Remarks on Convergence among Picard, Mann and Ishikawa Iteration for Complex Space

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ABSTRACT
In this paper we have drawn a comparative analysis among convergence of Picard, Mann and Ishikawa iteration for the complex space.

Keywords
Picard iteration, Mann iteration, Ishikawa iteration

1. INTRODUCTION
Several authors [1, 2, 4, 8, 9, 10] have studied the convergence of the well known Picard's, Mann and Ishikawa iteration and have drawn different results in their comparative analysis. Many of them have declared the convergence of Mann iteration as relatively faster than Ishikawa iteration.

Our research presents an exciting result that is entirely different from the previously declared results in this field.

We have undergone a comparative analysis among Picard, Mann and Ishikawa iteration and by using MatLab programme, we found that Picard's iteration converges faster than the rest, followed by Ishikawa while Mann iteration converges slowly.

2. PRELIMINEARIES
Let X be the metric space of complex numbers, D be nonempty convex subset of X and T be a selfmap of D, let $x_0 \in D$ [9].

The Picard's iteration is defined by

$$x_{n+1} = Tx_n, \quad n \geq 0 \tag{2.1}$$

The Mann iteration [6] is defined by

$$x_{n+1} = (1-s)x_n + s.Tx_n, \ n \geq 0 \tag{2.2}$$

where $0 < s < 1$

The Ishikawa iteration [5] is defined by

$$y_n = (1-s')x_n + s'.Tx_n, \ n \geq 0$$
$$x_{n+1} = (1-s)x_n + s.Ty_n, \ n \geq 0 \tag{2.3}$$

where $0 < s < 1$ and $0 < s' < 1$

Obviously for s =1, Mann iteration reduces to Picard's iteration and for s' =0 the Ishikawa iteration reduces to Mann Iteration.

In this paper, we have taken values of s and s' as $0 < s < 1$ and $0 < s' \leq \frac{1}{2}$ for our comparative analysis and have derived fixed points for quadratic, cubic and biquadratic polynomials. Here we have the following functions:

Quadratic functions: $f(z) = z^2 + c$

Cubic function: $f(z) = z^3 + c$

Biquadractic function: $f(z) = z^4 + c$

Starting with $z = (0,0)$ and $c = 0.1$, we have the following reults

3. FIXED POINTS
3.1 Fixed points of quadratic polynomial
Table 1: Picard iteration

Number of iteration *i*	**|F(z)|**	**Number of iteration *i***	**|F(z)|**
1.	0	6.	0.11267
2.	0.1	7.	0.11269
3.	0.11	8.	0.1127
4.	0.1121	9.	0.1127
5.	0.11257	10.	0.1127

Here we observe that the value converges to a fixed point after 07 iterations

Figure 1: Picard iteration

Table 2: Manniteration for s=0.1

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
102.	0.11267	112.	0.11269
103.	0.11268	113.	0.11269
104.	0.11268	114.	0.11269
105.	0.11268	115.	0.11269
106.	0.11268	116.	0.11269
107.	0.11268	117.	0.11269
108.	0.11268	118.	0.11269
109.	0.11269	119.	0.11269
110.	0.11269	120.	0.1127
111.	0.11269	121.	0.1127

Here we skipped 101iteration and the value converges to a fixed point after 119 iterations

Figure 2: Mann iteration for s=0.1

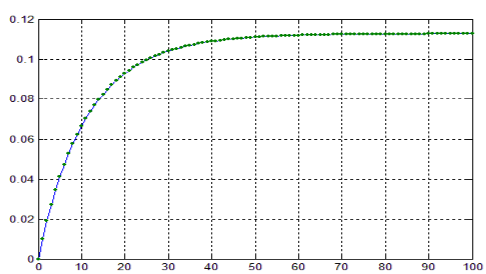


Table 3: Ishikawa iteration for s=0.6, s'=0.1

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
61.	0.34539	71.	0.34547
62.	0.3454	72.	0.34547
63.	0.34542	73.	0.34547
64.	0.34542	74.	0.34548
65.	0.34543	75.	0.34548
66.	0.34544	76.	0.34548
67.	0.34545	77.	0.34548
68.	0.34545	78.	0.34549
69.	0.34546	79.	0.34549
70.	0.34546	80.	0.34549

Here we skipped 60 iteration and the value converges to a fixed point after 77 iterations

Figure 3: Ishikawa iteration for s=0.6, s'=0.1

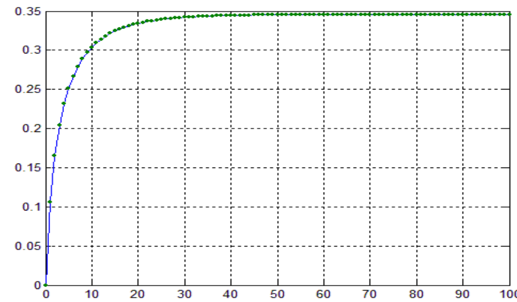


Table 4: Mann iteration fors=0.3

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
20.	0.11208	30.	0.11266
21.	0.11223	31.	0.11267
22.	0.11234	32.	0.11268
23.	0.11242	33.	0.11268
24.	0.11249	34.	0.11269
25.	0.11254	35.	0.11269
26.	0.11257	36.	0.11269
27.	0.1126	37.	0.11269
28.	0.11263	38.	0.1127
29.	0.11264	39.	0.1127

Here we skipped 20 iteration and the value converges to a fixed point after 37 iterations

Figure 4: Mann iteration for s=0.3

Table 5: Ishikawa iteration for s=0.6, s'=0.3

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
14.	0.25432	24.	0.25592
15.	0.2548	25.	0.25593
16.	0.25515	26.	0.25594
17.	0.25539	27.	0.25595
18.	0.25556	28.	0.25596
19.	0.25568	29.	0.25596
20.	0.25576	30.	0.25596

21.	0.25582	31.	0.25596
22.	0.25587	32.	0.25597
23.	0.2559	33.	0.25597

Here we skipped 13 iteration and the value converges to a fixed point after 31 iterations

Figure 5: Ishikawa iteration for s=0.6, s'=0.3

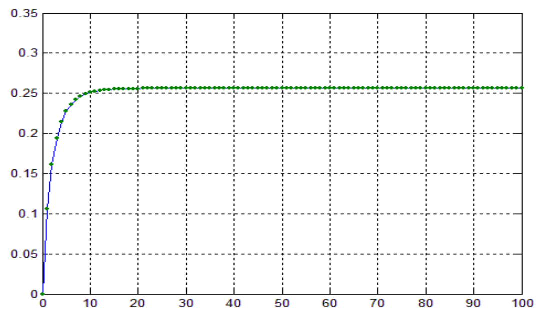


Table 6: Mann iteration for s=0.5

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
4.	0.091032	14.	0.11255
5.	0.099659	15.	0.11261
6.	0.1048	16.	0.11264
7.	0.10789	17.	0.11267
8.	0.10976	18.	0.11268
9.	0.11091	19.	0.11269
10.	0.1116	20.	0.11269
11.	0.11203	21.	0.1127
12.	0.11229	22.	0.1127
13.	0.11245	23.	0.1127

Here we skipped 03 iteration and the value converges to a fixed point after 20 iterations

Figure 6: Mann iteration for s=0.5

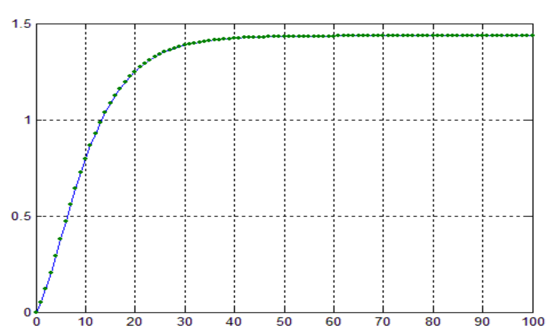


Table 7: Ishikawa iteration for s=0.6, s'=0.5

Number of iteration i	**|F(z)|**	**Number of iteration i**	**|F(z)|**
4.	0.18492	14.	0.22192
5.	0.20032	15.	0.22207
6.	0.20923	16.	0.22216
7.	0.21447	17.	0.22221
8.	0.21759	18.	0.22224
9.	0.21946	19.	0.22226
10.	0.22059	20.	0.22228
11.	0.22126	21.	0.22228
12.	0.22167	22.	0.22229
13.	0.18492	23.	0.22229

Here we skipped 03 iteration and the value converges to a fixed point after 21 iterations

Figure 7: Ishikawa iteration for s=0.6, s'=0.5

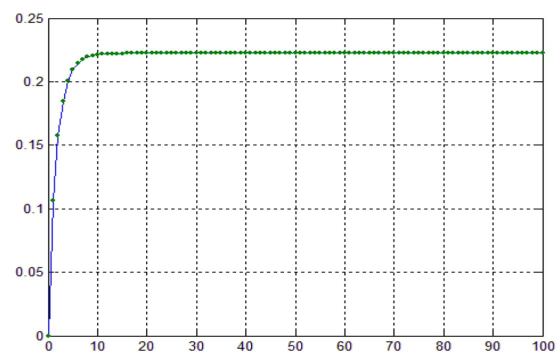


Table 8: Mann iteration for s=0.8

Number of iteration i	**|F(z)|**	**Number of iteration i**	**|F(z)|**
1.	0	11.	0.1127
2.	0.08	12.	0.1127
3.	0.10112	13.	0.1127
4.	0.1084	14.	0.1127
5.	0.11108	15.	0.1127
6.	0.11209	16.	0.1127
7.	0.11247	17.	0.1127
8.	0.11261	18.	0.1127
9.	0.11267	19.	0.1127
10.	0.11269	20.	0.1127

Here we observe that the value converges to a fixed point after 11 iterations

Figure 8: Mann iteration for s=0.8

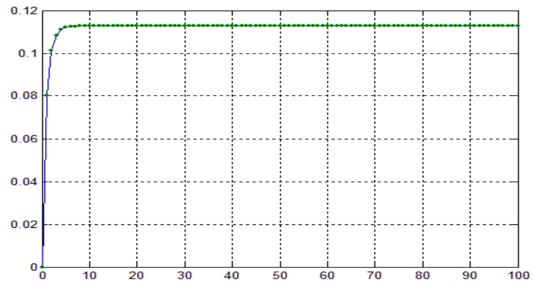


Table 9: Ishikawa iteration for s=0.6, s'=0.8

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
5.	0.18496	15.	0.19544
6.	0.19019	16.	0.19544
7.	0.1928	17.	0.19544
8.	0.19412	18.	0.19545
9.	0.19478	19.	1.95E-01
10.	0.19511	20.	1.95E-01
11.	0.19528	21.	1.95E-01
12.	0.19536	22.	0.19545
13.	0.1954	23.	0.19545
14.	0.19543	24.	0.19545

Here we skipped 04 iteration and the value converges to a fixed point after 21 iterations

Figure 9: Ishikawa iteration for s=0.6, s'=0.8

3.2 Fixed points of Cubic polynomial

Table 10: Picard iteration

Number of iteration *i*	|F(z)|	Number of iteration *i*	|F(z)|
1.	0	6.	0.10103
2.	0.1	7.	0.10103
3.	0.101	8.	0.10103
4.	0.10103	9.	0.10103
5.	0.10103	10.	0.10103

Here we observe that the value converges to a fixed point after 03 iterations

Figure 10: Picard iteration

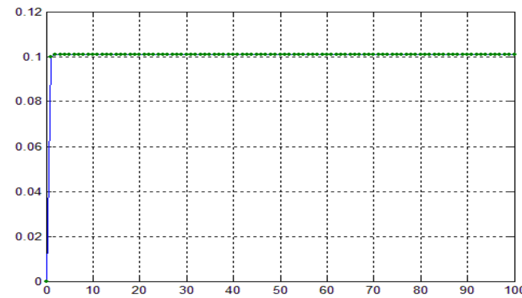


Table 11: Mann iteration for s=0.1

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
79.	0.101	89.	0.10102
80.	0.101	90.	0.10102
81.	0.101	91.	0.10102
82.	0.10101	92.	0.10102
83.	0.10101	93.	0.10102
84.	0.10101	94.	0.10102
85.	0.10101	95.	0.10102
86.	0.10101	96.	0.10103
87.	0.10102	97.	0.10103
88.	0.10102	98.	0.10103

Here we skipped 78 iteration and the value converges to a fixed point after 95 iterations

Figure 11: Mann iteration for s=0.1

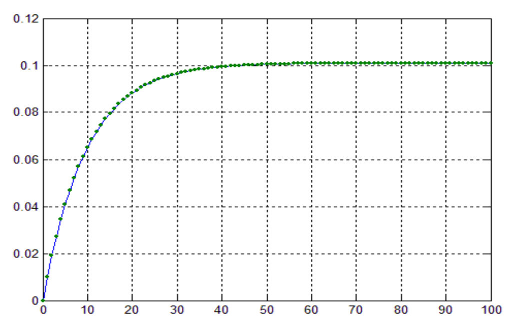


Table 12: Ishikawa iteration for s=0.6, s'=0.1

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.18571
2.	0.1006	12.	0.1858
3.	0.1444	13.	0.18585
4.	0.16508	14.	0.18588
5.	0.1753	15.	0.18589

6.	0.18046	16.	0.1859
7.	0.1831	17.	0.1859
8.	0.18446	18.	0.1859
9.	0.18516	19.	0.18591
10.	0.18552	20.	0.18591

Here we observe that the value converges to a fixed point after 19 iterations

Figure 12: Ishikawa iteration for s=0.6, s'=0.1

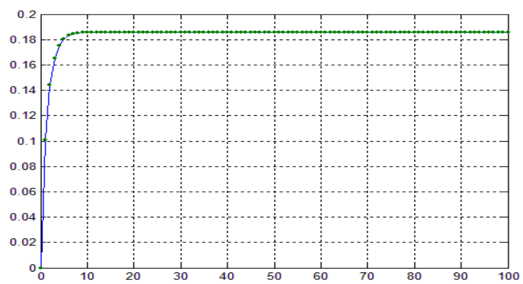


Table 13: Mann iteration for s=0.3

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
12.	0.098804	22.	0.10096
13.	0.099452	23.	0.10098
14.	0.099911	24.	0.101
15.	0.10024	25.	0.10101
16.	0.10047	26.	0.10101
17.	0.10063	27.	0.10102
18.	0.10075	28.	0.10102
19.	0.10083	29.	0.10102
20.	0.10089	30.	0.10103
21.	0.10093	31.	0.10103

Here we skipped 11 iterations and the value converges to a fixed point after 29 iterations

Figure 13: Mann iteration for s=0.3

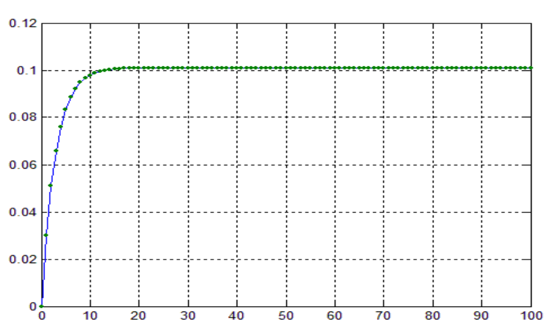


Table 14: Ishikawa iteration for s=0.6, s'=0.3

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.17821
2.	0.1006	12.	0.17825
3.	0.14323	13.	0.17827
4.	0.16217	14.	0.17828
5.	0.17082	15.	0.17828
6.	0.17481	16.	0.17829
7.	0.17666	17.	0.17829
8.	0.17753	18.	0.17829
9.	0.17793	19.	0.17829
10.	0.17812	20.	0.17829

Here we observe that the value converges to a fixed point after 15 iterations

Figure 14: Ishikawa iteration for s=0.6, s'=0.3

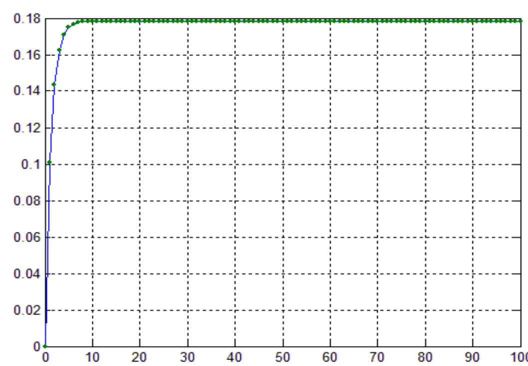


Table 15: Mann iteration for s=0.5

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.1009
2.	0.05	12.	0.10097
3.	0.075063	13.	0.101
4.	0.087743	14.	0.10101
5.	0.094209	15.	0.10102
6.	0.097523	16.	0.10103
7.	0.099225	17.	0.10103
8.	0.1001	18.	0.10103
9.	0.10055	19.	0.10103
10.	0.10078	20.	0.10103

Here we observe that the value converges to a fixed point after 15 iterations

Figure 15: Mann iteration for s=0.5

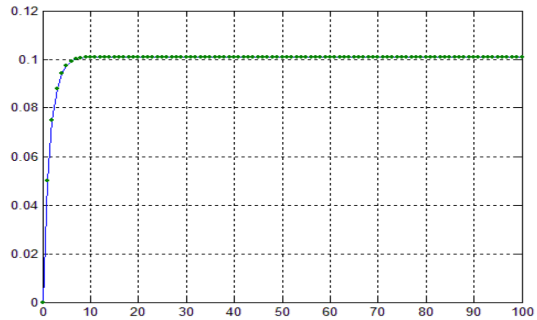


Table 16: Ishikawa iteration for s=0.6, s'=0.5

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.17341
2.	0.1006	12.	0.17344
3.	0.1423	13.	0.17345
4.	0.16	14.	0.17345
5.	0.16762	15.	0.17345
6.	0.17092	16.	0.17345
7.	0.17235	17.	0.17345
8.	0.17297	18.	0.17345
9.	0.17325	19.	0.17345
10.	0.17336	20.	0.17345

Here we observe that the value converges to a fixed point after 13 iterations

Figure 16: Ishikawa iteration for s=0.6, s'=0.5

Table 17: Mann iteration for s=0.8

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.10103
2.	0.08	12.	0.10103
3.	0.09641	13.	0.10103
4.	0.099999	14.	0.10103
5.	0.1008	15.	0.10103
6.	0.10098	16.	0.10103
7.	0.10102	17.	0.10103
8.	0.10103	18.	0.10103
9.	0.10103	19.	0.10103
10.	0.10103	20.	0.10103

Here we observe that the value converges to a fixed point after 07 iterations

Figure 17: Mann iteration for s=0.8

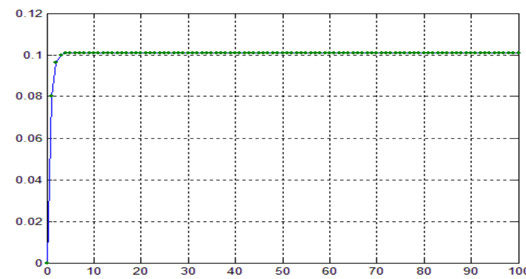
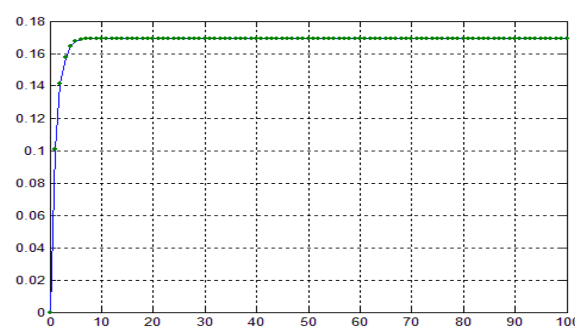


Table 18: Ishikawa iteration for s=0.6, s'=0.8

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.16926
2.	0.1006	12.	0.16927
3.	0.1413	13.	0.16928
4.	0.15785	14.	0.16928
5.	0.16461	15.	0.16928
6.	0.16737	16.	0.16928
7.	0.1685	17.	0.16928
8.	0.16896	18.	0.16928
9.	0.16915	19.	0.16928
10.	0.16923	20.	0.16928

Here we observe that the value converges to a fixed point after 12 iterations

Figure 18: Ishikawa iteration for s=0.6, s'=0.8



3.3 Fixed points of Bi-quadratic polynomial

Table 19: Picard iteration

Number of iteration *i*	|F(z)|	Number of iteration *i*	|F(z)|
1.	0	6.	0.1001
2.	0.1	7.	0.1001
3.	0.1001	8.	0.1001
4.	0.1001	9.	0.1001
5.	0.1001	10.	0.1001

Here we observe that the value converges to a fixed point after 02 iterations

Figure 19: Picard iteration

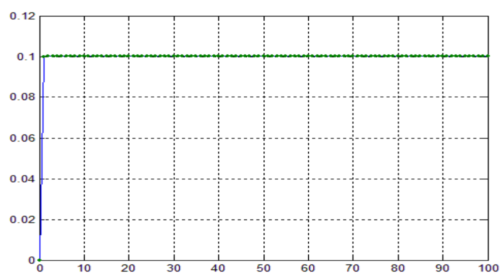


Table 20: Mann iteration for s=0.1

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
77.	0.10007	87.	0.10009
78.	0.10007	88.	0.10009
79.	0.10007	89.	0.10009
80.	0.10008	90.	0.10009
81.	0.10008	91.	0.10009
82.	0.10008	92.	0.10009
83.	0.10008	93.	0.10009
84.	0.10008	94.	0.10009
85.	0.10009	95.	0.1001
86.	0.10009	96.	0.1001

Here we skipped 76 iteration and the value converges to a fixed point after 95 iterations

Figure 20: Mann iteration for s=0.1

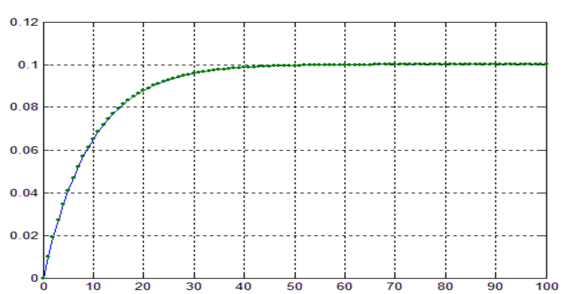


Table 21: Ishikawa iteration for s=0.6, s'=0.1

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.17078
2.	0.10006	12.	0.1708
3.	0.14081	13.	0.17081
4.	0.15791	14.	0.17081
5.	0.16523	15.	0.17082
6.	0.16839	16.	0.17082
7.	0.16976	17.	0.17082
8.	0.17036	18.	0.17082
9.	0.17062	19.	0.17082
10.	0.17073	20.	0.17082

Here we observe that the value converges to a fixed point after 14 iterations

Figure 21: Ishikawa iteration for s=0.6, s'=0.1

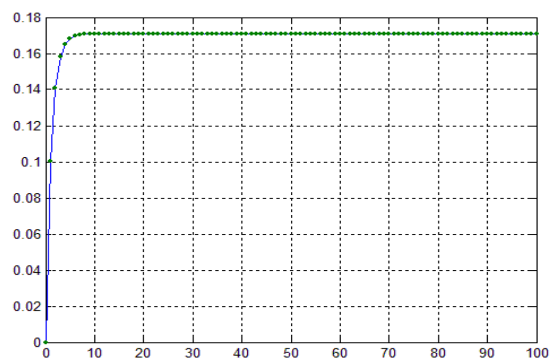


Table 22: Mann iteration for s=0.3

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
11.	0.09724	21.	0.10002
12.	0.098095	22.	0.10004
13.	0.098694	23.	0.10006
14.	0.099114	24.	0.10007
15.	0.099409	25.	0.10008
16.	0.099616	26.	0.10009
17.	0.09976	27.	0.10009
18.	0.099862	28.	0.10009
19.	0.099933	29.	0.1001
20.	0.099983	30.	0.1001

Here we skipped 10 iterations and the value converges to a fixed point after 28 iterations

Figure 22: Mann iteration for s=0.3

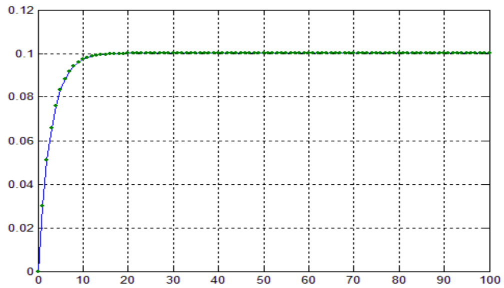


Table 23: Ishikawa iteration for s=0.6, s'=0.3

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.16892
2.	0.10006	12.	0.16894
3.	0.14053	13.	0.16894
4.	0.15714	14.	0.16894
5.	0.16403	15.	0.16895
6.	0.16689	16.	0.16895
7.	0.16809	17.	0.16895
8.	0.16859	18.	0.16895
9.	0.1688	19.	0.16895
10.	0.16888	20.	0.16895

Here we observe that the value converges to a fixed point after 14 iterations

Figure 23: Ishikawa iteration for s=0.6, s'=0.3

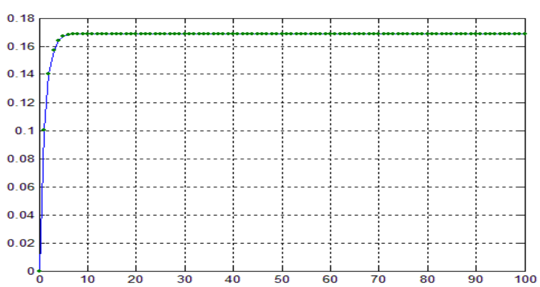


Table 24: Mann iteration for s=0.5

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.099999
2.	0.05	12.	0.10005
3.	0.075003	13.	**0.10007**
4.	0.087517	14.	0.10009
5.	0.093788	15.	0.10009
6.	0.096933	16.	0.1001
7.	0.09851	17.	0.1001
8.	0.099302	18.	0.1001
9.	0.0997	19.	0.1001
10.	0.099899	20.	0.1001

Here we observe that the value converges to a fixed point after 15 iterations

Figure 24: Mann iteration for s=0.5

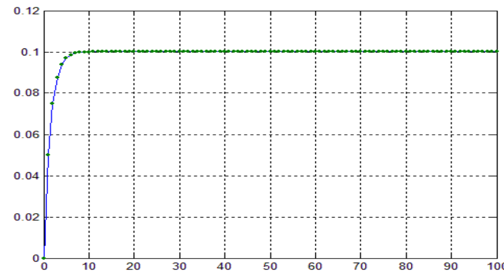


Table 25: Ishikawa iteration for s=0.6, s'=0.5

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.1678
2.	0.10006	12.	0.16781
3.	0.14033	13.	0.16782
4.	0.15664	14.	0.16782
5.	0.16327	15.	0.16782
6.	0.16596	16.	0.16782
7.	0.16706	17.	0.16782
8.	0.16751	18.	0.16782
9.	0.16769	19.	0.16782
10.	0.16777	20.	0.16782

Here we observe that the value converges to a fixed point after 25 iterations

Figure 25: Ishikawa iteration for s=0.6, s'=0.5

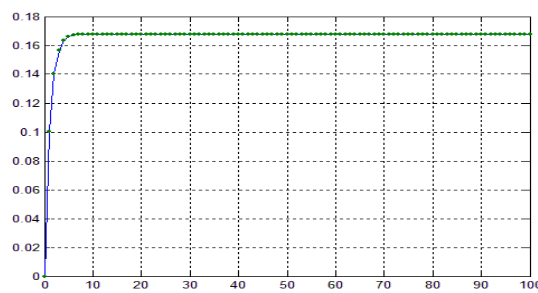


Table 26: Mann iteration for s=0.8

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.1001
2.	0.08	12.	0.1001
3.	0.096033	13.	0.1001
4.	0.099275	14.	0.1001
5.	0.099933	15.	0.1001

6.	0.10007	16.	0.1001
7.	0.10009	17.	0.1001
8.	0.1001	18.	0.1001
9.	0.1001	19.	0.1001
10.	0.1001	20.	0.1001

Here we observe that the value converges to a fixed point after 07 iterations

Figure 26: Mann iteration for s=0.8

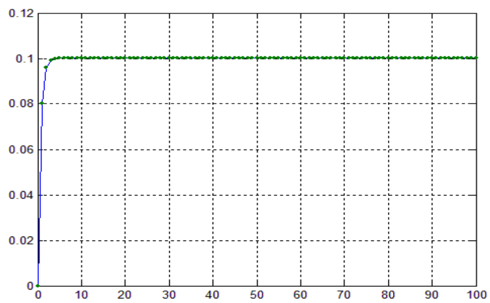
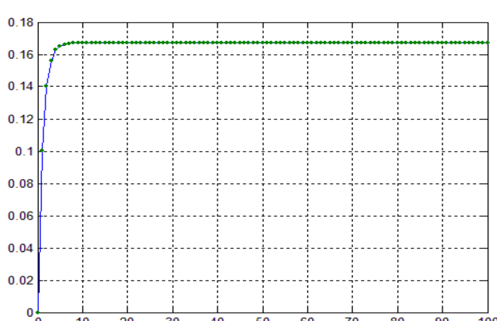


Table 27: Ishikawa iteration for s=0.6, s'=0.8

Number of iteration i	|F(z)|	Number of iteration i	|F(z)|
1.	0	11.	0.16697
2.	0.10006	12.	0.16698
3.	0.14015	13.	0.16699
4.	0.15622	14.	0.16699
5.	0.16267	15.	0.16699
6.	0.16526	16.	0.16699
7.	0.16629	17.	0.16699
8.	0.16671	18.	0.16699
9.	0.16688	19.	0.16699
10.	0.16694	20.	0.16699

Here we observe that the value converges to a fixed point after 12 iterations

Figure 27: Ishikawa iteration for s=0.6, s'=0.8



4. COMPARATIVE ANALYSIS

For Quadratic polynomial we observe that Picard iteration converges very fast to fixed point after 07 iterations. On the other hand for s=0.6, s'=0.1, Ishikawa iteration converges faster to fixed point (after 77 iterations) as comparative to Mann iteration which converges after 119 iterations.

Similarly for s=0.6, s'=0.3 Ishikawa converges after 31 iterations to fixed point while Mann iteration for s=0.3 converges after 37 iterations.

But for s=0.6, s'=0.5 Ishikawa iteration and Mann iteration for s=0.5 shows equivalence as both converges after 20 iterations.

For Cubic polynomial we observe that Picard again converges (after 03 iterations) faster than Mann and Ishikawa iterations. On the other hand s=0.6, s'=0.1, Ishikawa iteration converges to fixed point after 19 iterations which is comparatively faster than Mann iteration for s=0.1, which converges to fixed point after 95 iterations.

Similarly for s=0.6, s'=0.3 Ishikawa iteration converges after 15 iterations while Mann iteration for s=0.3, converges after 29 iterations.

Likewise for s=0.6, s'=0.5 Ishikawa iteration converges after 12 iterations while Mann iteration for s=0.5, converges after 15 iterations.

For Biquadratic polynomial, we again observe that Picard iteration converges to fixed point very fast after 02 iterations. On the other hand, for s=0.6, s'=0.1, Ishikawa iteration converges after 14 iterations while Mann iteration for s=0.1 converges after 95 iterations.

Similarly for s=0.6, s'=0.3 Ishikawa iteration converges after 14 iterations while Mann iteration for s=0.3, converges after 28 iterations.

Likewise for s=0.6, s'=0.5 Ishikawa iteration converges after 12 iterations while Mann iteration for s=0.5, converges after 15 iterations.

In this paper we have not considered higher values of s because if we consider s=0.8 or more higher, than Mann iteration starts behaving like Picard and so it will converges comparatively faster than Ishikawa iteration. Table 8 and Table 9 of 3.1, 3.2 and 3.3 shows the following behavior.

5. CONCLUSION

The unprecedented graphical study done previously by us shows that for the above mentioned conditions $0 < s < 1, 0 < s' \leq \frac{1}{2}$, we have Ishikawa iteration converging faster than Mann iteration while Picard’s iteration converges faster than both.

Besides this, we also observe that as the power of the polynomial increases, the convergence goes on faster and faster for all iterations (Picard’s, Mann and Ishikawa).

6. REFERENCES

[1] G.V.R Babu and K.N.V.V. Vara Prashad, "*Mann iteration converges faster than Ishikawa iteration for the class of Zamfirescu Operators*", Fixed Point Theory and Applications, Vol. 2006, Art. ID 49615, 6 pages.

[2] V. Berinde, "Picard's iteration converges faster than Mann iteration for a class of Quasi-Contractive Operators", Fixed Point Theory and Application, Vol. 2004, No. 2. 97-105.

[3] Y. S. Chauhan, R. Rana and A. Negi, "New Julia Sets of Ishikawa Iterates", International Journal of Computer Applications 7(13):34–42, October 2010.

[4] K. Deimling, "Nonlinear Functional Analysis", Springer Verlag, Berlin, 1985.

[5] S. Ishikawa, "Fixed points by a new iteration method", Proc. Amer. Math. Soc.44 (1974), 147-150.

[6] W. R. Mann, "Mean value methods in iteration", Proc. Amer. Math. Soc. 4(1953), 506-510.

[7] R. Rana, Y. S. Chauhan and A. Negi, Non Linear dynamics of Ishikawa Iteration, International Journal of Computer Applications 7(13):43–49, October 2010.

[8] B. E. Rhoades and S. M. Soltuz, "The convergence of mean value iteration for a family of maps", I. J. M. M. S., (2005), 3479-3485.

[9] Xue Zhiqun, " Remarks of equivalence among Picard, Mann and Ishikawa iterations in Normed Spaces", Fixed Point Theory and Applications, Vol. 2007, ArticleId 61434, 5 pages.

[10] Xue Zhiqun, "On the convergence between Mann iteration and Ishikawa iteration for the Generalized Lipschitizan and ϕ-strongly pseudocontractive mappings", Bull. Korean Math. Soc. 45 (2008), No. 4, pp- 635-644.