Anti M-Fuzzy Sub-Bigroup and its Bi Lower Level M-Sub Bigroups

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ABSTRACT
In this paper, we introduce the concept of anti M-fuzzy sub-bigroup of an M-bigroup and bi lower level subset of an anti M-fuzzy sub-bigroup and discussed some of its properties.

Keywords
M-group, anti M-fuzzy subgroup, anti M-fuzzy sub-bigroup of an M-bigroup, bi lower level subset.

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1. INTRODUCTION
The concept of fuzzy sets was initiated by Zadeh[12]. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld[6] gave the idea of fuzzy subgroups and Ranjith Biswas[1] gave the idea of anti fuzzy subgroups.
The notion of bigroup was first introduced by P.L.Maggu in 1994. W.B. Vasantha Kandasamy and D.Meiyappan introduced concept of fuzzy sub-bigroup of a bigroup. Author N. Jacobson[5] introduced the concept of M-group, M-subgroup.

2. PRELIMINARIES
This section contains some definitions and results to be used in the sequel.

2.1 Definition
A set (G, + , •) with two binary operations + and • is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that

i. $G = G_1 \cup G_2$,
ii. $(G_1 , +)$ is a group,
iii. $(G_2 , \bullet)$ is a group.

A non-empty subset H of a bigroup (G , + , •) is called a sub-bigroup, if H itself is a bigroup under the operations + and • defined on G.

2.2 Definition
A group with operators is an algebraic system consisting of a group G , a set M and a function defined in the product set M × G and having values in G such that, if ma denotes the element in G determined by the element a of G and the element m of M, then m(ab) = (ma)(mb) holds for all a, b ∈ G and m ∈ M. We shall use the phrases "G is an M-group" to a group with operators.
A subgroup H of an M-group G is said to be an M-subgroup if mx ∈ H for all m ∈ M and x ∈ H.

2.3 Definition
A set (G , + , •) with two binary operation + and • is called an M-bigroup if there exist two proper subsets G1 and G2 of G such that

i. $G = G_1 \cup G_2$
ii. $(G_1 , +)$ is an M-group.
iii. $(G_2 , \bullet)$ is an M-group.

A non-empty subset H of an M-bigroup (G , + , •) is called an M-sub bigroup, if H itself is a M-bigroup under + and • operations defined on G.

2.4 Definition
Let G be a group. A fuzzy subset A of G is called an anti fuzzy subgroup if for x, y ∈G,

i. $A(xy) \leq \max \{ A(x) , A(y)\}$,
ii. $A(x^{-1}) = A(x)$.

2.5 Definition
Let A be a fuzzy subset of S. For t ∈ [0, 1], the lower level subset of A is the set, $\bar{A}t = \{ x \in S : A(x) \leq t \}$.

2.6 Definition
Let G be a finite group of order n and A be an anti fuzzy subgroup of G .
Let Im (A) = { ti : A(x) = ti for some x ∈ G }. Then { $\bar{A}ti$ } are the only lower level subgroups of A.

2.7 Definition
Let G be an M-group and A be an anti fuzzy subgroup of G . Then A is called an anti M-fuzzy subgroup of G if for all x ∈ G and m ∈ M , then $A(mx) \leq A(x)$.

2.8 Definition
Let G be an M-group and A be an anti M-fuzzy subgroup of G .
Let Im (A) = { ti : A(x) = ti for some x ∈ G }. Then { $\bar{A}ti$ } are the only lower level M-subgroups of A.

2.9 Definition

Let G = (G1 ∪ G2 , + , •) be a bigroup. Then a fuzzy subset A is said to be an anti fuzzy sub-bigroup of G if there exist two fuzzy subsets A1 of G1 and A2 of G2 such that

i. $A = A_1 \cup A_2$.

ii. (A_1 , +) is an anti fuzzy subgroup of (G_1 , +)

iii. (A_2 , •) is a an anti fuzzy subgroup (G_2 , •).

2.10 Definition

Let G = (G1 ∪ G2 , + , •) be an M-bigroup. Then an anti fuzzy sub-bigroup A of G is said to be an anti M-fuzzy sub-bigroup of G if

i. A (m + x) ≤ A(x) for all $x \in G_1$ and m ∈ M.

ii. A (m • x) ≤ A(x) for all $x \in G_2$ and m ∈ M.

3. PROPERTIES OF BI LOWER LEVEL SUBSETS OF AN ANTI M-FUZZY SUB-BIGROUP OF AN M-BIGROUP

In this section, we introduce the concept of lower level subset of an anti M-fuzzy sub-bigroup of an M-bigroup and discuss some of its properties.

3.1 Definition

Let G = (G1 ∪ G2 ,+, •) be an M-bigroup and A = (A1 ∪ A2) be an anti M-fuzzy sub-bigroup of G. The bi lower level subset of the anti M-fuzzy sub-bigroup A of G is defined as

Āt = Ā1t ∪ Ā2t , for every t ∈ [max {A1(e1), A2(e2) },1], where e1 denotes the identity element of the group (G1, +) and e2 denotes the identity element of the group (G2, •).

Remark

The condition t ∈ [max {A1(e1), A2(e2) },1] is essential for the bi lower level to be an M-sub bigroup, for if t ∉ [max {A1(e1), A2(e2) },1], the bi lower level subset need not in general be an M-sub bigroup of an M-bigroup G.

3.1 Theorem

Every bi lower level Subset of an anti M-fuzzy sub-bigroup A of an M-bigroup G is an M-sub bigroup of G.

Proof

Let A= (A1 ∪ A2) be an anti M-fuzzy sub-bigroup of an M-bigroup G = (G1 ∪ G2 ,+, •). Consider the bi lower level subset Āt of an anti M-fuzzy sub-bigroup A, for every t ∈ [max {A1(e1), A2(e2) },1], where e1 denotes the identity of (G1,+) and e2 denotes the identity element of the group (G2, •). Then Āt = Ā1t ∪ Ā2t where Ā1t and Ā2t are M-subgroups of G1 and G2 respectively. Hence by the definition of M-sub bigroup Āt is an M-sub bigroup of G.

3.2 Theorem

Let G be an M-bigroup and A1, A2 be fuzzy subsets of A such that A = (A1 ∪ A2). The bi lower level subset Āt of A is an M-sub bigroup of G ,t ∈ [max {A1(e1), A2(e2) },1], where e1 denotes the identity element of G1 and e2 denotes the identity element of G2 respectively. Then A is an anti M-fuzzy sub-bigroup of G.

Proof

Let G = (G1 ∪ G2) be an M-bigroup.
Given that the bi lower M-level subset Āt = Ā1t ∪ Ā2t is an M-sub bigroup of G.
Clearly Ā1t is an M-subgroup of G1 , A1 is an anti M-fuzzy subgroup of G1.
Clearly Ā2t is a subgroup of G2 , A2 is an anti M-fuzzy subgroup of G2.
Clearly A = (A1 ∪ A2) and hence A is an anti M-fuzzy sub-bigroup of G.

3.1 Definition

Let A = (A1 ∪ A2) be an anti M-fuzzy sub-bigroup of an M-bigroup G = (G1 ∪ G2). The M-sub bigroups Āt, for t ∈[max {A1(e1), A2(e2) },1] ,where e1 denotes the identity element of G1 and e2 denotes the identity element of G2 respectively, are called bi lower level M-sub bigroups of A.

3.3 Theorem

Let A = (A1 ∪ A2) be an anti M-fuzzy sub-bigroup of an M-bigroup G = (G1 ∪ G2). Two bi lower level M-sub bigroups Āα, Āβ , α, β ∈[max {A1(e1), A2(e2) },1] , where e1 denotes the identity element of G1 and e2 denotes the identity element of G2 respectively with α < β are equal iff there is no x in G such that α < A(x) ≤ β.

Proof

Let A = (A1 ∪ A2) be an anti M-fuzzy sub-bigroup of an M-bigroup G = (G1 ∪ G2). Consider the two bi lower level M-sub bigroups Āα, Āβ, α, β ∈[max {A1(e1), A2(e2) },1] where e1 denotes the identity element of G1 and e2 denotes the identity element of G2 respectively with α < β.
Let Āα = Āβ.
We have to prove that there is no x in G such that α < A(x) ≤ β.
Suppose that there is an x in G such that α < A(x) < β , then x ∈ Āβ and x ∉ Āα .
This implies Āα ⊂ Āβ, which contradicts the assumption that Āα = Āβ.
Hence there is no x in G such that α < A(x) ≤ β.
Conversely, suppose that there is no x in G such that α < A(x) ≤ β.
Then , by definition, Āα ⊂ Āβ.
Let x ∈ Āβ and there is no x in G such that α < A(x) ≤ β.
Hence x ∈ Āα.
That is, Āβ ⊂ Āα.
Hence Āα = Āβ.

3.4 Theorem

A fuzzy subset A of G is an anti M-fuzzy sub-bigroup of G iff the bi lower level Subsets Āt, , t ∈ Image A, are M-sub bigroups of G.

Proof : It is clear.

3.5 Theorem

Any M-sub bigroup H of an M-bigroup G can be realized as a bi lower level M-sub bigroup of some anti M-fuzzy sub-bigroup of G.

Proof

Let $G = (G_1 \cup G_2, +, \bullet)$ be an M-bigroup.

Let $H = (H_1 \cup H_2, +, \bullet)$ be an M-sub bigroup of G.

Let A_1 and A_2 be a fuzzy subsets of A defined by

$$A_1(x) = \begin{cases} 0 & \text{if } x \in H_1 \\ t & \text{if } x \notin H_1 \end{cases}$$

$$A_2(x) = \begin{cases} 0 & \text{if } x \in H_2 \\ t & \text{if } x \notin H_2, \end{cases}$$

where $t \in [\max\{A1(e1), A2(e2)\}, 1]$, and e1 denotes the identity element of G1 and e2 denotes the identity element of G2 respectively.

We shall prove that $A = (A1 \cup A2)$ is an anti M-fuzzy sub-bigroup of G.

Suppose $x, y \in H$, then

i. $x, y \in H_1 \Rightarrow x + y \in H_1$ and $x + (-y) \in H_1$.

$A_1(x)=0$, $A_1(y)=0$ and $A_1(x + (-y)) = 0$, then $A_1(x + (-y)) \leq \max\{A_1(x), A_2(y)\}$.

ii. $x, y \in H_2 \Rightarrow xy \in H_2$ and $xy^{-1} \in H_2$.

$A_2(x) = 0$, $A_2(y) = 0$ and $A_2(xy^{-1}) = 0$, then

$A_2(xy^{-1}) \leq \max\{A_2(x), A_2(y)\}$.

iii. $x \in H_1$ and $y \notin H_1 \Rightarrow x + y \notin H_1$ and $x + (-y) \notin H_1$.

$A_1(x) = 0$, $A_1(y) = t$ and $A_1(x + (-y)) = t$, then $A_1(x + (-y)) \leq \max\{A_1(x), A_2(y)\}$.

iv. $x \in H_2$ and $y \notin H_2 \Rightarrow xy \notin H_2$ and $xy^{-1} \notin H_2$.

$A_2(x) = 0$, $A_2(y) = t$ and $A_2(xy^{-1}) = t$, then

$A_2(xy^{-1}) \leq \max\{A_2(x), A_2(y)\}$.

Suppose $x, y \notin H$, then

i. $x, y \notin H_1$ then $x + y \in H_1$ or $x + y \notin H_1$.

$x, y \notin H_1$ then $x + (-y) \in H_1$ or $x + (-y) \notin H_1$.

$A_1(x) = t$, $A_1(y) = t$ and $A_1(x + (-y)) = 0$ or t , then $A_1(x + (-y)) \leq \max\{A_1(x), A_2(y)\}$.

ii. $x, y \notin H_2 \Rightarrow xy \in H_2$ or $xy \notin H_2$.

$x, y \notin H_2 \Rightarrow xy^{-1} \in H_2$ or $xy^{-1} \notin H_2$.

$A_2(x) = t$, $A_2(y) = t$ and $A_2(xy^{-1}) = 0$ or t , then $A_2(xy^{-1}) \leq \max\{A_2(x), A_2(y)\}$.

Thus in all cases,

$(A_1, +)$ is an anti fuzzy subgroup of $(G_1, +)$ and

$(A_2, \bullet)$ is an anti fuzzy subgroup of $(G_2, \bullet)$.

Hence $A = (A_1 \cup A_2)$ is an anti fuzzy sub-bigroup of G.

Suppose, for $m \in M$ and $x \in H_1$, we have $m + x \in H_1$.

Then, $A_1(x) = 0$ and $A_1(m + x) = 0$.

Hence, $A_1(m + x) \leq A_1(x)$.

Now, for $m \in M$ and $x \notin H_1$, we have $m + x \in H_1$ or $m + x \notin H_1$.

Then, $A_1(x) = t$ and $A_1(m + x) = 0$ or t .

Hence, $A_1(m + x) \leq A_1(x)$.

Clearly, A_1 is an anti M-fuzzy subgroup of G_1.

Suppose, for $m \in M$ and $x \in H_2$, we have $mx \in H_2$.

Then, $A_2(x) = 0$ and $A_2(mx) = 0$.

Hence, $A_2(mx) \leq A_2(x)$.

Now, for $m \in M$ and $x \notin H_2$, we have $mx \in H_2$ or $mx \notin H_2$.

Then, $A_2(x) = t$ and $A_2(mx) = 0$ or t .

Hence, $A_2(mx) \leq A_2(x)$.

Clearly, A_2 is an anti M-fuzzy subgroup of G_2.

Hence $A = (A_1 \cup A_2)$ is an anti M-fuzzy sub-bigroup of G.

For this anti M-fuzzy sub-bigroup, $\bar{A}_t = \bar{A}_{1t} \cup \bar{A}_{2t} = H$.

Remark

As a consequence of the Theorem 2.3, the bi lower level M-sub bigroups of an anti M-fuzzy sub-bigroup A form a chain. Since $A(e_1) \leq A(x)$ or $A(e_2) \leq A(x)$ for all x in G, therefore $\bar{A}_{t0}$, where $\max\{A_1(e_1), A_2(e_2)\} = t_0$ is the smallest and we have the chain :

$\{e_1, e_2\} = \bar{A}_{t0} \subset \bar{A}_{t1} \subset \bar{A}_{t2} \subset \ldots \subset \bar{A}_{tn} = G$,

where $t_0 < t_1 < t_2 < \ldots\ldots < t_n$.

4. CONCLUSION

In this paper , we define a new bialgebraic structure of of anti M-fuzzy sub-bigroup of an M-bigroup and bi lower level subset of an anti M-fuzzy sub-bigroup.Futher, we wish to define the relation between of anti M-fuzzy normal sub-bigroup of an M-bigroup and bi lower level subset of an anti M-fuzzy normal sub-bigroup also the same in Intuitionistic fuzzy and other some groups are in progress.

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