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(k,r) - Semi Strong Chromatic Number of a Graph

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ABSTRACT
Let G = (V, E) be a simple, finite, undirected graph. Let k, r be positive integers. A set S ⊆ V (G) is called (k, r)-semi strongly stable set if $|N_r(u) \cap S| \le k$, for all $u \in V(G)$. A partition of V(G) into (k, r)-semi strongly stable sets is called (k, r)-semi strong coloring of G. The minimum order of a (k, r)-semi strong coloring of G is called (k, r)-semi strong chromatic number of G and it is denoted by $\chi^s_{(k,r)}$(G). The number $\chi^s_{(k,r)}$(G) is determined for various known graphs and some bounds are obtained for it.

Keywords— (k, r)-semi strongly stable set, (k, r)-chromatic number.

1. INTRODUCTION
Consider a network and a coloring scheme for the nodes. Two nodes are compatible if they receive the same color. The usual coloring scheme stipulates that adjacent nodes should not receive the same color. Such a scheme is helpful in storage problem of chemicals where two non compatible chemicals (two chemicals which when placed nearby will cause danger) cannot be stored in the same room. The chromatic number of such a scheme will give the minimum number of storage spaces required for keeping all the chemicals without any problem.

In a Institution groups may exist. People in the same group may have some common interest. People in different groups may not be well disposed to each other. The affinity of people within a group may be determined by the closeness between the people and the number of people in the group. For example we may say that a group may consist of at most k −1 persons who are within a prescribed mental distance from a person. When we model this situation graphically, all the nodes of the same group may be given the same color and different groups are to be given different colors. Such a situation leads to what is called (k, r)-coloring [14]. What we stipulate is that given a node u utmost k − 1 nodes can receive the same color as u provided they are at a distance utmost r from u.

There are many generalizations of chromatic number existing in the literature [1-3] [6-9] [11]. The purpose of this paper is to study yet another generalized coloring called (k, r)-semi strong coloring of a given graph G.

We present in the beginning the preliminary notions and results.

We consider only finite simple graphs. Let r ≥ 1 be an integer and G = (V, E) be a graph.

The open r- neighborhood $N_r(v)$ of a vertex v in a graph G is defined by

$N_r(v) = \{u: 0 < d(u, v) \le r\}$ and its closed r-neighbourhood is $N_r[v] = N_r(v) \cup \{v\}$ The r-degree $deg_r(v)$ of v in G is given by $|N_r(v)|$, while Δ_r(G) and δ_r(G) denote the maximum and minimum r-degree among all the vertices of G, respectively.

In this paper, (k, r)-Semi strong coloring is defined and new results are proved .Dr. E. Sampath kumar and L. Pushpalatha introduced the concept of semi strong coloring [13].

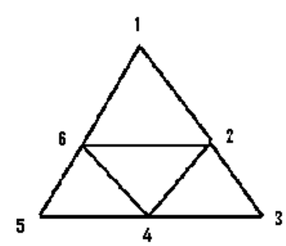
Definition: 1.2

A set S ⊆ V (G) is called **(k, r) strongly stable set** if $|N_r(u) \cap S| \le k$, for all $u \in V(G)$.A set S ⊆ V (G) is called **(k, r)-semi strongly stable set** if $|N_r(u) \cap S| \le k$, for all u∈V(G).A partition of V (G) into (k, r)-semi strongly stable sets is called **(k, r)-semi strong coloring of G**. The minimum order of a (k, r)-semi strong coloring of G is called **(k, r)-semi strong chromatic number of G** and is denoted by **$\chi^s_{(k,r)}$(G).**

Result:1.3

Any graph admits a (k, r)-semi strongly stable partition containing all singleton subsets of V(G). Therefore the existence of $\chi^s_{(k,r)}(G)$ is guaranteed.

Example:1.4



H

Let r=1, k=1. Let S = {1}. Any singleton set is a (1, 1) - Semi Strongly stable set.
Let r=1, k=2. Let S= {6, 2}.
Let r =1, k=3 .Let S= {1, 3, 5}.
Let r=2, k=1 .Let S= {1}. Any singleton set is a (1, 2) - Semi Strongly stable set.

Let r=2, k=2 .Let S={1, 4}, {5, 2}, {3, 6}
Let r=2, k=3 .Let S={1, 3, 5}, {2, 4, 6}.

$$\chi^{s}_{(k,r)}(H) = \begin{cases} 6 \ \ if \ \ r = 1, k = 1 \\ 3 \ \ if \ \ r = 1, k = 2 \\ 2 \ \ if \ \ r = 1, k \geq 3 \\ 6 \ \ if \ \ r = 2, k = 1 \\ 3 \ \ if \ \ r = 2, k = 2 \\ 2 \ \ if \ \ r = 2, k \geq 3 \end{cases}$$

Now let us find (k, r)-semi strong chromatic number for some standard graphs.

1.5 (k, r)-semi strong chromatic number for some standard graphs

1. $\chi^{s}_{(k,r)}(K_n) = \left\{ \left\lceil \frac{n}{k} \right\rceil \quad for \ all \ k \ and \ r \right.$

2.

$$\chi^{s}_{(k,r)}(K_{(1,n)}) = \begin{cases} n & if \ \ k = 1 \ and \ r = 1 \\ \left\lfloor \frac{n+1}{k} \right\rfloor & if \ \ r = 1 \ and \ 2 \leq k < n \ and \ \left\lfloor \frac{n+1}{k} \right\rfloor > 1 \\ \left\lceil \frac{n+1}{k} \right\rceil & if \ \ r = 1 \ and \ 2 \leq k < n \ and \ \left\lfloor \frac{n+1}{k} \right\rfloor = 1 \\ 1 & if \ \ r = 1 \ and \ k \leq n \\ \left\lceil \frac{n+1}{k} \right\rceil & if \ \ r \geq 2 \ and \ for \ all \ k \end{cases}$$

3. When $2 \leq m \leq n$

$$\chi^{s}_{(k,r)}(K_{m,n}) = \begin{cases} \left\lceil \frac{n}{k} \right\rceil & if \ \ r = 1 \ and \ k \geq 1 \\ \left\lceil \frac{n+m}{k} \right\rceil & if \ \ r \geq 2 \ and \ for \ all \ k \end{cases}$$

4. $\chi^{s}_{(k,r)}(W_n) = \left\{ \left\lceil \frac{n}{k} \right\rceil \quad for \ all \ k \ and \ r \right.$

5.

$$\chi^{s}_{(k,r)}(C_n) = \begin{cases} when \ \ r = k = 1 \\ \quad \begin{cases} 2 & if \ \ n \equiv 1 \bmod 3 \\ 3 & otherwise \end{cases} \\ when \ \ k = 1 \ \ and \ \ r \geq 2 \\ \quad \begin{cases} n & if \ \ n \leq 4r + 1 \\ \left\lceil \frac{n}{2} \right\rceil & if \ \ n > 4r + 1 \\ n & if \ \ r \geq \left\lfloor \frac{n}{2} \right\rfloor \end{cases} \\ when \ \ k = r \ \ and \ \ if \ \ 1 \leq r < \left\lfloor \frac{n}{2} \right\rfloor \\ \quad \begin{cases} 2 & if \ \ n \equiv 1 \bmod (2r + 1) \\ 3 & otherwise \\ \left\lceil \frac{n}{k} \right\rceil & if \ \ r \geq \left\lfloor \frac{n}{2} \right\rfloor \ \ and \ \ k = r \end{cases} \\ \quad \begin{cases} \left\lceil \frac{n}{k} \right\rceil & if \ \ 2 \leq k < r \\ 2 & if \ \ r < k < 2r \\ 1 & if \ \ k \geq 2r \end{cases} \end{cases}$$

6.

$$\chi^{s}_{(k,r)}(P_n) = \begin{cases} \quad when \ r = k \\ \quad \begin{cases} 1, \ if \ r \geq d \\ 2, \ if \ r < d \end{cases} \\ \begin{cases} when \ k = 1 \ and \ r \geq 2 \\ 2r + 1, \ if \ n > 2r + 1 \\ n, \quad if \ n \leq 2r + 1 \end{cases} \\ \begin{cases} r - k + 2, \ if \ n > 2r - 1 \ and \ k < r \\ 1, \quad if \left\lfloor \frac{n}{k} \right\rfloor = 1, k < r \ an \ n \leq 2r - 1 \\ \left\lceil \frac{n}{k} \right\rceil, \quad if \left\lfloor \frac{n}{k} \right\rfloor > 1, k < r \ an \ n \leq 2r - 1 \end{cases} \\ \begin{cases} 1, \ if \ k > r \ and \ k \geq d \\ 2, \ if \ r < k < d \end{cases} \end{cases}$$

Remark:1.6

1. If either r ≥ diam(G) then $\chi^s_{(k,r)}$(G) = $\left\lceil \frac{n}{k} \right\rceil$

2. If k > Δ_r(G) ,then $\chi^s_{(k,r)}$(G) = 1.

Result 1.7

For any spanning subgraph H of G, then $\chi^s_{(k,r)}$ (G) ≥ $\chi^s_{(k,r)}$(H)

Proposition: 1.8

If Δ_r(G) is the maximum r-degree of a graph G, then ⌈ Δ_r(G) / k ⌉ ≤ $\chi^s_{(k,r)}$(G) Equality holds when K > Δ_r(G) .

Proof:

Let v be a vertex of G with deg_r(G) =Δ_r(G), then there exists $\left\lceil \frac{\Delta_r(G)}{k} \right\rceil$ sets of neighbours of v having distinct colors.

Therefore $\lceil \Delta_r(G)/k \rceil \leq$ $\chi^s_{(k,r)}$(G).

If k > Δ_r(G), then $\chi^s_{(k,r)}$(G) = 1 and $\lceil \Delta_r(G)/k \rceil$ =1. Hence the equality holds.

Proposition: 1.9

If $\chi_{(k,r)}$ (G) is the (k, r)-chromatic number of a connected graph G, then $\chi_{(k,r)}$ (G) ≤ $\chi^s_{(k,r)}$(G), if G ≠ K_2

Proof:

It is well known that $\chi_{(k,r)}$ (G) ≤ $\lceil \Delta_r(G)/k \rceil$ + 1 [14].

Hence $\chi_{(k,r)}$ (G) – 1 ≤ $\lceil \Delta_r(G)/k \rceil$ ≤ $\chi^s_{(k,r)}$(G).

$$\chi_{(k,r)}(K_2) = \begin{cases} 1, \ if \ k \geq 2 \ and \ r \geq 1 \\ 2, \ if \ k = 1 \ and \ r = 1 \end{cases}$$

Proposition: 1.10

$$\chi^s_{(k,r)}(k_2) = 1, \ \forall \ k, r \geq 1$$

$$\chi^s_{(k,r)}(k_2) = \chi_{(k,r)}(k_2), \ \forall \ k \geq 2 \ and \ r \geq 1$$

$$\chi_{(1,1)}(K_2) > \chi^s_{(1,1)}(K_2)$$

Remark 1.11

If G =P_{2r} or G=C_{2r} (r ≥ 2), then $\chi_{(2r,r)}$(G) = $\chi^s_{(2r,r)}$(G)

2. (k, r)-COMPLETE GRAPHS

In this section we introduce the definition of (k, r)-Complete graph and develop some basic results.

For any fixed k, $\chi^s_{(k,d)}$ (G) ≤$\lceil n/k \rceil$, where the equality is reached when r is the diameter of the graph G. But there are graphs for which $\chi^s_{(k,r)}$(G) = $\lceil n/k \rceil$.but r is less than the diameter of the graph G.

These observations motivated the definition of (k, r)-Complete graph.

Definition: 2.1

For a fixed k and r, a graph G is said to be (k, r)-Complete graph iff $\chi^s_{(k,r)}$(G) = $\lceil n/k \rceil$.

The Cardinality of maximal (k, r)-Complete sub graph of G is the (k, r)-clique number of G, denoted by $\omega_{(k,r)}$(G).

Observation: 2.2

1. Any graph G is (k, d)-complete for all the values of k and r
2. A graph G is (k, r)-complete for all the values of k and r iff G is K_n.

Theorem: 2.3

The following are equivalent for graphs G on n ≥ 3 vertices.

1. $\chi^s_{(k,r)}$ (G) = $\lceil n/k \rceil$.
2. G is (k, r)-complete.
3. Either G is trivially (k, r)-complete or non-trivially (k, r) complete.

Theorem: 2.4

For any graph G, k. $\omega_{(k,r)}$(G) ≤ $\chi_{(k,r)}$ (G) ≤ $\chi^s_{(k,r)}$ (G)

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