

New Construction of Fuzzy Soft Lattices

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ABSTRACT

In this paper, we study Molodtsov notion of soft set and fuzzy soft set considering the fact that the parameters are mostly fuzzy hedges or fuzzy parameters. We also, study the notion of fuzzy soft lattice on groups, homomorphic image, pre-image of fuzzy soft lattices, arbitrary family of fuzzy soft lattices and fuzzy normal soft lattices using T-norms. Also the notion of sensible fuzzy soft lattice in groups and some related properties on it are investigated.

Key Words: Soft set, Soft lattice, fuzzy soft lattice, Sensible fuzzy normal lattice, direct product.

1. INTRODUCTION

In 1999 Molodtsov [9], initiated the novel concept of soft set theory which is completely new approach for modeling, vagueness and uncertainties. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in [9]. After Molodtsov work, some different applications of soft set theory were studied in [5].

Furthermore Maji, Biswas and Roy worked on soft set theory in [7]. Also Maji et.al [6] presented the definition of fuzzy soft set and Roy et.al presented some applications of the notion in decision making problems. The algebraic structures of set theories dealing with uncertainties has also been studied by some authors. Rosenfield [10] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets. Rough groups were defined by Biswas and some authors have studied the algebraic properties of rough sets as well. Recently many authors discuss the soft set research on the soft set theory is progressing rapidly. For example, the concept of soft semi ring [4], soft groups [5], soft BCK / BCI algebras, soft BL-algebras and fuzzy soft groups.

In this paper, we study Molodtsov notion of soft set and fuzzy soft set considering the fact that the parameters are mostly fuzzy hedges or fuzzy parameters. We introduce the notion of fuzzy soft lattice on groups, homomorphic image, pre-image of fuzzy soft lattices, arbitrary family of fuzzy soft lattices and fuzzy normal soft lattices using T-norms. Also the notion of sensible fuzzy soft lattices in groups and some related properties on it are investigated.

2. PRELIMINARIES

Definition 2.1 : A pair (f, A) is called a soft set over the lattice L , if $f : A \rightarrow p(L)$. Here L be the initial universe and E

be the set of parameters. Let $p(L)$ denotes the power set of L and IL denotes the set of all fuzzy set on L .

Definition 2.2: Let $M : (f, A)$ be a soft set and $f : A \rightarrow p(L)$ is a mapping. Where L is a complete lattice then the soft set M is called soft lattice.

Definition 2.3: A pair (f, A) is called a fuzzy soft set over L . Where $f : A \rightarrow IL$ (ie) for each $a \in A$, $f : L \rightarrow I$ is a fuzzy set in L .

Definition 2.4: Let (f, A) be a fuzzy soft set and $f_a : A \rightarrow G$ is called fuzzy soft lattice if

$$(FSL1) \quad f_a(x + y) \geq T \{f_a(x), f_a(y)\}$$

$$(FSL2) \quad f_a(-x) \geq f_a(x)$$

$$(FSL3) \quad f_a(x \vee y) \geq T \{f_a(x), f_a(y)\}$$

$$(FSL4) \quad f_a(x \wedge y) \geq T \{f_a(x), f_a(y)\} \quad , \quad \text{for all } x, y \in G$$

Definition 2.5: Let A be a fuzzy set and $f_a : A \rightarrow G$ be a lattice ordered group of A then f_a is called fuzzy soft lattice ordered group (FSLG) If

$$(FSLG1) \quad f_a(x + y) \geq T \{f_a(x), f_a(y)\}$$

$$(FSLG2) \quad f_a(-x) \geq f_a(x)$$

$$(FSLG3) \quad f_a(0) = 1, \text{ for all } x, y \in G$$

Definition 2.6 : By a t-norm T , we mean a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfying the following conditions

$$(T1) \quad T(0, X) = 0$$

$$(T2) \quad T(x, y) \leq T(x, z) \text{ if } y \leq z$$

$$(T3) \quad T(x, y) = T(y, x)$$

$$(T4) \quad T(x, T(y, z)) = T(T(x, y), z), \quad \text{for all } x, y, z \in [0, 1]$$

Definition 2.7: For any fuzzy soft set f_a in G and $t \in [0, 1]$, we define the set

$U(f_a; t) = \{x \in G / f_a(x) \geq t\}$, which is called an upper cut of f_a and can be used to the characterization of f_a .

Definition 2.8: Let $f : G \rightarrow G'$ be a lattice group homomorphism and A be a fuzzy soft lattice of G' then $A \circ f(x) = (A \circ f)(x) = f^{-1}(A)(x)$

Definition 2.9 : Let $\theta : X \rightarrow Y$ be a map. A and B are fuzzy soft lattices in X and Y respectively. Then the inverse image of B under θ is a fuzzy soft set defined by

$$\theta^{-1}(B) = (f_a)_{\theta^{-1}(B)} \text{ where } (f_a)_{\theta^{-1}(B)}(x) = (f_a)_B(\theta(x))$$

Definition 2.10: Let G be a group and A be a fuzzy soft lattice of G , then A is called Fuzzy Normal soft lattice (FNSL) if $A(xy) = A(yx)$, for all $x, y \in G$

Definition 2.11: Let $(f_a)_A$ be a fuzzy soft set of G . Let $\theta: G \rightarrow G'$ be a map. Define the map $(f_a)_A(\theta): G \rightarrow [0, 1]$ by $(f_a)_{A\theta}(x) = (f_a)_A(\theta(x))$.

3 CHARACTERIZATION OF FUZZY SOFT LATTICES

Proposition 3.1 : Let G and G' be two Fuzzy Soft Lattice ordered group and $\theta: G \rightarrow G'$ be a homomorphism If B is a Fuzzy Soft Lattice of G' then the pre-image $\theta^{-1}(B)$ is Fuzzy Soft Lattice of G .

Proof : Assume that B is a Fuzzy Soft Lattice of G . Let $x, y \in G$.

$$\begin{aligned} \text{(FSL1)} \quad (f_a)_{\theta^{-1}(B)}(x+y) &= (f_a)_B(\theta(x+y)) \\ &= (f_a)_B(\theta(x) + \theta(y)) \\ &\geq T \{ (f_a)_B(\theta(x)), (f_a)_B(\theta(y)) \} \end{aligned}$$

$$\geq T \{ (f_a)_{\theta^{-1}(B)}(x), (f_a)_{\theta^{-1}(B)}(y) \}$$

$$\begin{aligned} \text{(FSL2)} \quad (f_a)_{\theta^{-1}(B)}(-x) &= (f_a)_B(\theta(-x)) \\ &\geq (f_a)_B(\theta(x)) \\ &\geq (f_a)_{\theta^{-1}(B)}(x) \end{aligned}$$

$$\begin{aligned} \text{(FSL3)} \quad (f_a)_{\theta^{-1}(B)}(x \vee y) &= (f_a)_B(\theta(x \vee y)) \\ &= (f_a)_B(\theta(x) \vee \theta(y)) \\ &\geq T \{ (f_a)_B(\theta(x)), (f_a)_B(\theta(y)) \} \\ &\geq T \{ (f_a)_{\theta^{-1}(B)}(x), (f_a)_{\theta^{-1}(B)}(y) \} \end{aligned}$$

$$\begin{aligned} \text{(FSL4)} \quad (f_a)_{\theta^{-1}(B)}(x \wedge y) &= (f_a)_B(\theta(x \wedge y)) \\ &= (f_a)_B(\theta(x) \wedge \theta(y)) \\ &\geq T \{ (f_a)_B(\theta(x)), \mu_B(\theta(y)) \} \\ &\geq T \{ (f_a)_{\theta^{-1}(B)}(x), (f_a)_{\theta^{-1}(B)}(y) \} \end{aligned}$$

$\therefore \theta^{-1}(B)$ is a Fuzzy Soft Lattice of G .

Proposition 3.2 : Let $\theta: G \rightarrow G'$ be an epimorphism and B is a Fuzzy soft set in G' . If $\theta^{-1}(B)$ is Fuzzy Soft Lattice of G then B is Fuzzy Soft Lattice of G' .

Proof : Let $x, y \in G$, there exist an element $a, b \in G'$ such that $\theta(a) = x, \theta(b) = y$

$$\begin{aligned} \text{(FSL1)} \quad (f_a)_B(x+y) &= (f_a)_B(\theta(a) + \theta(b)) \\ &= (f_a)_B\theta(a+b) \end{aligned}$$

$$\begin{aligned} &= (f_a)_{\theta^{-1}(B)}(a+b) \\ &\geq T \{ (f_a)_{\theta^{-1}(B)}(a), (f_a)_{\theta^{-1}(B)}(b) \} \\ &\geq T \{ (f_a)_B\theta(a), (f_a)_B\theta(b) \} \\ &\geq T \{ (f_a)_B(x), (f_a)_B(y) \} \end{aligned}$$

$$\begin{aligned} \text{(FSL2)} \quad (f_a)_B(-x) &= (f_a)_B(-\theta(a)) \\ &= (f_a)_B(\theta(-a)) \\ &= (f_a)_{\theta^{-1}(B)}(-a) \\ &\geq (f_a)_{\theta^{-1}(B)}(a) \\ &\geq (f_a)_B\theta(a) \\ &\geq (f_a)_B(x) \end{aligned}$$

$$\begin{aligned} \text{(FSL3)} \quad (f_a)_B(x \vee y) &= (f_a)_B(\theta(a) \vee \theta(b)) \\ &= (f_a)_B\theta(a \vee b) \\ &= (f_a)_{\theta^{-1}(B)}(a \vee b) \end{aligned}$$

$$\begin{aligned} &\geq T \{ (f_a)_{\theta^{-1}(B)}(a), (f_a)_{\theta^{-1}(B)}(b) \} \\ &\geq T \{ (f_a)_B\theta(a), (f_a)_B\theta(b) \} \\ &\geq T \{ (f_a)_B(x), (f_a)_B(y) \} \end{aligned}$$

$$\begin{aligned} \text{(FSL4)} \quad (f_a)_B(x \wedge y) &= (f_a)_B(\theta(a) \wedge \theta(b)) \\ &= (f_a)_B(\theta(a \wedge b)) \\ &= (f_a)_{\theta^{-1}(B)}(a \wedge b) \end{aligned}$$

$$\begin{aligned} &\geq T \{ (f_a)_{\theta^{-1}(B)}(a), (f_a)_{\theta^{-1}(B)}(b) \} \\ &\geq T \{ (f_a)_B\theta(a), (f_a)_B\theta(b) \} \\ &\geq T \{ (f_a)_B(x), (f_a)_B(y) \} \end{aligned}$$

$\therefore B$ is a Fuzzy Soft Lattice of G' .

Proposition 3.3 : If $\{A_i\}$ is a family of Fuzzy Soft Lattice of G then $\cap A_i$ is a Fuzzy Soft Lattice of G . Where $\cap A_i = \{x, \wedge (f_a)_{A_i}(x) / x \in G\}$, here $i \in A$.

Proof : Let $x, y \in G$ then for $i \in A$ it follows that

$$\begin{aligned} \text{(FSL1)} \quad (\cap (f_a)_{A_i})(x+y) &= \wedge (f_a)_{A_i}(x+y) \\ &\geq \wedge T \{ (f_a)_{A_i}(x), (f_a)_{A_i}(y) \} \end{aligned}$$

$$\geq \wedge T \{ (\cap (f_a)_{A_i})(x), (\cap (f_a)_{A_i})(y) \}$$

$$\begin{aligned} \text{(FL2)} \quad (\cap (f_a)_{A_i})(-x) &= \wedge (f_a)_{A_i}(-x) \\ &\geq \wedge (f_a)_{A_i}(x) \\ &\geq (\cap (f_a)_{A_i})(x) \end{aligned}$$

$$(FL3) (\cap(f_a)_{A_i})(x \vee y) = \Lambda(f_a)_{A_i}(x \vee y)$$

$$\geq \Lambda T \{(\cap(f_a)_{A_i})(x), (\cap(f_a)_{A_i})(y)\}$$

$$\geq T \{(\cap(f_a)_{A_i})(x), (\cap(f_a)_{A_i})(y)\}$$

$$(FL4) (\cap(f_a)_{A_i})(x \wedge y) = \Lambda(f_a)_{A_i}(x \wedge y)$$

$$\geq \Lambda T \{(f_a)_{A_i}(x), (f_a)_{A_i}(y)\}$$

$$\geq T \{(\cap(f_a)_{A_i})(x), (\cap(f_a)_{A_i})(y)\}$$

$\therefore \cap A_i$ is a Fuzzy Soft Lattice of G.

Proposition 3.4 : If A is Fuzzy Soft set in G such that all non empty level subject $U(A; t)$ is a Fuzzy Soft Lattice of G then A is Fuzzy Soft Lattice of G.

Proof : Let $x, y \in U(A; t)$, we have $A(x) \geq t$ and $A(y) \geq t$. So that $x + y \in U(A; t)$ we have $A(x + y) \geq t$.

$$(FSL1) A(x + y) \geq t \geq \min\{t, t\}$$

$$\geq T \{A(x), A(y)\}$$

$$(FSL2) A(-x) \geq t = A(x) \geq A(x)$$

$$(FSL3) A(x \vee y) \geq t \geq \min\{t, t\} \geq T \{A(x), A(y)\}$$

$$(FSL4) A(x \wedge y) \geq t \geq \min\{t, t\} \geq T \{A(x), A(y)\}$$

$\therefore A$ is a Fuzzy Soft Lattice of G.

Proposition 3. : Let A be a Fuzzy Soft Lattice of G. Let A^\oplus be a Fuzzy set in G defined by $A^\oplus(x) = A(x) + 1 - A(0)$, for all $x \in G$. then A^\oplus is Normal Fuzzy Soft Lattice of G which contains A.

Proof : For any $x \in G$,

$$A^\oplus = A(x) + 1 - A(0)$$

$$(FSL1) A^\oplus(x + y)$$

$$= A(x + y) + 1 - A(0)$$

$$\geq T \{A(x), A(y)\} + 1 - A(0)$$

$$\geq T \{A(x) + 1 - A(0), A(y) + 1 - A(0)\}$$

$$\geq T \{A^\oplus(x), A^\oplus(y)\}$$

$$(FSL2) A^\oplus(-x) = A(-x) + 1 - A(0)$$

$$\geq A(x) + 1 - A(0)$$

$$\geq A^\oplus(x)$$

$$(FSL3) A^\oplus(x \vee y) = A(x \vee y) + 1 - A(0)$$

$$\geq T \{A(x), A(y)\} + 1 - A(0)$$

$$\geq T \{A(x) + 1 - A(0), A(y) + 1 - A(0)\}$$

$$\geq T \{A^\oplus(x), A^\oplus(y)\}$$

$$(FSL4) A^\oplus(x \wedge y) = A(x \wedge y) + 1 - A(0)$$

$$\geq T \{A(x), A(y)\} + 1 - A(0)$$

$$\geq T \{A(x) + 1 - A(0), A(y) + 1 - A(0)\}$$

$$\geq T \{A^\oplus(x), A^\oplus(y)\}$$

$\therefore A^\oplus$ is a Fuzzy Soft Lattice of G and $A^\oplus \subset A$.

Proposition 3.6 : If A is a Fuzzy Soft Lattice of G and θ is a homomorphism of G then the Fuzzy Soft set A^θ of G given by $A^\theta = \{ \langle x; (f_a)_{A^\theta}(x) \rangle, x \in G \}$ is a Fuzzy Soft Lattice of G.

Proof : For any $x, y \in G$, we have

$$(FSL1) (f_a)_{A^\theta}(x + y) = (f_a)_A \theta(x + y)$$

$$= (f_a)_A (\theta(x) + \theta(y))$$

$$\geq T \{(f_a)_A \theta(x), (f_a)_A \theta(y)\}$$

$$\geq T \{(f_a)_{A^\theta}(x), (f_a)_{A^\theta}(y)\}$$

$$(FSL2) (f_a)_{A^\theta}(-x) = (f_a)_{A^\theta}(-x)$$

$$\geq (f_a)_{A^\theta}(x)$$

$$(FSL3) \mu_{A^\theta}(x \vee y) = (f_a)_A \theta(x \vee y)$$

$$= (f_a)_A (\theta(x) \vee \theta(y))$$

$$\geq T \{(f_a)_A \theta(x), (f_a)_A \theta(y)\}$$

$$\geq T \{(f_a)_{A^\theta}(x), (f_a)_{A^\theta}(y)\}$$

$$(FSL4) (f_a)_{A^\theta}(x \wedge y) = (f_a)_{A^\theta}(x \wedge y)$$

$$= (f_a)_A (\theta(x) \wedge \theta(y))$$

$$\geq T \{(f_a)_A \theta(x), (f_a)_A \theta(y)\}$$

$$\geq T \{(f_a)_{A^\theta}(x), (f_a)_{A^\theta}(y)\}$$

$\therefore A^\theta$ is a Fuzzy Soft Lattice of G.

Proposition 3.7 : Let T be a continuous t-norm and let m be a homomorphism on G. If f_a is Fuzzy Soft Lattice of G then $(f_a)^m$ is Fuzzy Soft Lattice of f(G).

Proof : Let $A_1 = f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$ and $A_{12} = f^{-1}(y_1 + y_2)$. Where $y_1 + y_2 \in f(G)$. Consider the set

$$A_1 + A_2 = \{x \in G / x = a_1 + a_2\}$$

For some $a_1 \in A_1$ and $a_2 \in A_2$. If $x \in A_1 + A_2$ then $x = x_1 + x_2$. So that, we have

$$f(x) = f(x_1 + x_2) = f(x_1) + f(x_2) = y_1 + y_2$$

So $x \in f^{-1}(y_1 + y_2) = A_{12}$, thus $A_1 + A_2 \subset A_{12}$. It follows that

$$(FSL1) (f_a)^m(y_1 + y_2)$$

$$= \sup \{f_a(x) / x \in f^{-1}(y_1 + y_2)\}$$

$$= \sup \{f_a(x) / x \in A_{12}\}$$

$$\geq \sup \{f_a(x) / x \in A_1 + A_2\}$$

$$\geq \sup \{f_a(x_1 + x_2) / x_1 \in A_1, x_2 \in A_2\}$$

$$\geq \sup \{T\{f_a(x_1), f_a(x_2) / x_1 \in A_1, x_2 \in A_2\}\}$$

Since T is continuous for $\epsilon > 0$, We see that

$$\text{If } \sup \{f_a(x_1) / x_1 \in A_1\} + x_1^* \leq \delta \text{ and } \sup \{f_a(x_2) / x_2 \in A_2\} + x_2^* \leq \delta$$

$$T \{ \sup \{ f_a(x_1) / x_1 \in A_1 \}, \sup \{ f_a(x_2) / x_2 \in A_2 \} \} + T(x_1^*, x_2^*) \leq \dots (1)$$

Choose $a_1 \in A_1$ and $a_2 \in A_2$, such that

$$\sup \{ f_a(x_1) / x_1 \in A_1 \} + f_a(a_1) \leq \delta \text{ and } \sup \{ f_a(x_2) / x_2 \in A_2 \} + f_a(a_2) \leq \delta$$

Then

$$T \{ \sup \{ f_a(x_1) / x_1 \in A_1 \}, \sup \{ f_a(x_2) / x_2 \in A_2 \} \} + T(f_a(a_1), f_a(a_2)) \leq \dots (2)$$

Thus we have

$$(f_a)^m(x+y) \geq \sup \{ T(f_a(x_1), f_a(x_2)) / x_1 \in A_1, x_2 \in A_2 \} \\ = T \{ \sup \{ f_a(x_1) / x_1 \in A_1 \}, \sup \{ f_a(x_2) / x_2 \in A_2 \} \} \geq T \{ (f_a)^m(x), (f_a)^m(y) \}$$

Similarly, we can show

$$(FSL2) (f_a)^m(-x) \geq (f_a)^m(x)$$

$$(FSL3) (f_a)^m(x \vee y) \geq T \{ (f_a)^m(x), (f_a)^m(y) \}$$

and

$$(FSL4) (f_a)^m(x \wedge y) \geq T \{ (f_a)^m(x), (f_a)^m(y) \}$$

Thus $(f_a)^m$ is fuzzy soft lattice of G.

Proposition 3.8 : Let T be a t-norm. Then every sensible Fuzzy Soft Lattice A of G is Fuzzy Soft Lattice of G.

Proof : Assume that A is sensible Fuzzy Soft Lattice of G then we have

$$(FSL1) A(x+y) \geq T \{ A(x), A(y) \}$$

$$(FSL2) A(-x) \geq A(x)$$

$$(FSL3) A(x \vee y) \geq T \{ A(x), A(y) \}$$

$$(FSL4) A(x \wedge y) \geq T \{ A(x), A(y) \}, \text{ for all } x, y \in G.$$

Since A is sensible, we have

$$T \{ A(x), A(y) \}$$

$$= T \{ \{ S(A(x), A(y)) \}, \{ S(A(x), A(y)) \} \}$$

$$\leq T \{ A(x), A(y) \},$$

$$T(A(x), A(y)) = T \{ A(x), A(y) \}$$

It follows that

$$A(x+y) \geq T(A(x), A(y))$$

$$= T \{ A(x), A(y) \}, \text{ for all } x, y \in G.$$

Similarly, we can show (FSL2), (FSL3) and (FSL4) of Definition (2.4)

So A is a Fuzzy Soft Lattice of G.

Proposition 3.9: An onto homomorphic image of Fuzzy Soft Lattice with sup property is Fuzzy Soft Lattice.

Proof : Let $f: G \rightarrow G'$ be an onto homomorphism of G and let A be Fuzzy Soft Lattice of G with sup property.

Given $x, y \in G$, we let $x_0 \in f^{-1}(x')$ and $y_0 \in f^{-1}(y')$ be such that

$$A(x_0) = \sup A(h), \quad A(y_0) = \sup A(h)$$

$$h \in f^{-1}(x') \quad h \in f^{-1}(y')$$

Respectively. Then we can deduce that

$$(FSL1) A^f(x' + y') = \sup A(z) \\ z \in f^{-1}(x_1 + y_1) \\ \geq T \{ A(x_0), A(y_0) \} \geq T \{ \sup A(h), \sup A(h) \} \\ \left. \begin{array}{l} h \in f^{-1}(x') \quad h \in f^{-1}(y') \\ \geq T \{ A^f(x'), A^f(y') \} \end{array} \right\} \\ A^f(x' + y') \geq T \{ A^f(x'), A^f(y') \}$$

$$(FSL2) A^f(-x) = \sup \{ A(z) \\ z \in f^{-1}(-x') \} \\ \geq A(y_0) \\ = \sup A(h) \\ h \in f^{-1}(y') \\ = A^f(y').$$

Similarly, we can prove (FSL3), (FSL4) of Definition (2.4)

Proposition 3.10 : Let $f: G \rightarrow G'$ be a Lattice group homomorphism and A be a Fuzzy Soft Lattice of G' then $f^{-1}(A)$ is a Fuzzy Soft Lattice of G.

Proof : Let $x, y \in G$ and A is a Fuzzy Soft Lattice of G' .

$$(FSL1) f^{-1}(A)(x+y) = A f(x+y) \\ = A(f(x) + f(y))$$

$$\geq T \{ A(f(x)), A(f(y)) \}$$

$$\geq T \{ f^{-1}(A)(x), f^{-1}(A)(y) \}$$

$$(FSL2) f^{-1}(A)(-x) = A f(-x)$$

$$\geq A f(x)$$

$$\geq f^{-1}(A)(x)$$

$$(FSL3) f^{-1}(A)(x \vee y) = A f(x \vee y)$$

$$= A(f(x) \vee f(y))$$

$$\geq T \{ A f(x), A f(y) \}$$

$$\geq T \{ f^{-1}(A)(x), f^{-1}(A)(y) \}$$

$$(FSL4) f^{-1}(A)(x \wedge y) = A f(x \wedge y)$$

$$= A(f(x) \wedge f(y))$$

$$\geq T \{ A f(x), A f(y) \}$$

$$\geq T \{ f^{-1}(A)(x), f^{-1}(A)(y) \}$$

$\therefore f^{-1}(A)$ is a Fuzzy Soft Lattice of G.

Proposition 3.11: Let A be a fuzzy normal soft lattice of G. Then for all $x, y \in G$. $A([x, y]) = A(0)$.

Proof :

Since A is fuzzy normal soft lattice of G. Then we have

$$A(x) = A(y \ x \ y^{-1}) \text{ for all } x, y \in G.$$

Replacing x by x^{-1} and y by y^{-1} , it gives

$$A(y^{-1}) = A(x^{-1} \ y^{-1} \ xy) \text{ or } A(x^{-1} \ y^{-1} \ x \ yy^{-1}) = A(y^{-1}) \text{ or } A([x, y]y^{-1})$$

$$= A(y^{-1}) \text{ or } A[x, y] = A(0)$$

4 : Direct product of Fuzzy Soft Lattices

Definition 4.1 : Let A_i be a Fuzzy Soft Lattice of G_i , for $i = 1, 2, 3 \dots n$. Then the product of A_i ($i = 1, 2, 3, \dots n$) is the function $A_1 \times A_2 \times A_3 \times \dots \times A_n : G_1 \times G_2 \times \dots \times G_n \rightarrow L$ defined by

$$(A_1 \times A_2 \times A_3 \times \dots \times A_n)(x_1, x_2, \dots, x_n) = T \{A_1(x_1), A_2(x_2) \dots A_n(x_n)\}$$

Proposition 4.2 : The Direct product of Fuzzy Soft Lattices is a Fuzzy Soft Lattices.

Proof :

$$\text{Let } x = (x_1, x_2, \dots, x_n), y = (y_1, y_2, \dots, y_n)$$

$$\text{Let } (x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n) \in A_1 \times A_2 \times A_3 \times \dots \times A_n = f_a$$

$$\begin{aligned} \text{(FSL1)} \quad f_a(x+y) &= f_a((x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n)) \\ &= A_1 \times A_2 \times A_3 \times \dots \times A_n((x_1+y_1), (x_2+y_2) \dots (x_n+y_n)) \\ &= f_a((x_1+y_1), (x_2+y_2), \dots, (x_n+y_n)) \\ &\geq T\{A_1(x_1+y_1), A_2(x_2+y_2), A_n(x_n+y_n)\} \\ &\geq T\{T\{A_1(x_1), A_1(y_1)\}, T\{A_2(x_2), A_2(y_2)\}, \dots, T\{A_n(x_n), A_n(y_n)\}, \} \end{aligned}$$

$$\geq T \{T \{A_1(x_1), A_2(x_2), \dots, A_n(x_n)\}, T \{A_1(y_1), A_2(y_2) \dots, A_n(y_n)\}\}$$

$$\geq T \{(A_1 \times A_2 \times \dots \times A_n)(x_1, x_2, \dots, x_n), (A_1 \times A_2 \times \dots \times A_n)(y_1, y_2, \dots, y_n)\}$$

$$\geq T \{\lambda(x), \lambda(y)\}$$

$$\begin{aligned} \text{(FSL2)} \quad f_a(-x) &= f_a(-x) \\ &= (A_1 \times A_2 \times \dots \times A_n)((-x_1), (-x_2), \dots, (-x_n)) \\ &= T \{A_1(-x_1), A_2(-x_2), \dots, A_n(-x_n)\} \\ &\geq T \{A_1(x_1), A_2(x_2) \dots A_n(x_n)\} \\ &\geq (A_1 \times A_2 \times \dots \times A_n)(x_1, x_2, \dots, x_n) \\ &\geq f_a(x) \end{aligned}$$

$$\begin{aligned} \text{(FSL3)} \quad f_a(x \vee y) &= f_a((x_1, x_2, \dots, x_n) \vee (y_1, y_2, \dots, y_n)) \\ &= f_a(x_1 \vee y_1, x_2 \vee y_2, \dots, x_n \vee y_n) \\ &= (A_1 \times A_2 \times \dots \times A_n)(x_1 \vee y_1, x_2 \vee y_2, \dots, x_n \vee y_n) \\ &= T \{A_1(x_1 \vee y_1), A_2(x_2 \vee y_2), \dots, A_n(x_n \vee y_n)\} \\ &\geq T\{T\{A_1(x_1), A_1(y_1)\}, T\{A_2(x_2), A_2(y_2)\}, \dots, T \{A_n(x_n), A_n(y_n)\}\} \end{aligned}$$

$$\geq T \{\{T \{A_1(x_1), A_2(x_2) \dots A_n(x_n)\}, T \{A_1(y_1), A_2(y_2) \dots A_n(y_n)\}\}\}$$

$$\geq T \{(A_1 \times A_2 \times \dots \times A_n)(x_1, x_2, \dots, x_n), (A_1 \times A_2 \times \dots \times A_n)(y_1, y_2, \dots, y_n)\}$$

$$\geq T \{f_a(x), f_a(y)\}. \text{Similarly, we can show (FSL4) of definition (2.4)}$$

Proposition 4.3 : The intersection of two Fuzzy Soft Lattices is a Fuzzy Soft Lattice.

Proof : Since A and B are Fuzzy Soft Lattices in G.

$$\begin{aligned} \text{(FSL1)} \quad (f_a \cap f_b)(x+y) &= T \{f_a(x+y), f_b(x+y)\} \\ &\geq T \{T \{f_a(x), f_a(y)\}, T \{f_b(x), f_b(y)\}\} \\ &\geq T \{T \{f_a(x), f_b(x)\}, T \{f_a(y), f_b(y)\}\} \\ &\geq T \{(f_a \cap f_b)(x), (f_a \cap f_b)(y)\} \end{aligned}$$

$$\begin{aligned} \text{(FSL2)} \quad (f_a \cap f_b)(-x) &= T \{f_a(-x), f_b(-x)\} \\ &\geq T \{f_a(x), f_b(x)\} \\ &\geq (f_a \cap f_b)(x) \end{aligned}$$

$$\begin{aligned} \text{(FSL3)} \quad (f_a \cap f_b)(x \vee y) &= T \{f_a(x \vee y), f_b(x \vee y)\} \\ &\geq T \{T \{f_a(x), f_b(y)\}, T \{f_b(x), f_b(y)\}\} \\ &\geq T\{T\{f_a(x), f_b(x)\}, T \{f_a(y), f_b(y)\}\} \\ &\geq T \{(f_a \cap f_b)(x), (f_a \cap f_b)(y)\} \end{aligned}$$

Similarly, we can show (FSL4 of Definition (2.4))

Proposition 4.4: If f_a is a Fuzzy Soft Lattice then f_a^c is also a Fuzzy Soft Lattice.

Proof : For any $x, y \in G$.

$$\begin{aligned} \text{(FSL1)} \quad f_a^c(x+y) &= 1 - f_a(x+y) \\ &\leq S \{1 - f_a(x), 1 - f_a(y)\} \\ &\leq S \{f_a^c(x), f_a^c(y)\} \end{aligned}$$

$$\begin{aligned} \text{(FSL2)} \quad f_a^c(-x) &= 1 - f_a(-x) \\ &\leq 1 - f_a(x) \\ &\leq f_a^c(x) \end{aligned}$$

$$\begin{aligned} \text{(FSL3)} \quad f_a^c(x \vee y) &= 1 - f_a(x \vee y) \\ &\leq S \{1 - f_a(x), 1 - f_a(y)\} \\ &\leq S \{f_a^c(x), f_a^c(y)\} \end{aligned}$$

$$\begin{aligned} \text{(FSL4)} \quad f_a^c(x \wedge y) &= 1 - f_a(x \wedge y) \\ &\leq S \{1 - f_a(x), 1 - f_a(y)\} \\ &\leq S \{f_a^c(x), f_a^c(y)\}. \text{Hence } f_a^c \text{ is fuzzy} \end{aligned}$$

soft lattice.

5. CONCLUSION

FULI [3] introduced the concept of soft lattices and P.K.. Maji et. al [6] investigated the concept of fuzzy soft sets (2001). In this paper we introduce the notion of fuzzy soft lattices with respect to soft lattice L under the triangular norms. To extend this work on could study the properties of fuzzy soft sets, in other algebraic structures such as soft G-modules and fields.

6. APPLICATIONS

A Lattice structure has been found to be extremely important in the areas of Medical sciences, social sciences, environmental sciences, economics, and decision making problems. Also, it will be very useful to make all types of mathematical modeling.

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