\beta|h| - 1 + C + D\right] \quad for \quad \beta l = \frac{\pi}{2} \quad (5)$$

where
$$C = X'_X\cos\beta h$$
$$D = X'_Y\left(\cos\frac{\beta h}{2} - \cos\frac{\pi}{4}\right)$$

In the above equations (4) and (5), V is the applied voltage, h is the distance along the dipole axis measured from the feeding point, l is the half-length of the dipole, a is the radius of dipole, $\beta = 2\pi/\lambda$ and the other symbols are as defined below[11].

$$\Gamma_{dA} = 2C1\overline{C}_c(\beta l) - 2\cos\beta l\overline{C}_c(2\beta l)$$
$$- 2\cot\beta lC1C_s(\beta l) + EXP.C_s(2\beta l) \quad (6)$$

where
$$C1 = 1 + \cos\beta l$$
$$EXP = (\cot\beta l\cos\beta l - \sin\beta l)$$

For βl ≥ π/2

$$\Gamma_{dA} = \overline{C}_c(0.5\pi) + C2\overline{C}_c(E) - \cos2\beta l\overline{C}_c(2E)$$
$$+ \sin2\beta l\left[\overline{C}_c(\beta l) - \overline{C}_c(2\beta l)\right] - C2C_s(\beta l) \quad (7)$$
$$+ \cos2\beta lC_s(2\beta l) + \sin2\beta l\left[C_s(E) - C_s(2E)\right]$$

where $C2 = 1 + \cos2\beta l$

when βl = π/2

$$\Gamma_{dA}\left(\beta l = \frac{\pi}{2}\right) = 2\overline{C}_c(0.5\pi) - C_s(\pi) \quad (8)$$

The complex functions X_X, X_Y, X'_X and X'_Y in equations (4) and (5) are expressed as

$$X_X = Z^{-1}(\Gamma_{dB}\Gamma_V - j\Gamma_{dC}\Gamma_A) \quad (9)$$

$$X_Y = -jZ^{-1}\left[\begin{array}{c}\Gamma_{dC}(\Gamma_{dE}\cos\beta l - \Gamma_U) \\ + \Gamma_{dF}\Gamma_V\end{array}\right] \quad (10)$$

$$X'_X = \frac{\Gamma_{dB}\Gamma_{dA}}{\Gamma_{dB}\Gamma_V - j\Gamma_{dC}\Gamma_A} \quad (11)$$

$$X'_Y = \frac{j\Gamma_{dC}\Gamma_{dE}}{\Gamma_{dB}\Gamma_V - j\Gamma_{dC}\Gamma_A} \quad (12)$$

where $Z = \Gamma_{dB}(\Gamma_{dE}\cos\beta l - \Gamma_U) + j\Gamma_{dF}\Gamma_A \quad (13)$

The other symbols in the above equations are as follows:

$$\Gamma_{dE} = \frac{2}{C3}\overline{C}_c(\beta l) - \frac{\cos\beta l}{C3}\overline{C}_c(2\beta l)$$
$$- \frac{2\cos\beta l}{C3}\overline{C}(\beta l) + \frac{\cos\beta l}{C3}\overline{C}(2\beta l) \quad (14)$$
$$- \frac{\sin\beta l}{C3}C_s(2\beta l)$$

$$\Gamma_{dF} = \frac{1}{C4}\left[\begin{array}{c}-2S_c(\beta l) + \cos\beta lS_c(2\beta l) \\ + 2\cos\beta lS(\beta l) - \cos\beta lS(2\beta l) \\ + \sin\beta lS_s(2\beta l)\end{array}\right] \quad (15)$$

$$\Gamma_{dC} = \frac{1}{C4}\left\{\begin{array}{c}-2\sin\beta lS_c(\beta l) + 2\cos\beta lS_s(\beta l) \\ -\sin2\beta l[S_c(\beta l) - S_c(2\beta l)] \\ + C2S_s(\beta l) - \cos2\beta lS_s(2\beta l)\end{array}\right\} \quad (16)$$

$$\Gamma_{dB} = \frac{1}{C4}\left[G_1 + jH_1\right] \quad (17)$$

With

$$G_1 = 2\sinh^{-1}\left(\frac{l}{a}\right) - C(P) - C(Q)$$
$$- \sinh^{-1}\left(\frac{2l}{a}\right)\cos\frac{\beta l}{2} + \frac{1}{2}\cos\frac{\beta l}{2}C(R)$$
$$- \frac{1}{2}\sin\frac{\beta l}{2}S(R) + \cos\frac{\beta l}{2}\overline{C}(2\beta l) \quad (18)$$
$$+ \frac{1}{2}\cos\frac{\beta l}{2}C(T) + \frac{1}{2}\sin\frac{\beta l}{2}S(T)$$
$$- 2\cos\frac{\beta l}{2}\overline{C}(\beta l)$$

$$H_1 = -S(P) - S(Q) + \frac{1}{2}\sin\frac{\beta l}{2}C(R)$$
$$+ \frac{1}{2}\cos\frac{\beta l}{2}S(R) + \frac{1}{2}\cos\frac{\beta l}{2}S(T)$$
$$- \frac{1}{2}\sin\frac{\beta l}{2}C(T) + 2\cos\frac{\beta l}{2}S(\beta l)$$
$$- \cos\frac{\beta l}{2}S(2\beta l)$$
(19)

$$\Gamma_V = G_2 + jH_2$$
(20)

$$G_2 = -\sin 2\beta l\left[\overline{C}_c(\beta l) - \overline{C}_c(2\beta l)\right]$$
$$+ \cos 2\beta l\left[C_s(\beta l) - C_s(2\beta l)\right] + C_s(\beta l)$$
(21)

$$H_2 = \sin 2\beta l\left[S_c(\beta l) - S_c(2\beta l)\right]$$
$$- \cos 2\beta l\left[S_s(\beta l) - S_s(2\beta l)\right] - S_s(\beta l)$$
(22)

$$\Gamma_U = \cos\beta l\left[\overline{C}_c(2\beta l) - \overline{C}_c(2\beta l)\right]$$
$$+ \sin khC_s(2\beta l)$$
$$+ j\left\{ \begin{array}{l} \cos\beta l[S(2\beta l) - S_c(2\beta l)] \\ -\sin\beta l S_s(2\beta l) \end{array} \right\}$$
(23)

$$\Gamma_A = G_3 + jH_3$$
(24)

With
$$G_3 = \cos\frac{\beta l}{2}\left[\begin{array}{l} \sinh^{-1}\left(\frac{l}{a}\right) - \frac{1}{2}C(R) \\ -\frac{1}{2}C(T) - \overline{C}(2\beta l) \end{array} \right]$$
$$+ \frac{1}{2}\sin\frac{\beta l}{2}[S(R) - S(T)]$$
(25)

$$H_3 = \cos\frac{\beta l}{2}\left[S(2\beta l) - \frac{1}{2}S(R) - \frac{1}{2}S(T) \right]$$
$$+ \frac{1}{2}\sin\frac{\beta l}{2}[C(T) - C(R)]$$
(26)

For a set of values for *a* and *l* all the functions shown above can be calculated. Thus from equations (4) and (5) the self impedance of the dipole is computed as

$$Z_s = \frac{-j60\Gamma_{dA}\cos\beta l}{\sin\beta l + X_x C3 + X_Y C4}$$
(27)
$$for \quad \beta l \neq \frac{\pi}{2}$$

$$Z_s = \frac{j60\Gamma_{dA}}{-1 + X'_X - (1 - \sqrt{2}/2)X'_Y}$$
(28)
$$for \quad \beta l = \frac{\pi}{2}$$

In the equations from (4) through (28) the various symbols used are defined as follows. The generalized sine and cosine integrals are defined as:

$$C_s(\beta l) = C_s(\beta a, \beta l)$$
$$C_s(2\beta l) = C_s(\beta a, 2\beta l)$$
$$C_s(0.5\pi) = C_s(\beta a, \pi/2)$$

$$C_s(E) = C_s(\beta a, \beta l - \pi/2)$$
$$C_s(2E) = C_s(\beta a, 2\beta l - \pi/2)$$
$$C_s(\pi) = C_s(\beta a, \pi)$$

$$C_s(E) = C_s(\beta a, \beta l - \pi/2)$$
$$C_s(2E) = C_s(\beta a, 2\beta l - \pi/2)$$

$$C_s(P) = C_s\left(\beta a\frac{\sqrt{3}}{2}, \frac{\beta l}{2}\right)$$

$$C_s(Q) = C_s\left(\beta a\frac{\sqrt{3}}{2}, \frac{3\beta l}{2}\right)$$

$$C_s(R) = C_s\left(\beta a\frac{\sqrt{3}}{2}, 3\beta l\right)$$

$$C_s(T) = C_s\left(\beta a\frac{\sqrt{3}}{2}, \beta l\right)$$

The expressions hold good for other integrals shown in equations (4) through (28). These generalized integrals are given in the reference [11] and Appendix. The other symbols used are defined as follows:

$$C1 = (1 + \cos\beta l)$$
$$C2 = (1 + \cos 2\beta l)$$
$$C3 = (1 - \cos\beta l)$$
$$C4 = (1 - \cos\frac{\beta l}{2})$$
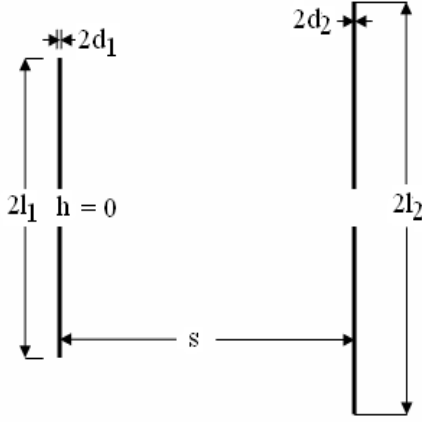
3. ANALYSIS OF ARRAY FOR MUTUAL IMPEDANCE

Figure. 2 Two non-identical element array

The open circuit mutual impedance between two parallel dipoles of half-lengths l_1 and l_2 spaced a distance s apart as shown in Figure. 2 is calculated by

$$Z_m = \frac{-1}{I_1(0)I_2(0)} \int_{-l2}^{l1} E_{s1}(h) I_2(h) dh \qquad (29)$$

where $I_1(0)$ and $I_2(0)$ are the input currents of No.1 and No.2 antennas respectively. The **h** component of electric field $E_{s1}(h)$ produced at No.2 antenna by the current on No. 1 antenna is given by [11]

$$E_{h1} = \frac{-j30}{\beta}\left(\beta^2 + \frac{\partial^2}{\partial h^2}\right) \int_{-l1}^{l1} I_1(h') K(h,h') dh' \qquad (30)$$

where $K(h,h') = \dfrac{e^{-j\beta R}}{R} \qquad (31)$

$$R = \left[(h-h')^2 + s^2\right]^{1/2} \qquad (32)$$

Substituting equations (35) into (34) and using (4) and (5) for I(h') and I(h) the expression for mutual impedance can be obtained as

a) for $\beta l_1 \neq \pi/2$ and $\beta l_2 \neq \pi/2$,

$$Z_m = \frac{-j30\left\{\begin{array}{l} \beta M_a + 2(\cos \beta l_1)M_b \\ -\left[\begin{array}{l}1 + X_{X1}\sin \beta l_1 \\ +\frac{1}{2}X_{Y1}\sin(\beta l_1/2)\end{array}\right] \\ M_c\end{array}\right\}}{Y_a Y_b} \qquad (33)$$

where

$$Y_a = \sin \beta l_1 + X_{X1}(1 - \cos \beta l_1) \\ + X_{Y1}\left(1 - \cos \frac{\beta l_1}{2}\right) \qquad (34)$$

$$Y_b = \sin \beta l_2 + X_{X2}(1 - \cos \beta l_2) \\ + X_{Y2}\left(1 - \cos \frac{\beta l_2}{2}\right) \qquad (35)$$

$$M_a = \int_{-l2}^{l2}\left[\begin{array}{l}\sin \beta(l_2 - |h|) + X_{X2}C5 \\ + X_{Y2}C6\end{array}\right] \\ \times \int_0^{l1}\left(\begin{array}{l}X_{X1}\cos \beta l_1 - \frac{3}{4}X_{Y1}\cos \frac{\beta h'}{2} \\ + X_{Y1}\cos \frac{\beta l_1}{2}\end{array}\right) \\ KK\ dh'dh \qquad (36)$$

$$M_b = \int_{-l2}^{l2} K(h,0)\left[\begin{array}{l}\sin \beta(l_2 - |h|) \\ + X_{X2}C5 \\ + X_{Y2}C6\end{array}\right]dh \qquad (37)$$

$$M_c = \int_{-l2}^{l2} KK\left[\begin{array}{l}\sin \beta(l_2 - |h|) \\ + X_{X2}C5 + \\ X_{Y2}C6\end{array}\right]dh \qquad (38)$$

b) for $\beta l_1 = \pi/2$ and $\beta l_2 \neq \pi/2$,

$$Z_m = \frac{j30\left\{\begin{array}{l}\beta M_d + 2M_b + \\ \left[X'_{X1} - \frac{1}{2}X'_{Y1}\sin(\pi/4)\right] \\ M_c\end{array}\right\}}{Y_c Y_b} \qquad (39)$$

where

$$Y_c = -1 + X'_{X1} - X'_{Y1}\left(1 - \cos\frac{\pi}{4}\right) \qquad (40)$$

$$M_d = \int_{-l2}^{l2}\left[\begin{array}{l} \sin\beta(l_2 - |h|) + X_{X2}C5 + \\ X_{Y2}C6 \end{array}\right] \qquad$$

$$\times \int_{0}^{l1}\left(\begin{array}{l} -1 + \frac{3}{4}X'_{X1}\cos\frac{kh'}{2} \\ + X'_{Y1}\cos\frac{\pi}{4} \end{array}\right) KK\, dh'\, dh \qquad (41)$$

c) for $\beta l_1 = \beta l_2 = \pi/2$,

$$Z_m = \frac{j30\left\{\begin{array}{l} \beta M_e + 2M_f \\ + \left[X'_{X1} - \frac{1}{2}X'_{Y1}\sin(\pi/4)\right]M_g \end{array}\right\}}{Y_c Y_d} \qquad (42)$$

where

$$Y_d = -1 + X'_{X2} - X'_{Y2}\left(1 - \cos\frac{\pi}{4}\right) \qquad (43)$$

(which will be identical to M_c if $d_2 = d_1$)

$$M_e = \int_{-l2}^{l2}\left[\begin{array}{l} \sin\beta|h| - 1 + X'_{X2}\cos\beta h \\ - X'_{Y2}C7 \end{array}\right]$$

$$\times \int_{0}^{l1}\left(-1 + \frac{3}{4}X'_{X1}\cos\frac{kh'}{2} + X'_{Y1}\cos\frac{\pi}{4}\right)$$

$$KK\, dh'\, dh \qquad (44)$$

$$M_f = \int_{-l2}^{l2} K(h,0)\left[\begin{array}{l} \sin\beta|h| - 1 \\ + X'_{X2}\cos\beta h \\ - X'_{Y2}C7 \end{array}\right] dh \qquad (45)$$

$$M_g = \int_{-l2}^{l2} KK\left[\begin{array}{l} \sin\beta|h| - 1 \\ + X'_{X2}\cos\beta h - X'_{Y2}C7 \end{array}\right] dh \qquad (46)$$

In the above equations the symbols used are defined as:

$$C5 = \cos\beta h - \cos\beta l_2$$

$$C6 = \cos\frac{\beta h}{2} - \cos\frac{\beta l_2}{2}$$

$$C7 = \cos\frac{\beta h}{2} - \cos\frac{\pi}{4}$$

$$KK = K(h,h') + K(h,-h')$$

Since X_X, X_Y, X'_X and X'_Y are given in equations (9) to (12), the integrals Ma through Mg can all be computed numerically, once the values for d_1, d_2, l_1, l_2 and s are specified. Hence the mutual impedance in (33), (39) or (42) can also be computed. All the equations from (4) to (46) are given in reference [11]. Also, when $X_{X1} = X_{X2} = X_{Y1} = X_{Y2} = 0$, equation (33) reduces to familiar expression for the case of simple sinusoidal current distribution. In such case the mutual impedance is calculated as [12]

$$Z_m = -30\int_{0}^{l_2}\left(\begin{array}{l} \dfrac{-je^{-j\beta r_1}}{r_1} - \dfrac{je^{-j\beta r_2}}{r_2} \\[4mm] + \dfrac{2j\cos kh_1 e^{-j\beta r_0}}{r_0} \end{array}\right) \qquad (47)$$

$$\sin\beta(l_2 - h)dh$$

$$r_0 = \sqrt{s^2 + h^2}$$

where $\quad r_1 = \sqrt{s^2 + (l_1 - h)^2} \qquad (48)$

$$r_2 = \sqrt{s^2 + (l_1 + h)^2}$$

4. RESULTS

In order to validate the impedances, an array of Log-Periodic antenna is considered with **Gain=9dB.** The frequency range was selected as 54 MHz to 600 MHz From Isbell curves for 9 dB gain, **τ=0.861 & σ=0.162.** With this data obtained the lengths, spacings and diameters are evaluated. The number of dipole elements is taken as n=10. The numerical integrations of the generalized sine and cosine integrals and the impedances were computed using MATLAB. The impedances were computed for frequency of 54 MHz were plotted as shown in the figures [3-6] that follow.

The variation of self impedances with lengths of elements for a fixed frequency of 54 MHz is shown in Figure 3 below.
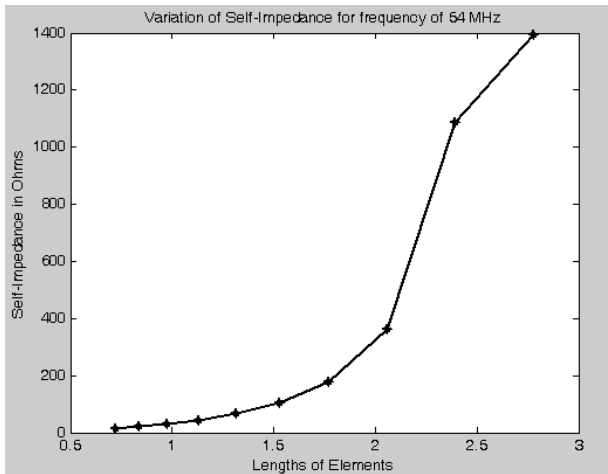

Figure 3 Self Impedance for frequency=54 MHz

The variation of Mutual Impedances with spacings between elements is shown in Figure. 4. Note that $Z_{12}= Z_{21}$, $Z_{13}=Z_{31}$, $Z_{23}=Z_{32}$……., $Z_{910}=Z_{109}$.
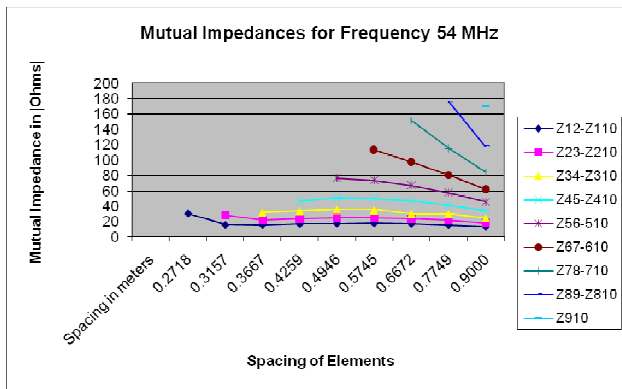

Figure. 4. Variation of Mutual Impedance

Figure 5 illustrates the variation of Self Impedances with different frequencies.

The variation of absolute self impedance with logarithm of frequency for the first three elements is shown in Figure. 6.
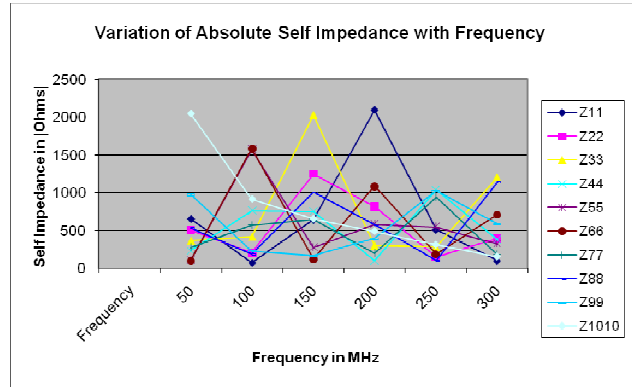

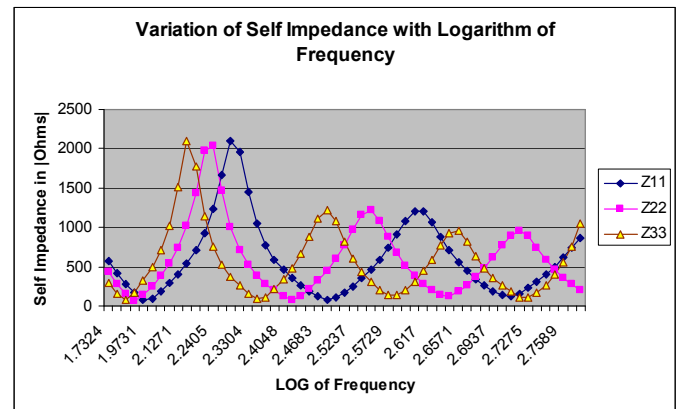**Figure 5 Variation of Self Impedance with Frequency**

Figure 6. Variation of Self Impedance with Log of Frequency

5. CONCLUSIONS

The following conclusions are evident from results presented above.

5.1 Self Impedance

- The self impedance is purely dependent on frequency of operation
- As it can be seen from results the self impedance for frequency of 54 MHz is increasing from smallest element to largest element but it may not be true for all frequencies.
- The variation of self impedance with frequency is periodic and hence the name **Log-Periodic**.

5.2 Mutual Impedance

- For a fixed operating frequency the absolute value of mutual impedance seems to decrease from smallest element to largest element, but it also depends on spacing of elements.

- For a fixed operating frequency the absolute value of mutual impedances depends on length and spacing of the elements between each element.

6. REFERENCES

[1] R.H. DuHamel and D. E. Isbell, *Broadband Logarithmically Periodic Antenna Structures, 1957 IRE National Convention Record,* pt. 1, pp. 119-128

[2] Constantine A. Balanis, *Antenna Theory,* John Wiley & Sons, 1997

[3] Dr. G.S.N. Raju, *Antennas & Wave Propagation,* Pearson Education, 2005

[4] D. E. Isbell, *Log Periodic Dipole Arrays, IRE Transactions, Antennas Propagation.*, Vol, AP-8, pp. 260-267, May 1960

[5] Banos, A., *Dipole Radiation in the presence of a conducting half space,* Peragamon Press, Oxford England, 1966.

[6] King R. W. P., *Theory of Linear Antennas, Harvard University Press,* Cambridge, Mass, 1956.

[7] Bhattacharya B.K., *Input resistances of horizontal electric and vertical magnetic dipoles over a homogeneous ground, IEEE Transactions, Antennas & Propagation,* Vol. AP-11, No.3 pp. 261-266, May 1963.

[8] King. R.W.P. and T.T. Wu, *Currents, Charges near fields of antennas, Radio Sci.*, Vol. 69D, No. 3, pp 429-446, March, 1965.

[9] Johnson J. H. Wang, *The Physical Foundation,Developmental History, and Ultra-wideband Performance of SMM (Spiral-Mode Microstrip) Antennas, IEEE Symposium, Antennas & Propagation,* pp. 586-589,July 2005

[10] Y. Mushiake, "*A Report on Japanese Development of Antennas: From the Yagi-Uda Antenna to Self-complementary Antennas," IEEE Antennas and Prop. Symp. Dig*, June 2003.

[11] M. T. Maa, *Theory and Applications of Antenna Arrays*, John Wiley & Sons, 1974

[12] Edward C. Jordan, Keith G. Balmin, *Electro-Magnetic waves and radiating systems*, PHI, 2001

7. APPENDIX

The generalized sine and cosine integrals appearing in equations (6) to (28) are expressed as [9]

$$S(b,x) = \int_0^x \frac{\sin\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A1}$$

$$C(b,x) = \int_0^x \frac{1-\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A2}$$

$$\overline{C}(b,x) = \int_0^x \frac{\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A3}$$

$$S_s(b,x) = \int_0^x \frac{\sin y\sin\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A4}$$

$$S_c(b,x) = \int_0^x \frac{\cos y\sin\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A5}$$

$$C_s(b,x) = \int_0^x \frac{\sin y\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A6}$$

$$C_c(b,x) = \int_0^x \frac{(1-\cos y)\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A7}$$

$$\overline{C}_c(b,x) = \int_0^x \frac{\cos y\cos\sqrt{y^2+b^2}}{\sqrt{y^2+b^2}}dy \tag{A8}$$

The numerical integrations are carried out using MATLAB in all the analysis carried out for impedance calculations.

8. AUTHORS PROFILE

B. I. Neelgar received his B.E. in Electronics and Communication Engineering from Karnataka University, Dharawad, Karnataka State, India in the year 1984 and M. Tech in Digital Electronics and Advanced Communication from Karnataka Regional Engineering College (NITK), Surathkal, Mangalore University, Karnataka State, India in the year 1995. Presently he is pursuing his Ph.D from Jawaharlal Nehru Technological University (JNTU), Hyderabad. He has an Industry experience of 4.5 years and Teaching experience of 20 years. His areas of interest include Antennas, Antenna Arrays and Communication.

Dr. G.S.N. Raju received his B.E., M.E. with distinction and first rank from Andhra University and Ph.D. from IIT, Kharagpur. He is Professor of Electronics and Communication Engineering in Andhra University, College of Engineering, Visakhapatnam, India. He is in teaching and research for the last 30 years in Andhra University. He guided 16 Ph.D.s in the fields of Antennas, Electromagnetics, EMI/EMC and Microwave, Radar Communications, Electronic circuits. Published about 260 technical papers in National/International Journals/Conference Journals and transactions.

He is the recipient of The State Best Teacher Award' from the Government of Andhra Pradesh in 1999, 'The Best Researcher Award' in 1994 ,'Prof. Aiya Memorial National IETE Award' for his best Research guidance in 2008 and Dr. Sarvepalli

Radhakrishnan Award for the Best Academician of the year 2007, He was a visiting Professor in the University of Paderborn and also in the University Karlsruhe, Germany in 1994.

At present he holds the positions of Principal, Andhra University College of Engineering (A), Visakhapatam, Chief Editor of National Journal of Electromagnetic Compatibility.

Prof. Raju has published five textbooks Antennas and Wave Propagation, Electromagnetic Field Theory and Transmission Lines, Electronics Devices and Circuits, Microwave Engineering, Radar Engineering and Navigational Aids

Prof. Raju has been the best faculty performer in Andhra University with the performance index of 99.37%.