

# Backstepping Controller Design using a High Gain Observer for Induction Motor

Haj Brahim Imen  
University of Sfax,  
Automatic Control Unit,  
National Engineering School  
of Sfax, BP. 1173, 3038 Sfax,  
Tunisia

Soufien Hajji  
University of Sfax,  
Automatic Control Unit,  
National Engineering School  
of Sfax, BP. 1173, 3038 Sfax,  
Tunisia

Abdessattar Chaari  
University of Sfax,  
Automatic Control Unit,  
Department of Electrical  
Engineering,  
National Engineering School  
of Sfax, BP. 1173, 3038 Sfax,  
Tunisia

## ABSTRACT

In this paper, a backstepping controller of Induction Motor (IM) is proposed using the fifth order model in fixed two frame reference axis with rotor flux and stator currents as state variables. The approach of backstepping requires, generally, that the nonlinear system is in strict feed-back loop. To implement this strategy over the IM, some transformations on the model  $(\alpha, \beta)$  of the machine have been carried out without recourse to the oriented flux hypothesis which allows a triangular state representation. The overall system stability is proved by Lyapunov theory. Indeed the controller relationship depends on the unmeasured states of the IM, and a nonlinear observer to high gain is used in order to reconstruct the motor speed, the rotor flux and the load torque. Simulation results are provided to illustrate the effectiveness of the proposed approach and the robustness to uncertainties, such as rotor resistance variations.

## General Terms

Backstepping controller design based high gain observer, Stability using Lyapunov theory, robustness of the backstepping controller to uncertainties in IM.

## Keywords

Backstepping control, induction motor, high gain observer.

## 1. INTRODUCTION

In recent years, Backstepping approach became one of the most popular design methods for large scale nonlinear systems. The idea of backstepping design is to select recursively some appropriate functions of state variables as pseudo-control inputs for lower dimension subsystems of the overall system. Each backstepping stage results in a new pseudo-control design, expressed in terms of the pseudo-control designs from preceding design stages. When the procedure terminates, a feedback design for the true control input is the result which achieves the original design objective by virtue of final Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage [1]. This control scheme can successfully guarantee the global asymptotic stability [2].

Applied to the control of IM, the strategy of backstepping can be used in two different ways. The first method utilizes the entire IM model without any simplifying assumption, which leads to a tedious analysis to construct a regression matrix [3]. These problems are taken care of by the introduction of the neuronal

networks techniques to design the fictitious controller [4-5]. The second method works in combination with field oriented control FOC. Many versions of backstepping control have been developed. In [6-7-8], the PI controllers used in conventional FOC for speed and current regulation are replaced by backstepping controllers. In [9-10-11], the authors have extended the method to the adaptive neuronal network control systems in order to compensate the parameters variations and reject the external load torque disturbance.

One may note that the FOC methods represent a type of partial feedback linearization control technique in which the zero dynamic stability cannot be proved. As a result, it is not guaranteed that the system model is robust to parameters variation [4].

The interest of the backstepping controller, which is adopted in this paper, can be summarized in the fact that we have carried out some transformations on the model  $(\alpha, \beta)$  of the machine allowing the state representation for a strict feed-back form. Based on the algorithm presented in [12], which requires that the nonlinear system must be in a triangular form, a novel approach of backstepping design is presented to control the speed and the rotor flux.

This paper is organized as follows: The detailed induction motor is presented in section 2. The nonlinear Backstepping control is summarized in section 3. In section 4, a high gain observer is given in order to estimate the unmeasured states. The simulation results, for performances evaluation, are illustrated and discussed in section 5.

## 2. INDUCTION MOTOR MODEL

The fifth order IM model, in two fixed axis reference frame with rotor fluxes and stator currents as state variables [14], is given as:

$$\begin{cases} \dot{i}_s = -\gamma i_s + K \left( \frac{1}{T_r} I_2 - n_p \Omega J_2 \right) \psi_r + \frac{1}{\sigma L_s} u_s \\ \dot{\psi}_r = \frac{M}{T_r} i_s - \left( \frac{1}{T_r} I_2 - n_p \Omega J_2 \right) \psi_r \\ \dot{\Omega} = \frac{n_p M}{J L_r} i_s^T J_2 \psi_r - \frac{1}{J} T_l \end{cases} \quad (1)$$

where  $i_s = [i_{s\alpha} \ i_{s\beta}]^T$ ,  $\psi_r = [\varphi_{r\alpha} \ \varphi_{r\beta}]^T$ ,  $u_s = [u_{s\alpha} \ u_{s\beta}]^T$  are, respectively, the stator current, the rotor fluxes and the stator voltages;  $\Omega$  and  $T_i$  respectively denote the motor speed and the load torque;  $f(\Omega) = \frac{1}{T_r} I_2 - n_p \Omega J_2$ ,  $I_2$  is the  $2 \times 2$  matrix

identity and  $J_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ;  $n_p$  is the number of pole pairs;  $J$  is the motor moment of inertia. The parameters  $T_r$ ,  $\sigma$ ,  $K$  and  $\gamma$  are defined as:

$$T_r = \frac{L_r}{R_r}, \sigma = 1 - \frac{M^2}{L_s L_r}, K = \frac{M}{\sigma L_s L_r}, \gamma = \frac{R_s}{\sigma L_s} + \frac{M^2}{\sigma L_s L_r^2}$$

$L_s, L_r$  are per-phase stator and rotor inductances.  $M$  is the mutual inductance.  $R_s, R_r$  are stator and rotor resistances.

## 2. BACKSTEPPING CONTROL

The control objective consists in regulating the square of the fluxes vector norm at a desired constant value and forced the speed to follow a reference profile. Let us denote by

$$z^1 = \begin{pmatrix} z_1^1 \\ z_2^1 \end{pmatrix} = \begin{pmatrix} \Omega \\ \|\psi_r\|^2 \end{pmatrix} = \begin{pmatrix} \Omega \\ \varphi_{r\alpha}^2 + \varphi_{r\beta}^2 \end{pmatrix} \text{ the vector variable to be}$$

controlled and let  $z_d^1 = \begin{pmatrix} z_{d1}^1 \\ z_{d2}^1 \end{pmatrix}$  be the corresponding desired

trajectory. With these notations, the control input should be defined to achieve asymptotically the following equation:

$$\lim_{t \rightarrow \infty} (z_1^1(t) - z_{d1}^1(t)) = \lim_{t \rightarrow \infty} (z_2^1(t) - z_{d2}^1(t)) = 0$$

### 2.1 Preliminary stage

The backstepping approach provides a recursive method for stabilizing the origin of a system in strict-feedback form. In order to implement this strategy over the IM, we must go through a preliminary step in which we will make some transformations that bring the model of the IM in strict feedback form [13].

The new introduced coordinates are expressed by:

$$\begin{cases} z_1^1 = \Omega \\ z_2^1 = \|\psi_r\|^2 = \varphi_{r\alpha}^2 + \varphi_{r\beta}^2 = \psi_r^T \psi_r \\ z_3^1 = \frac{n_p M}{L_r} i_s^T J_2 \psi_r \\ z_4^1 = i_s^T \psi_r \\ \xi = \arctan \left( \frac{\varphi_{r\beta}}{\varphi_{r\alpha}} \right) \end{cases} \quad (2)$$

In a condensed form, system (1) can be written as:

$$\begin{cases} \dot{z}^1 = A_1 z^2 + \Phi^1(C_r(t), z^1) \\ \dot{z}^2 = b(\xi, z) u_s + g(z) \\ \dot{\xi} = \frac{L_r}{n_p T_r} \frac{z_1^2}{z_2^1} + n_p z_4^1 \end{cases} \quad (3)$$

where:

$$A_1 = \begin{pmatrix} \frac{1}{J} & 0 \\ 0 & \frac{2M}{T_r} \end{pmatrix}, \Phi^1(C_r(t), z^1) = \begin{pmatrix} -\frac{1}{J} T_i \\ -\frac{2}{T_r} z_1^2 \end{pmatrix}$$

$$b(\xi, z) = \frac{1}{\sigma L_s} \sqrt{z_2^1} \begin{pmatrix} -\frac{n_p M}{L_r} \sin(\xi) & \frac{n_p M}{L_r} \cos(\xi) \\ \cos(\xi) & \sin(\xi) \end{pmatrix}$$

$$g(z) = \begin{pmatrix} g_1(z) \\ g_2(z) \end{pmatrix}$$

$$g_1(z) = -n_p^2 \frac{KM}{L_r} z_1^1 z_2^1 - \left( \gamma + \frac{1}{T_r} \right) z_1^2 - n_p^2 \frac{M}{L_r} z_1^1 z_2^2$$

$$g_2(z) = \frac{K}{T_r} z_1^2 + \frac{L_r}{M} z_1^1 z_2^1 - \left( \gamma + \frac{1}{T_r} \right) z_2^2 + \frac{M}{T_r} \frac{1}{z_2^1} \left( (z_2^2)^2 + \left( \frac{L_r}{n_p M} z_1^2 \right)^2 \right)$$

Let  $z_d = \begin{pmatrix} z_d^1 \\ z_d^2 \end{pmatrix}$  with  $z_d^2 = \begin{pmatrix} z_{d1}^2 \\ z_{d2}^2 \end{pmatrix}$  be the trajectory of  $z$

corresponding to the reference trajectory  $z_{d1}^1(t)$  and let  $u_d$  be the associated input i.e. the input which brings  $z(t)$  to the desired trajectory  $z_d(t)$ . According to system (3),  $z_d^2$  and  $u_d$  can be computed as:

$$\begin{cases} z_d^2 = A_1^{-1} \left( \dot{z}_d^1 - \Phi^1(T_i(t), z_d^1) \right) \\ b(\xi, z_d) u_d = \dot{z}_d^2 - g(z_d) \end{cases} \quad (4)$$

$$\text{This gives } z_d^2 = \begin{pmatrix} J z_{d1}^1 + C_r \\ \frac{T_r}{2M} z_{d2}^1 + \frac{1}{M} z_{d2}^1 \end{pmatrix}, \dot{z}_d^2 = \begin{pmatrix} J \dot{z}_{d1}^1 + \dot{C}_r \\ \frac{T_r}{2M} \dot{z}_{d2}^1 + \frac{1}{M} \dot{z}_{d2}^1 \end{pmatrix}$$

We define the following variables of errors:

$$e^1 = \begin{pmatrix} e_1^1 \\ e_2^1 \end{pmatrix} = \begin{pmatrix} z_1^1 - z_{d1}^1 \\ z_2^1 - z_{d2}^1 \end{pmatrix}; e^2 = \begin{pmatrix} e_1^2 \\ e_2^2 \end{pmatrix} = \begin{pmatrix} z_1^2 - z_{d1}^2 \\ z_2^2 - z_{d2}^2 \end{pmatrix} \quad (5)$$

Based on (3), we can easily determine the errors dynamic:

$$\begin{cases} \dot{e}^0 = e^1 \\ \dot{e}^1 = A_1 e^2 + \Phi^1(C_r(t), z^1) + A_1 z_d^2 - \dot{z}_d^1 \\ \dot{e}^2 = b(\xi, z) u_s + g(z) - \dot{z}_d^2 \end{cases} \quad (6)$$

After some mathematical operations, the IM model in errors space can be simplified as:

$$\begin{cases} \dot{e}^0 = e^1 \\ \dot{e}^1 = A_1 e^2 + A_2 e^1 \\ \dot{e}^2 = b(\xi, z) u_s + g(z) - \dot{z}_d^2 \end{cases} \quad (7)$$

with  $A_2 = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{T_r} \end{pmatrix}$ .

## 2.2 Control design

Consider the following third order strict-feedback system.

$$\begin{cases} \dot{e}^0 = e^1 \\ \dot{e}^1 = A_1 e^2 + A_2 e^1 \\ \dot{e}^2 = b(\xi, z) u_s + g(z) - \dot{z}_d^2 \end{cases} \quad (8)$$

The controller design procedure for induction motor can be given as follows.

**Step1:** Start with the first equation of (9), we define a new coordinate  $y_0 = e^0$  and derive its dynamic:

$$\dot{y}_0 = e^1 \quad (9)$$

We view  $e^1$  as a control variable and define a virtual control law for equation (9), denote  $\alpha_0$ , and let  $y_1$  be an error variable representing the difference between the actual and virtual controls:

$$y_1 = e^1 - \alpha_0 \quad (10)$$

Our objective, in this step, is to design the stabilizing function  $\alpha_0$  which makes  $y_0 \rightarrow 0$ . The first Lyapunov candidate function  $V_0$  is chosen as:

$$V_0 = \frac{1}{2} y_0^T y_0 \quad (11)$$

Its time derivative is given by:

$$\dot{V}_0 = y_0^T \dot{y}_0 = y_0^T \alpha_0 + y_0^T y_1 \quad (12)$$

To make the first order system stabilizable, it is necessary that the Lyapunov function derivative  $\dot{V}_0$  is negative, which will allow the following choice.

$$\alpha_0 = -c_0 y_0 \quad (13)$$

Then the time derivative of  $V_0$  becomes:

$$\dot{V}_0 = -c_0 y_0^T y_0 + y_0^T y_1 \quad (14)$$

where  $c_0$  is a positive constant.

In order to obtain  $\dot{V}_0 < 0$  and guaranteed that  $y_0$  converge to zero asymptotically, the residual term  $y_0^T y_1$  will be compensated in the second step.

**Step2:** We derive the error dynamics for  $y_1 = e^1 - \alpha_0$

$$\begin{aligned} \dot{y}_1 &= e^1 + c_0 e^1 \\ &= A_1 e^2 + A_2 e^1 + c_0 e^1 \end{aligned} \quad (15)$$

Define a second virtual control law  $\alpha_1$  and let  $y_2$  be an error variable representing the difference between the actual and virtual controls.

$$y_2 = e^2 - \alpha_1 \quad (16)$$

Then the error equation (15) can be expressed as:

$$\dot{y}_1 = A_1 y_2 + A_1 \alpha_1 + A_2 e^1 + c_0 e^1 \quad (17)$$

The control objective is to make  $y_1$  converge to zero. We extend the initial Lyapunov function to reflect the presence of the new state variable  $y_1$  as:

$$V_1 = V_0 + \frac{1}{2} y_1^T y_1 \quad (18)$$

Then, its derivative is:

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + y_1^T \dot{y}_1 \\ &= -c_0 y_0^T y_0 + y_0^T y_1 + y_1^T (A_1 y_2 + A_1 \alpha_1 + A_2 e^1 + c_0 e^1) \\ &= -c_0 y_0^T y_0 + y_1^T (y_0 + A_1 \alpha_1 + A_2 e^1 + c_0 e^1) + y_1^T A_1 y_2 \end{aligned} \quad (19)$$

We can now select an appropriate virtual control  $\alpha_1$  to cancel some terms related to  $y_0$  and  $e^1$ :

$$\alpha_1 = -A_1^{-1} (c_1 y_1 + y_0 + (A_2 + c_0 I_2) e^1) \quad (20)$$

where  $c_1$  is a positive constant. So the time derivative of  $V_1$  becomes:

$$\dot{V}_1 = -c_0 y_0^T y_0 - c_1 y_1^T y_1 + y_1^T A_1 y_2 \quad (21)$$

Clearly, if  $y_2 = 0$  we obtain  $\dot{V}_1 = -c_0 y_0^T y_0 - c_1 y_1^T y_1$ , and thus both  $y_0$  and  $y_1$  are guaranteed to converge to zero asymptotically.

**Step3:** We proceed in the same way with the last equation in (8) and we start by computing the error dynamics for  $y_2 = e^2 - \alpha_1$  proposed in step2:

$$\begin{aligned} \dot{y}_2 &= \dot{e}^2 - \dot{\alpha}_1 \\ &= b(\xi, z)u_s + g(z) - \dot{z}_d^2 - \dot{\alpha}_1 \end{aligned} \quad (22)$$

We note the appearance of the real control  $u_s$ . In this step, our objective is to design the actual control input such that  $y_2$  converge to zero. So, we need so to select a new Lyapunov function to design the final control.

$$V_2 = V_1 + \frac{1}{2} y_2^T y_2 \quad (23)$$

Its derivative is given by:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + y_2^T \dot{y}_2 \\ &= -c_0 y_0^T y_0 - c_1 y_1^T y_1 \\ &\quad + y_2^T (b(\xi, z)u_s + A_1 y_1 + g(z) - \dot{z}_d^2 - \dot{\alpha}_1) \end{aligned} \quad (24)$$

To stabilize the global system (9), the real control input is selected to remove the residual term and make  $\dot{V}_2 < 0$ .

$$u_s = (b(\xi, z))^{-1} (-c_2 y_2 - A_1 y_1 + \dot{z}_d^2 + \dot{\alpha}_1 - g(z)) \quad (25)$$

Where  $c_2$  is a positive constant. The derivative of the candidate Lyapunov function becomes:

$$\dot{V}_2 = -c_0 y_0^T y_0 - c_1 y_1^T y_1 - c_2 y_2^T y_2 \quad (26)$$

Therefore the asymptotic convergence to zero of  $y_0$ ,  $y_1$  and  $y_2$  is guaranteed. Since  $y_0 = e^0$ ,  $e^0$  is also bounded, and  $\lim_{t \rightarrow \infty} e^0 = 0$ . The boundedness of  $e^1$  follows from the boundedness of  $\alpha_0$  in (13) and the fact that  $e^1 = y_1 + \alpha_0$  so,  $\lim_{t \rightarrow \infty} e^1 = 0$ . Similarly, the boundedness of  $e^2$  then follows from boundedness of  $\alpha_1$  in (20) and the fact that  $e^2 = y_2 + \alpha_1$ . So, we have  $\lim_{t \rightarrow \infty} e^2 = 0$ .

From the above we can obtain the control law as:

$$u_s = (b(\xi, z))^{-1} (-c_2 y_2 - A_1 y_1 + \dot{z}_d^2 + \dot{\alpha}_1 - g(z)) \quad (27)$$

Now, it suffices to replace, in the final control law, input  $z$  and  $e$  by their respective expressions in the original coordinates, namely (2) and (5).

### 3. HIGH GAIN OBSERVER

The control law depends on unmeasured states; a nonlinear observer shall be synthesized in order to achieve the estimation objective. In fact the proposed observer [14] consists of two cascade observers. The first observer provides the estimation of rotor fluxes using measurement of stators currents and motor

speed. The second cascade observer provides the estimation of the load torque and its derivative.

The measured output vector is  $y = \begin{pmatrix} i_s \\ \Omega \end{pmatrix}$ . A high gain observer

can be synthesized in order to provide the estimation of  $\psi_r$ , and its equations are:

$$\begin{cases} \dot{\hat{i}}_s = Kf(\Omega)\hat{\psi}_r - \gamma\hat{i}_s + \frac{1}{\sigma L_s} u_s - 2\theta_1 (\hat{i}_s - i_s) \\ \dot{\hat{\psi}}_r = \frac{M}{T_r} \hat{i}_s - f(\Omega)\hat{\psi}_r - \frac{\theta_1^2}{K} f^{-1}(\Omega) (\hat{i}_s - i_s) \end{cases} \quad (28)$$

where  $\hat{i}_s$  and  $\hat{\psi}_r$  are the respective estimates of  $i_s$ ,  $\psi_r$ , and  $\theta_1$  is the parameter design.

The flux estimates are then used together with the motor speed measurements in order to estimate the load torque  $T_L$  and its time derivative  $T_{Lp}$  using the following nonlinear observer:

$$\begin{cases} \dot{\hat{\Omega}} = \frac{nM}{JL_s} \hat{i}_s^T J_2 \hat{\psi}_r - \frac{1}{J} \hat{T}_L - 3\theta_2 (\hat{\Omega} - \Omega) \\ \dot{\hat{T}}_L = \hat{T}_{Lp} + 3\theta_2^2 J (\hat{\Omega} - \Omega) \\ \dot{\hat{T}}_{Lp} = \theta_2^3 J (\hat{\Omega} - \Omega) \end{cases} \quad (29)$$

where  $\hat{T}_L$  and  $\hat{T}_{Lp}$  are the respective estimates of  $T_L$ ,  $T_{Lp}$ , and  $\theta_2$  is the parameter design.

Notice that the estimation error converges exponentially to zero for observer (30). The main characteristic of observer (31) lies in the fact that when the time derivative of the load torque is constant, the estimation error converges to zero exponentially. When this time derivative is not constant but remains bounded by a constant, the estimation error can be made as small as desired by choosing  $\theta_2$  high enough [13].

### 4. SIMULATION RESULTS

The proposed control algorithm has been simulated for a 1kW induction motor in order to prove the rightness and effectiveness of the designed controller.

The gains  $[\theta_1 \theta_2 c_0 c_1 c_2]$  are chosen as follows:

$[2500 \ 500 \ 5000 \ 500 \ 5000]$  to satisfy convergence conditions.

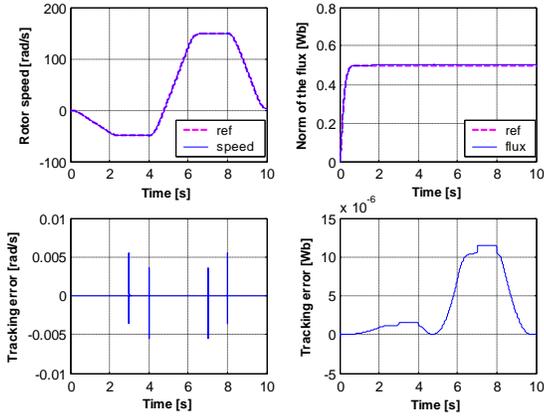
The simulation parameters of IM are givens as:

$$n_p = 2 ; J = 0.015 Kgm^2 ; M = 0.29H$$

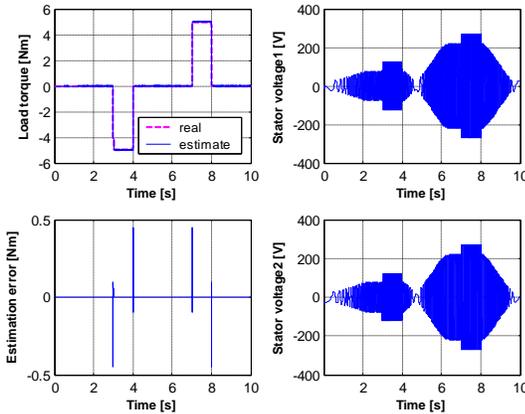
$$R_s = 10.6\Omega ; R_r = 2.88\Omega ; L_s = L_r = 0.3H$$

Simulation results, with nominal value of rotor resistance, are reported in figure 1 to 3. The obtained result shows that the tracking performance of speed and flux are very satisfactory for various forms of reference speed (low speed, high-speed and

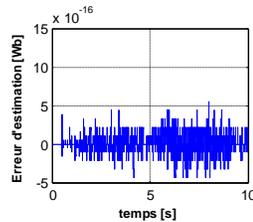
reverse speed). Notice that the tracking error related to the motor speed tends rapidly to zero after each sudden change of load torque. The performances of the observer are illustrated in figure 2 and 3 where the estimation error of  $\|\psi_r\|^2$  and the load torque are given.



**Fig1: Time evolution of the rotor speed, the fluxes norm and their tracking error**

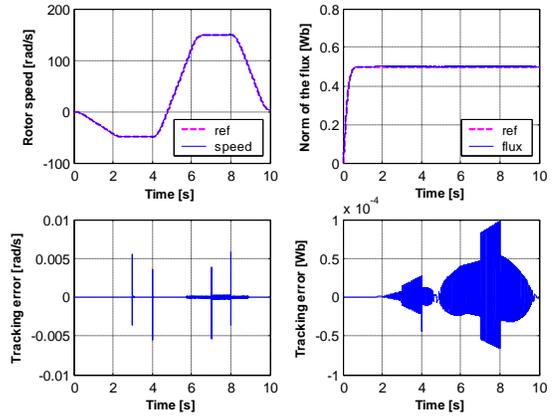


**Fig2: Time evolution of the load torque and the voltage input**

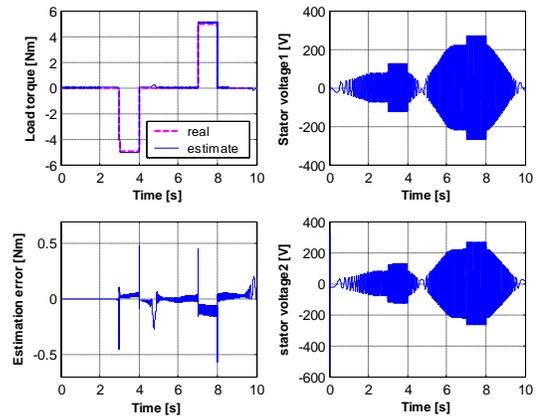


**Fig3: Estimation error of  $\|\psi_r\|^2$**

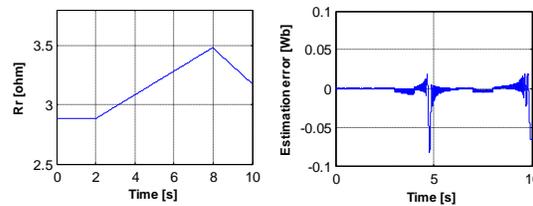
To test the effect of rotor resistance on the controller, its variation (figure 6-a) is injected into the model of IM while its nominal value is retained in the expression of the controller and the observer. The obtained results (figures 4, 5, 6-b) are quite similar to those obtained in the first case. This proves the robustness and effectiveness of the proposed control law.



**Fig4: Time evolution of the rotor speed, the fluxes norm and their tracking error with rotor resistance variation.**



**Fig5: Time evolution of the load torque and the voltage input with rotor resistance variation.**



**Fig6: Variation of rotor resistance (a), Estimation error of  $\|\psi_r\|^2$  with rotor resistance variation (b)**

## 5. CONCLUSION

In this paper, a backstepping controller has been proposed for IM rotor flux and speed tracking control. The nonlinear controller is designed based on the fifth order IM model in two fixed axis reference frame. The validity of this design is demonstrated through computer simulations. The obtained results confirm also the robustness of the proposed control law against the variation of rotor resistance.

## 6. APPENDIX

The expression of the final control input:

$$u_s = (b(\xi, z))^{-1} \left( \underbrace{-c_2 y_2 - A_1 y_1 + \dot{z}_d^2 + \dot{\alpha}_1}_{h} - g(z) \right)$$

$$(b(\xi, z))^{-1} = \frac{\sigma L_s L_r}{n_p M z_2^1} \begin{pmatrix} -\varphi_{r\beta} & \frac{n_p M}{L_r} \varphi_{r\alpha} \\ \varphi_{r\alpha} & \frac{n_p M}{L_r} \varphi_{r\beta} \end{pmatrix}$$

$$g(z) = \begin{pmatrix} g_1(z) \\ g_2(z) \end{pmatrix}$$

$$g_1(z) = -n_p^2 \frac{KM}{L_r} z_1^1 z_2^1 - \left( \gamma + \frac{1}{T_r} \right) z_1^2 - n_p^2 \frac{M}{L_r} z_1^1 z_2^2$$

$$g_2(z) = \frac{K}{T_r} z_2^1 + \frac{L_r}{M} z_1^1 z_2^1 - \left( \gamma + \frac{1}{T_r} \right) z_2^2 + \frac{M}{T_r} \frac{1}{z_2^1} \left( (z_2^2)^2 + \left( \frac{L_r}{n_p M} z_1^2 \right)^2 \right)$$

$$h = - \begin{pmatrix} k_{01} e_1^0 + k_{11} e_1^1 + k_{21} e_1^2 - \dot{C}_r - J \dot{z}_{d1}^1 \\ k_{02} e_2^0 + k_{12} e_2^1 + k_{22} e_2^2 - \frac{T_r}{2M} \dot{z}_{d2}^1 - \frac{1}{M} \dot{z}_{d2}^1 \end{pmatrix}$$

$$k_{01} = (J c_1 c_0 c_2 + J c_2 + c_0 \frac{1}{J})$$

$$k_{02} = \left( \frac{T_r}{2M} c_1 c_0 c_2 + \frac{T_r}{2M} c_2 + c_0 \frac{2M}{T_r} \right)$$

$$k_{11} = (J c_1 c_2 + J c_0 c_2 + \frac{1}{J} + J c_1 c_0 + J)$$

$$k_{12} = \frac{T_r}{2M} (c_1 c_2 + c_0 c_2 + c_1 c_0 + 1) + \frac{2M}{T_r} + \frac{2}{MT_r} - \frac{1}{M} (c_1 + c_0 + c_2)$$

$$k_{21} = (c_2 + c_1 + c_0); k_{22} = \left( c_2 + c_1 + c_0 - \frac{2}{T_r} \right)$$

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