# D<sup>3</sup>LS STAP Approach on Wideband Signals using Uniformly Spaced Real Elements

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# ABSTRACT

In this paper, Space Time Adaptive Processing (STAP) applied on wideband signals from uniformly spaced real elements using Direct Data Domain Least Squares (D<sup>3</sup>LS). Two antenna configurations are used: an array of dipole antenna (narrowband antenna) and an array of patch antenna (wideband antenna). A frequency transformation technique is used only for the case of narrowband antenna which transforms the steering vector at a wideband frequency to another steering vector at the desired frequency. The mutual coupling between elements will affect on the estimation of complex amplitude of Signal of Interest (SOI). It is necessary to compensate for the strong mutual coupling that exists between the real antenna elements. This is done by using coupling transformation matrix which converts the voltages that are induced at the real antenna elements to an equivalent set of voltages that will be induced by the same incident wave in uniform linear virtual array (ULVA). Then, we will apply STAP D<sup>3</sup>LS on the compensated voltages. Numerical simulations are done using the three main methods of  $D^{3}LS$  namely the forward. backward, and the forward-backward methods.

# **General Terms**

Adaptive signal Processing algorithm, Beamforming algorithm

## **Keywords**

Direct Data Domain Least Squares (D<sup>3</sup>LS), Space Time Adaptive Processing (STAP)

# 1. INTRODUCTION

The beamforming of adaptive antenna is a computationally intensive process at which each users signal is multiplied by complex weight vectors that adjust the magnitude and phase of the signal from each antenna element. The principle advantage of an adaptive array is the ability to electronically steer the main lobe of the antenna to any desired direction while also automatically placing deep pattern nulls in the specific direction of interference sources and the direct data domain algorithms used to overcome these drawbacks of statistical techniques which are adaptively minimize the interference power while maintaining array gain in the direction of the target signal [1,2]. Until recently, most of the works on smart antennas have concerned narrowband communication systems that are characterized by a fractional bandwidth in the order of one to a few percents. The beamforming techniques used in narrowband systems are inadequate for wideband systems because they are unable to track a desired user or form nulls or low sidelobes

towards interfering sources over a large frequency band. In order to overcome the problem of narrowband beamformers, several wideband beamforming techniques have been proposed recently [3]. In order to deal with wideband signals we have two solutions either using wideband antennas or using narrowband antennas with a frequency transformation technique. This transformation technique is done by transforming the steering vector at any frequency within the design frequency band into a new Steering vector at a specified reference frequency. In the practical case, if we are dealing with real elements. The elements spatially sample and reradiate the incident fields. The reradiated fields interact with the other elements resulting in mutual coupling between elements. EM principles are applied to compensate for the effects of mutual coupling between the antenna elements. This EM processing technique transforms the voltages that are induced in the uniformly spaced real elements due to all incoming signals to an equivalent set of voltages that will be produced in a ULVA containing isotropic point radiators by the same set of incident signals [4]. Once the corrected voltages are obtained, we can apply STAP based on D<sup>3</sup>LS approach. STAP is carried out by performing two-dimensional (2-D) filtering on signals that are collected by simultaneously combining signals from the elements of an antenna array (the spatial domain) as well as from the multiple pulses from coherent radar (the temporal domain) [5]. This algorithm is used for suppressing highly dynamic interference and enables the system to detect potentially weak target returns.

# 2. SYSTEM MODEL

Assume that U+1 source impinge on an array of real elements antenna from distinct azimuthal directions  $\varphi_0, \ldots, \varphi_U$ . So in addition to SOI there are U undesired signals. We will deal with two arrays of dipole and patch antennas. Fig.1 and Fig.2 show the configurations of the Uniform Linear Equal Spaced Dipole Array (ULESDA) and patch array respectively. Each configuration consists of N elements located along the x-axis, and the distance between any two adjacent elements is d.



Fig.1 Geometry of ULESDA and its equivalent ULVA



Fig.2 Geometry of Patch array and its equivalent ULVA

Using the complex envelope representation, a single snapshot of the voltages represents a  $N \times 1$  vector of phasor voltages [x(t)] received by the elements of the actual array at a particular time instance *t* and can be expressed by

$$\begin{bmatrix} x(t) \end{bmatrix} = \begin{bmatrix} x_0(t) \\ x_1(t) \\ \vdots \\ x_{N-1}(t) \end{bmatrix} = \sum_{u=0}^U s_u(t) [a(\varphi_u)] + [\zeta(t)]$$
(1)

Where  $s_u(t)$  denotes the signal at the elements of the array from the  $u^{th}$  source, for u = 0, ..., U.  $[a(\varphi)]$  denotes the far field pattern of the array toward the azimuth direction  $\varphi$  and  $[\zeta(t)]$  denotes the noise vector at each of the loaded antenna elements. By using a matrix representation (1) becomes [6]

$$[x(t)] = [A(\varphi)][s(t)] + [\zeta(t)]$$
<sup>(2)</sup>

Where  $[A(\varphi)]$  of size  $N \times (U+1)$  referred to as the array manifold corresponding to each one of the incident signals of unity amplitude and is represented by

$$[A(\varphi)] = [a(\varphi_0), a(\varphi_1), \dots, a(\varphi_U)]$$
(3)

#### 2.1 Frequency Transformation Technique

A wideband received signal is incident on a uniform linear array of N elements and d the spacing between elements. The procedure is based on transforming the steering vector at the wideband frequency  $f_w$  into a prespecified reference frequency  $f_n$ using the transformation matrix for U+1 uniformly spaced directions covering the angular region [3,7]. This technique is used for the case of first configuration as shown in fig.1 because this antenna is narrowband antenna and we will transform the wideband steering vector to another steering vector operating in the frequency range of this ULESDA. The array steering matrix whose columns are the array steering vectors at the frequency  $f_w$ corresponding to each of the U+1 directions is defined by:

$$\left[A(\varphi, f_w)\right] = \left[a(\varphi_0, f_w), a(\varphi_1, f_w), \dots, a(\varphi_U, f_w)\right] \quad (4)$$

Hence, we are going to select the best-fit frequency transformation,  $[\Im_f]$ , between The steering matrix at the frequency  $f_w$ ,  $[A(\varphi, f_w)]$ , and The steering matrix at the reference frequency  $f_n$ ,  $[A_v(\varphi, f_n)]$  such that

$$[\mathfrak{I}_{f}][A(\varphi, f_{w})] = [A_{v}(\varphi, f_{n})]$$
(5)

First, we define a set of uniformly defined angles to cover a sector, the azimuth angles spanning  $[\varphi_a, \varphi_{a+1}]$ 

$$[\Phi_{q}] = [\varphi_{q}, \varphi_{q} + \varphi, \varphi_{q} + 2\varphi, ...., \varphi_{q+1}]$$
(6)

where the angle  $\varphi$  represents the azimuth incremental step size. Then we measure the steering vectors associated with the set  $[\Phi_q]$ . The measured steering matrix at the frequency  $f_w$  is defined by

$$[A(\Phi_q, f_w)] = [a(\varphi_q, f_w), a(\varphi_q + \varphi, f_w), \dots, a(\varphi_{q+1}, f_w)]$$
(7)

Next, we calculate the measured steering matrix at the reference frequency  $f_n$  corresponding to the same set of angles  $[\Phi_a]$ 

$$[A_{v}(\Phi_{q}, f_{w})] = [a_{v}(\varphi_{q}, f_{w}), a_{v}(\varphi_{q} + \varphi, f_{w}), \dots, a_{v}(\varphi_{q+1}, f_{w})](8)$$

The frequency transformation matrix  $[\Im_f]$  is obtained by minimizing the mean square error function and the solution of it is as follows [7]

$$[\mathfrak{I}_{f}] = [A_{v}(\Phi_{q}, f_{n})][A(\Phi_{q}, f_{w})]^{H} \left\{ A(\Phi_{q}, f_{w})][A(\Phi_{q}, f_{w})]H \right\}^{-1} (9)$$

where the superscript H represents the conjugate transpose of a complex matrix. The frequency transformation matrix  $[\Im_f]$ 

needs to be computed only once *a priori* for the defined sector and this computation can be done off-line. So, now the actual steering vector at the wideband frequency is transformed to another one at the reference frequency.

#### **2.2 Coupling Compensation Technique**

In practice, the induced voltages in a real array (ULESDA or patch array) are contaminated by the effects of the mutual coupling between the elements of the array which will undermine the performance of a conventional adaptive signal processing algorithm [4]. By using a coupling transformation matrix, one can compensate for all the undesired electromagnetic effects. The voltages induced in an actual array are then transformed to a set of voltages that would be induced in an ULVA consisting of omnidirectional isotropic point radiators. It is based on transforming the real array into an ULVA operating in the absence of mutual coupling and other undesired electromagnetic effects. Hence, we are going to select the best-fit coupling transformation, [3], between the real array manifold,  $[A(\varphi)]$ , and the virtual array manifold corresponding to an ULVA,  $[A_v(\varphi)]$  such that

$$[\mathfrak{I}][A(\varphi)] = [A_{\nu}(\varphi)] \tag{10}$$

for all of the azimuth angles  $\varphi$  within a predefined sector.

First, we define a set of uniformly defined angles to cover a sector, the azimuth angles spanning  $[\varphi_q, \varphi_{q+1}]$ 

$$[\Phi_q] = [\varphi_q, \varphi_q + \varphi, \varphi_q + 2\varphi, \dots, \varphi_{q+1}]$$
(11)

where  $\varphi$  represents the azimuth incremental step size. Then we measure the steering vectors associated with the set  $[\Phi_q]$  of the real array. The measured real array manifold is defined by

$$A[\Phi_{q}] = [a(\varphi_{q}), a(\varphi_{q} + \varphi), a(\varphi_{q} + 2\varphi), ..., a(\varphi_{q+1})]$$
(12)

This can include all the undesired electromagnetic coupling effects. Next, we calculate the virtual array manifold corresponding to the same set of angles  $[\Phi_a]$ 

$$A_{v}[\Phi_{q}] = [a_{v}(\varphi_{q}), a_{v}(\varphi_{q} + \varphi), a_{v}(\varphi_{q} + 2\varphi), ..., a_{v}(\varphi_{q+1})]$$
(13)

Then the coupling transformation matrix is obtained using a least squares solution as follows:

$$[\mathfrak{I}_q] = [A_\nu(\Phi_q)][A(\Phi_q)]^H \left\{ A(\Phi_q)][A(\Phi_q)]^H \right\}^{-1}$$
(14)

Where the superscript *H* represents the conjugate transpose of a complex matrix. This transformation matrix needs to be computed only once *a priori* for the defined sector and this computation can be done off-line. Hence, once  $[\mathfrak{I}]$  is known we can compensate for the various undesired electromagnetic effects, in real time by carrying out a single matrix vector

multiplication. Finally, using (14), we can obtain the processed input voltages in which the mutual coupling effects have been eliminated. The compensated voltages  $[x_c(t)]$  will then be given by [6]

$$[x_c(t)] = [\Im][x(t)] \tag{15}$$

# 3. D<sup>3</sup>LS STAP APPROACH

Since the antenna platform is moving, there is a Doppler shift,  $f_d$ , in the received signal With M pulses received by a single antenna element [8]. So, the system processes M coherent pulses at a constant pulse repetition frequency,  $f_r$ ,

The compensated voltages are now applied to the STAP processor as shown in Fig.3 [9].



#### Antenna Elements

The  $D^3LS$  method has three different formulations namely the forward, backward, and the forward–backward methods.

#### 3.1 Forward Method

The idea of the  $D^3LS$  method is to use a single space-time snapshot of the data received by the array antenna to generate a cancellation matrix that does not contain the SOI component. In other words, the cancellation matrix contains only the interference and noise in the received data. Then, the weight vector that forces this matrix to be zero will be determined. By putting an additional constraint row for the SOI, the weight vector preserves the SOI gain while canceling the interference and noise in the data [8]. To generate the cancellation matrix, the element-to-element offset of the SOI in space and time, respectively, are defined as [6, 9]

$$Z_1 = \exp(j2\pi \frac{d}{\lambda}\cos\phi_s) \tag{16}$$

$$Z_2 = \exp(j2\pi \frac{f_d}{f_r}) \tag{17}$$

where  $\phi_s$  and  $\lambda$  are the angle of arrival and the wavelength for SOI and *d* is antenna spacing. Thus we can form three cancellation equations from the received signal and its adjacent data as follows:

$$X_{m,n} - X_{m,n+1} Z_1^{-1}, (18)$$

$$X_{m,n} - X_{m+1,n} Z_2^{-1}, (19)$$

$$X_{m,n} - X_{m+1,n+1} Z_1^{-1} Z_2^{-1}$$
(20)

By setting the number of weights to be  $N_a N_p$  according to [6], the cancellation matrix for (18) can be formed as

$$\begin{bmatrix} x_{1,1} - x_{1,2}Z_1^{-1} & x_{1,2} - x_{1,3}Z_1^{-1} & \dots & x_{1,N_a} - x_{1,N_a+1}Z_1^{-1} \\ x_{2,1} - x_{2,2}Z_1^{-1} & x_{2,2} - x_{2,3}Z_1^{-1} & \dots & x_{2,N_a} - x_{2,N_a+1}Z_1^{-1} \\ \vdots & \vdots & \vdots \\ x_{N_p,1} - x_{N_p,2}Z_1^{-1} & x_{N_p,2} - x_{N_p,3}Z_1^{-1} & \dots & x_{N_p,N_a} - x_{N_p,N_a+1}Z_1^{-1} \end{bmatrix}$$
(21)

Similarly, the cancellation matrix for (19) and (20) can be generated in the same manner. We have to know that (18) corresponds to spatial difference, (19) corresponds to temporal difference, and (20) corresponds to spatial-temporal difference of the received signal. Once three different cancellation matrices have been generated, we will arrange the elements of each cancellation matrix as a row vector of dimension  $1 \times N_p N_a$  by putting each row side by side. We call this row as a cancellation row. Now, we have generated three cancellation rows. To find a weight vector, we need to generate the total of  $N_p N_a - 1$  row vectors. To preserve the SOI from being canceled by the weight vector, we left 1 row for gain constraints along target direction as follows [10].

$$\left[1 \ z_1 \ z_1^2 \ \dots \ z_1^{N_a - 1} \ z_2 \ z_1^{Z_2} \ z_1^2 z_2 \ \dots \ z_1^{N_a - 1} z_2 \ z_2^2 \ z_1^2 z_2^2 \ \dots \ z_1^{N_a - 1} z_2^{N_p - 1}\right] (22)$$

After we put all cancellation and constraints rows, we obtain a cancellation matrix [T] of dimension  $N_p N_a \times N_p N_a$  and the weight vector, W, which cancels the interference and maintain the SOI can be found by solving the following equation:

$$[T][W] = \begin{bmatrix} C & 0 & \cdots & 0 \end{bmatrix}_{N_a N_p}^T$$
(23)

where *C* is a look-direction gain SOI. After obtaining W, the signal amplitude,  $\hat{\alpha}$ , can be estimated from[8]

$$\hat{\alpha} = \frac{1}{C} \sum_{m=1}^{N_p} \sum_{n=1}^{N_a} W[(N_a \times m - 1) + n] X_{m,n}$$
(24)

#### 3.2 Backward Method

The weight vector can also calculated in a backward direction; where the equations (18)-(20) are generated in the reverse order with complex conjugate starting from  $X_{N_p,N_a}$  [6,10]. The cancellation equations for backward method are as follows.

$$X_{m,n}^* - X_{m,n-1}^* Z_1^{-1}, (25)$$

$$X_{m,n}^* - X_{m-1,n}^* Z_2^{-1}, (26)$$

$$X_{m,n}^{*} - X_{m-1,n-1}^{*} Z_{1}^{-1} Z_{2}^{-1}$$
(27)

Where \* denotes the complex conjugate of the data. We, then, simply obtain a similar cancellation matrix as in (21) in a similar fashion for the forward method with the reversed order of the conjugated data. Once  $N_pN_a$  –1 cancellation rows have been generated, the look-direction constraint in (22) will be added to obtain a square matrix [B] of size  $N_pN_a \times N_pN_a$ . The weight vector can be obtained by solving the following equation [8]

$$[B][W] = \begin{bmatrix} C & 0 & \cdots & 0 \end{bmatrix}_{N_a N_p \times 1}^T$$
(28)

After obtaining W, the signal amplitude can be estimated from

$$\hat{\alpha} = \left[\frac{Z_1^{N_a - 1} Z_2^{N_p - 1}}{C} \sum_{m=1}^{N_p} \sum_{n=1}^{N_a} W[(N_a \times m - 1) + n] X_{N_p - m + 1, N_a - n + 1}^*\right]^*$$
(29)

### 3.3 Forward-Backward Method

To increase the number of degrees of freedom of the system, one need to use the forward-backward method that can be generated by putting both of the cancellation matrices from forward and backward directions together when calculating the weight vector. And the target signal complex amplitude can be estimated by either forward or backward direction. This increases the number of degrees of freedom and the accuracy of the system [10]. In the next section, the performance of the three methods will be shown via numerical simulations.

#### **4** SIMULATION RESULTS

The performance of D<sup>3</sup>LS STAP on the induced signals from ULESDA and Patch array will be illustrated through the next two examples. For the first example, consider the SOI to be arriving from  $\varphi_s = 100^{\circ}$  with amplitude 1 + j1 and one interferer with signal-to-interference ratio (SIR) –47 dB and arriving from  $\varphi_J = 70^{\circ}$ . All these signals will be received by ULESDA. The signal-to-noise (SNR) ratio at each antenna element is 30 dB. In this simulation, The SOI and the jammer are operating at the following frequencies: 2GHz, 2.5GHz,

3GHz, 3.5GHz, and 4GHz. The antenna element spacing *d* is set to  $0.5\lambda$  at the reference frequency. *N* is set to 10. The induced voltages are applied to frequency and coupling transformation techniques respectively then the compensated voltages are applied to D<sup>3</sup>LS STAP algorithm. fig. 4, fig.5, and fig.6 show the performance of the system for the three methods of D<sup>3</sup>LS.



Fig. 4 Beam Pattern of the forward method using ULESDA



Fig. 5 Beam Pattern of the backward method using ULESDA



Fig. 6 Beam Pattern of the forward-backward method using ULESDA

For the second example, consider the SOI to be arriving from  $\varphi_s = 100^\circ$  with amplitude 1 + j1 and one interferer with SIR= -47 dB and arriving from  $\varphi_J = 50^\circ$ . All these signals will be received by patch array. The SNR at each antenna element is 30 dB. In this simulation, The SOI and the jammer are operating at the same frequencies of the first example. The antenna element spacing  $d = 0.5\lambda$  and N is set to 7. The induced voltages are applied to the coupling transformation technique then the compensated voltages are applied to D<sup>3</sup>LS STAP algorithm. The performance of the system for the three main methods of D<sup>3</sup>LS can be shown in fig. 7, fig.8, and fig.9.



Fig. 7 Beam Pattern of the forward method using patch array



Fig. 8 Beam Pattern of the backward method using patch array



Fig. 9 Beam Pattern of the forward-backward method using patch array

As seen from the figures; the performance of  $D^3LS$  STAP is very good since the jammer is nulled correctly for the three cases while the main beam is constrained in the SOI direction. The results of patch array have the advantage of providing deep null than the results of ULESDA. This advantage will affect directly on the accuracy of complex amplitude estimation.

# **5** CONCLUSION

A  $D^3LS$  STAP approach applied on a wideband signals received by ULESDA and Patch array. We use only single snapshot in order to null the interference signal while maintaining the main beam directed towards SOI. The main beam is directed towards the target signal and the jammer is nulled correctly with deep null. The results of Patch array is better than that of ULESDA since it provides more deep null in the direction of the jammer and it will affect on the accuracy of the complex amplitude estimation.

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