

# Optimal Replenishment Policy for Ameliorating Item with Shortages under Inflation and Time Value of Money using Genetic Algorithm

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## ABSTRACT

The implementation of supply chain has to reduce the total cost of system, but generally each component of a supply chain tries to find the best policy for its company and consequently tries to find a local optimum. Knowing that the sum of local optimum cannot constitute the global optimum, it is necessary to consider all costs of system simultaneously to find the optimal replenishment policy for all the components of a supply chain. Demand rate of the item is assumed to be a function of time known as ramp type function. Shortages are permitted and partially back-ordered. The back-ordering fraction is taken to be decreasing function of waiting time. We consider inflation and apply discounted cash flow in the problem analysis. Total cost of the system is formulated and optimal replenishment policy is derived, keeping in view the above factors of the system. We use Genetic Algorithm (GA) to solve the models. A numerical example and sensitivity analysis is shown to illustrate the models.

## Keywords

Replenishment policy, ramp type demand, amelioration items, Optimization, Genetic algorithm

## 1. INTRODUCTION

The classical economic production quantity (EPQ) model assumes constant demand and infinite lifetime of items in inventory. Subsequent research efforts have led to the removal of these two restrictions in consideration of time varying demand functions and finite lifetimes for inventoried items. This is in line with common experience in day-to-day production and inventory management. The time-varying demand functions considered in most EPQ models are unidirectional, i.e. they are either continuously non-decreasing or continuously non-increasing function of time. The main objective of an inventory model is to calculate how much to replenish and when to replenish. After reviewing many inventory models of the past, it has been observed that many researchers included different factors to basic inventory model in order to study different environments/scenarios. One of the basic requirements to develop any replenishment policy for inventory system is its demand. The concept of both deterministic and probabilistic types of demand was discussed in much earlier phase of development of this study. In case of deterministic demand, constant demand was in vogue in earlier days. Since there are different inventory systems in real life, therefore researchers have had to choose demand patterns accordingly. One of the most common demand patterns that is suitable for almost all types of inventory systems is time varying demand pattern and a lot of work has been done in this direction till now. In order to represent the sales in

different phases of the product life cycle in market, time dependent demand is suitable, e.g. the demand for inventory items increases over the time in the growth phase and decreases in decline phase. Silver and Meal (1969), Datta and Pal(1991), Chakrabarty et al. (1997) are few examples who considered linearly time dependent demand. Goyal(1987), Hwang (1995), Hwang(1997), Skouri and Papachristos (2003), Moon et al. (2005), Lin and Lin (2006) considered demand as continuous function of time. Benkherhouf and Balki (1997), Zhou, Lang and Yang(2004) considered demand as continuously increasing function of time. Datta and Pal(1988) used power pattern in time dependent demand. As far as concerned two types of time dependent demand can be frequently found in papers: 1) linearly positive/negative trend in demand and 2) exponentially increasing/decreasing demand rate. This concept results in continuously increase/decrease in demand over time which is not realistic. Demand may increase during certain time periods and becomes constant after that. This type of demand can be represented by ramp type function. Hill (1995), Mondal and Pal(1998), Wu and Quyang (2000), Panda et al. (2008), Skouri et al. (2009) considered ramp type time demand pattern. Besides demand, there are other important factors which affect an inventory model. Shortages are one of them. Any inventory model which is developed with no shortage can be redeveloped taking shortage into consideration. In above cited papers, the concept of shortages is excluded in some while included in others. When shortages arise, question of backordering arise simultaneously. Again two types of backordering can be experienced in inventory models, namely full backordering and partial backordering. Models considered by Goyal (1987), Datta and Pal (1991), Hariga (1995,1997), Chakrabarty et al. (1997), Wu and Quyang (2000), Moon et al. (2005) are some of the examples in which shortages are fully backordered. When customers have to face shortages, their response is different according the type of commodity and market environment. For a competitive market partial backordering plays a practical role. Wu (2001), Skouri and Papachristos (2003), Yang (2004), Lin and Lin (2006), Skouri et al. (2009) are few of researchers who considered shortages as partially backordered.

The replenishment problem has been traditionally treated from a multi-echelon and multi-product perspective (Jen-Ming and Tsung-Hui 2005). A multi-echelon replenishment problem focuses on channel coordination issues for inventory replenishment, between upstream and downstream components of a supply chain, with the objective of minimizing total system costs (Sila et al. 2005). Moreover, multi-product replenishment problems aim to coordinate the replenishment of various items in the same family or same category in order to reduce the frequency of major setups and

the related costs. This can be obtained by choosing an appropriate common replenishment frequency and lot-sizes within the family of items (Bahloul et al. 2008). Several previous works have studied the problem of multi-echelon, multi-product Supply Chain. Chen et al. (Cheng-Liang et al. 2004) have studied a multi-item inventory and transport problem with joint setup costs, referred to a joint replenishment problem.

## 2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used in formulating the models:

### 2.1 Assumptions

1. The inventory system involves single item.
2. Demand rate is depending on time given by

$$D(t) = \begin{cases} f(t), & t < \mu \\ f(\mu), & t \geq \mu \end{cases}$$

where  $f(t)$  is a positive continuous function of  $t \in [0, T]$ .

The function defined above is known as ramp type function.

3. Shortages are permitted and backordered at a rate  $B(t)$ , which is a non-increasing function of  $t$  with  $0 \leq B(t) \leq 1$ , where  $t$  is the waiting time up to next replenishment.
4. Replenishment rate is infinite.
5. Deterioration and amelioration occur at constant rate
6. The deterioration and amelioration occur when the item is effectively in stock.
7. Inflation and money-value is considered.

### 2.2 Notations:

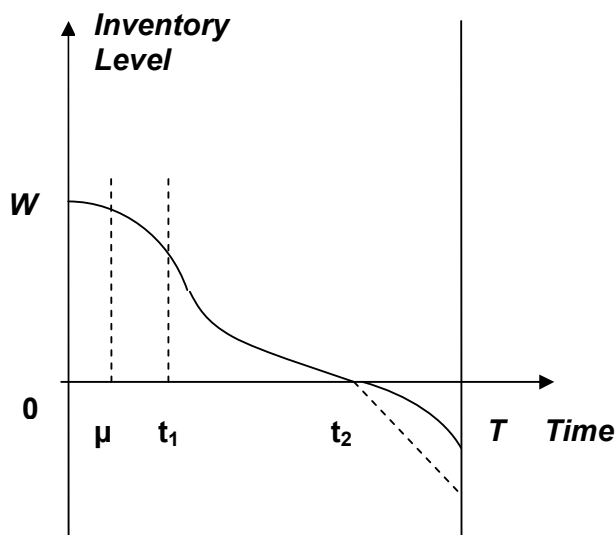
$D(t)$  : Demand rate  
 $\mu$  : Parameter for ramp type demand function

**Case1:** when  $t_1 > \mu$

$\theta(t)$  : Deterioration rate which is defined in terms of two parameter Weibull distribution as  $\theta(t) = \alpha\beta t^{\beta-1}$   
 $\alpha$  : Scale parameter of deterioration rate  
 $\beta$  : Shape parameter of deterioration rate  
 $\gamma(t)$  : Amelioration rate which is defined in terms of two parameter Weibull distribution as  $\gamma(t) = ght^{\beta-1}$   
 $g$  : Scale parameter of amelioration rate  
 $h$  : Shape parameter of amelioration rate  
 $r$  : Constant representing the difference between discount rate and inflation rate  
 $c_p$  : Purchase cost per unit  
 $c_o$  : Ordering cost per order  
 $c_h$  : Holding cost per unit per unit time  
 $c_a$  : Amelioration cost per unit  
 $c_d$  : Deterioration cost per unit  
 $c_s$  : Shortage cost per unit per unit time  
 $c_1$  : Lost sale cost per unit  
 $W$  : Maximum inventory level at initial point  
 $I_i(t)$  : Inventory level at any time  $t$  for  $i^{\text{th}}$  phase, where  $i=1,2,3,4$   
 $t_1$  : Time up to which inventory ameliorates.  
 $t_2$  : Time at which inventory level becomes zero  
 $T$  : Time for one replenishment cycle  
 $B(\tau)$  : Fraction of demand backordered, which is decreasing function of waiting time, given by  $1/1+\delta(\tau)$ ,  $0 < \delta < 1$

## 3. MODEL FORMULATION:

Replenishment is made at the beginning of the cycle i.e., at  $t=0$ , which brings the inventory level maximum equals to  $W$ . Amelioration/deterioration occurs as soon as item is received into inventory. Due to amelioration inventory accumulates till  $t=t_1$ . Due to demand and deterioration inventory level starts declining and reaches to zero at  $t=t_2$ . Shortage occurs during the interval  $[t_2, T]$ , which is partially backordered. Since the demand function is divided into two intervals, therefore three cases can be considered in order to represent the system



**Fig. 1 : Variation of inventory level with time for Case 1.**

Inventory system can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt} + (\alpha\beta t^{\beta-1} - ght^{h-1})I_1(t) = -(a + bt) \quad 0 \leq t \leq \mu \quad \dots (1)$$

with  $I_1(0) = W$

$$\frac{dI_2(t)}{dt} + (\alpha\beta t^{\beta-1} - ght^{h-1})I_2(t) = -(a + b\mu) \quad \mu \leq t \leq t_1 \quad \dots (2)$$

with  $I_2(\mu) = I_1(\mu)$

$$\frac{dI_3(t)}{dt} + \alpha\beta t^{\beta-1}I_3(t) = -(a + b\mu) \quad t_1 \leq t \leq t_2 \quad \dots (3)$$

with  $I_3(t_2) = 0$

$$\frac{dI_4(t)}{dt} = -\frac{(a + b\mu)}{1 + \delta(T - t)} \quad t_2 \leq t \leq T \quad \dots (4)$$

with  $I_4(t_2) = 0$

The solution of the above equations can be given as:

$$I_1(t) = e^{-\alpha t^\beta + gt^h} \left[ -a \left( t + \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{gt^{h+1}}{h+1} \right) - b \left( \frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{gt^{h+2}}{h+2} \right) + W \right], \quad 0 \leq t \leq \mu \quad \dots (5)$$

$$I_2(t) = e^{-\alpha t^\beta + gt^h} \left[ -(a + b\mu) \left( t + \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{gt^{h+1}}{h+1} \right) + b \left( \frac{\mu^2}{2} + \frac{\alpha \mu^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{g\mu^{h+2}}{(h+1)(h+2)} \right) + W \right]$$

$\mu \leq t \leq t_1 \quad \dots (6)$

$$I_3(t) = e^{-\alpha t^\beta} (a + b\mu) \left( -t - \frac{\alpha t^{\beta+1}}{\beta+1} + t_2 + \frac{\alpha t_2^{\beta+1}}{\beta+1} \right), \quad t_1 \leq t \leq t_2 \quad \dots (7)$$

$$I_4(t) = \frac{(a + b\mu)}{\delta} \log \left( \frac{1 + \delta(T - t_2)}{1 + \delta(T - t)} \right), \quad t_2 \leq t \leq T \quad \dots (8)$$

Since  $I_3(t_1) = I_2(t_1)$

$$\Rightarrow e^{-\alpha t_1^\beta} (a + b\mu) \left( -t_1 - \frac{\alpha t_1^{\beta+1}}{\beta+1} + t_2 + \frac{\alpha t_2^{\beta+1}}{\beta+1} \right) - e^{-\alpha t_1^\beta + gt_1^h} \left\{ -(a + b\mu) \left( t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{gt_1^{h+1}}{h+1} \right) + b \left( \frac{\mu^2}{2} + \frac{\alpha \mu^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{g\mu^{h+2}}{(h+1)(h+2)} \right) + W \right\} = 0 \quad \dots (9)$$

Now, the costs contributing the total cost of the system in case 1 are given as follows:

1) Present worth ordering and purchase cost :

$$PC = c_o + c_p W \quad \dots (10)$$

2) Present worth holding cost :

$$HC = c_h \left[ \int_0^\mu I_1(t) e^{-rt} dt + \int_\mu^{t_1} I_2(t) e^{-r(\mu+t)} dt + \int_{t_1}^{t_2} I_3(t) e^{-r(t_1+t)} dt \right] \quad \dots (11)$$

3) Present worth amelioration cost :

$$AC = c_a \left[ \int_0^\mu ght^{h-1} I_1(t) e^{-rt} dt + \int_\mu^{t_1} ght^{h-1} I_2(t) e^{-r(\mu+t)} dt \right] \quad \dots (12)$$

4) Present worth deterioration cost :

$$DC = c_d \left[ \int_0^\mu \alpha \beta t^{\beta-1} I_1(t) e^{-rt} dt + \int_\mu^{t_1} \alpha \beta t^{\beta-1} I_2(t) e^{-r(\mu+t)} dt + \int_{t_1}^{t_2} \alpha \beta t^{\beta-1} I_3(t) e^{-r(t_1+t)} dt \right] \quad \dots (13)$$

5) Present worth shortage cost :

$$SC = c_s \int_{t_2}^T (-I_4(t)) e^{-r(t_2+t)} dt \quad \dots (14)$$

6) Present worth lost sales cost :

$$LC = c_l \int_{t_2}^T \left( 1 - \frac{1}{1 + \delta(T-t)} \right) (a + b\mu) e^{-r(t_2+t)} dt \quad \dots (15)$$

Our problem is to minimize

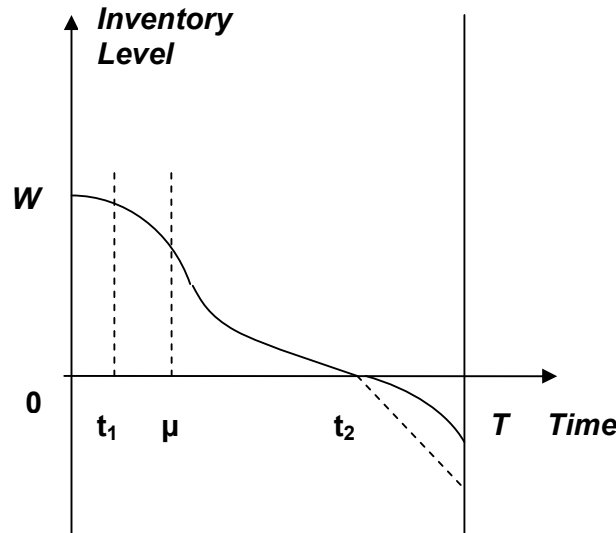
$$TC_1(\mu, t_1, t_2, W) = PC + HC + AC + DC + SC + LC \quad \dots (16)$$

subject to  $\mu < t_1 < t_2 < T$

and constraint given by equ.(9).

$$\mu, t_1, t_2, W \geq 0$$

Case 2: when  $t_1 < \mu$



**Fig. 2 : Variation of inventory level with time for Case 2**

Inventory system can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt} + (\alpha \beta t^{\beta-1} - ght^{h-1}) I_1(t) = -(a + bt) \quad 0 \leq t \leq t_1 \quad \dots (17)$$

with  $I_1(0) = W$

$$\frac{dI_2(t)}{dt} + \alpha \beta t^{\beta-1} I_2(t) = -(a + bt) \quad t_1 \leq t \leq \mu \quad \dots (18)$$

with  $I_2(t_1) = I_1(t_1)$

$$\frac{dI_3(t)}{dt} + \alpha \beta t^{\beta-1} I_3(t) = -(a + b\mu) \quad \mu \leq t \leq t_2 \quad \dots (19)$$

with  $I_3(t_2) = 0$

$$\frac{dI_4(t)}{dt} = -\frac{(a+b\mu)}{1+\delta(T-t)} \quad t_2 \leq t \leq T \quad \dots (20)$$

with  $I_4(t_2) = 0$

The solution of the above equations can be given as:

$$I_1(t) = e^{-\alpha t^\beta + g t^h} \left[ -a \left( t + \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{g t^{h+1}}{h+1} \right) - b \left( \frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{g t^{h+2}}{h+2} \right) + W \right], \quad 0 \leq t \leq t_1 \quad \dots (21)$$

$$I_2(t) = e^{-\alpha t^\beta} \left[ -(a+bt) \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{(\beta+1)(\beta+2)} - at - \frac{bt^2}{2} - e^{-g t_1^h} \left\{ a \left( t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{g t_1^{h+1}}{h+1} \right) + b \left( \frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{(\beta+2)} - \frac{g t_1^{h+2}}{h+2} \right) - W \right\} + (a+bt_1) \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{b\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} + at_1 + \frac{bt_1^2}{2} \right] \quad t_1 \leq t \leq \mu \quad \dots (22)$$

$$I_3(t) = e^{-\alpha t^\beta} (a+b\mu) \left( -t - \frac{\alpha t^{\beta+1}}{\beta+1} + t_2 + \frac{\alpha t_2^{\beta+1}}{\beta+1} \right), \quad \mu \leq t \leq t_2 \quad \dots (23)$$

$$I_4(t) = \frac{(a+b\mu)}{\delta} \log \left( \frac{1+\delta(T-t_2)}{1+\delta(T-t)} \right), \quad t_2 \leq t \leq T \quad \dots (24)$$

Since  $I_3(\mu) = I_2(\mu)$

$$\Rightarrow -(a+b\mu) \left( t_2 + \frac{\alpha t_2^{\beta+1}}{\beta+1} \right) + \frac{b\mu^2}{2} - e^{g t_1^h} \left\{ -a \left( t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{g t_1^{h+1}}{h+1} \right) + b \left( \frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{\beta+2} - \frac{g t_1^{h+2}}{h+2} \right) - W \right\} + (a+bt_1) \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{b\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} + at_1 + \frac{bt_1^2}{2} = 0 \quad \dots (25)$$

Now, the costs contributing the total cost of the system in case 1 are given as follows:

1) Present worth ordering and purchase cost :

$$PC = c_o + c_p W \quad \dots (26)$$

2) Present worth holding cost :

$$HC = c_h \left[ \int_0^{t_1} I_1(t) e^{-rt} dt + \int_{t_1}^{\mu} I_2(t) e^{-r(t_1+t)} dt + \int_{\mu}^{t_2} I_3(t) e^{-r(\mu+t)} dt \right] \quad \dots (27)$$

3) Present worth amelioration cost :

$$AC = c_a \int_0^{t_1} g h t^{h-1} I_1(t) e^{-rt} dt \quad \dots (28)$$

4) Present worth deterioration cost :

$$DC = c_d \left[ \int_0^{t_1} \alpha \beta t^{\beta-1} I_1(t) e^{-rt} dt + \int_{t_1}^{\mu} \alpha \beta t^{\beta-1} I_2(t) e^{-r(t_1+t)} dt + \int_{\mu}^{t_2} \alpha \beta t^{\beta-1} I_3(t) e^{-r(\mu+t)} dt \right] \quad \dots (29)$$

5) Present worth shortage cost :

$$SC = c_s \int_{t_2}^T (-I_4(t)) e^{-r(t_2+t)} dt \quad \dots (30)$$

6) Present worth lost sales cost :

$$LC = c_l \int_{t_2}^T \left( 1 - \frac{1}{1+\delta(T-t)} \right) (a+b\mu) e^{-r(t_2+t)} dt \quad \dots (31)$$

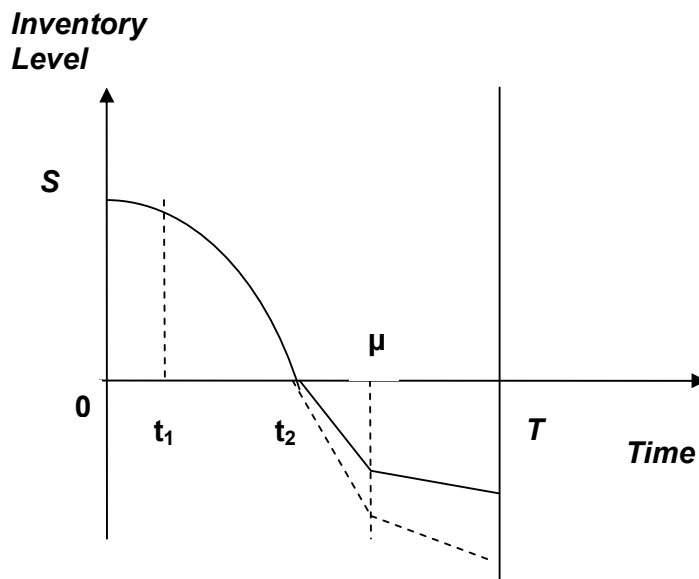
Our problem is to minimize

$$TC_2(\mu, t_1, t_2, W) = PC + HC + AC + DC + SC + LC \quad \dots (32)$$

subject to  $t_1 < \mu < t_2 < T$   
 and constraint given by equ.(25).

$$\mu, t_1, t_2, W \geq 0$$

Case 3: when  $t_1 < t_2 < \mu$



**Fig. 3 : Variation of inventory level with time for Case3**

Inventory system can be represented by the following differential equations:

$$\frac{dI_1(t)}{dt} + (\alpha\beta t^{\beta-1} - ght^{h-1})I_1(t) = -(a + bt) \quad 0 \leq t \leq t_1 \quad \dots (33)$$

with  $I_1(0) = W$

$$\frac{dI_2(t)}{dt} + \alpha\beta t^{\beta-1}I_2(t) = -(a + bt) \quad t_1 \leq t \leq t_2 \quad \dots (34)$$

with  $I_2(t_2) = 0$

$$\frac{dI_3(t)}{dt} = -\frac{(a + bt)}{1 + \delta(T - t)} \quad t_2 \leq t \leq \mu \quad \dots (35)$$

with  $I_3(t_2) = 0$

$$\frac{dI_4(t)}{dt} = -\frac{(a + b\mu)}{1 + \delta(T - t)} \quad \mu \leq t \leq T \quad \dots (36)$$

with  $I_4(\mu) = I_3(\mu)$

The solution of the above equations can be given as:

$$I_1(t) = e^{-\alpha t^\beta + gt^h} \left[ -a \left( t + \frac{\alpha t^{\beta+1}}{\beta+1} - \frac{gt^{h+1}}{h+1} \right) - b \left( \frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{gt^{h+2}}{h+2} \right) + W \right], \quad 0 \leq t \leq t_1 \quad \dots (37)$$

$$I_2(t) = e^{-\alpha t^\beta} \left[ -(a+bt) \frac{\alpha t^{\beta+1}}{\beta+1} + \frac{b\alpha t^{\beta+2}}{(\beta+1)(\beta+2)} - at - \frac{bt^2}{2} + (a+bt_2) \frac{\alpha t_2^{\beta+1}}{\beta+1} - \frac{b\alpha t_2^{\beta+2}}{(\beta+1)(\beta+2)} + at_2 + \frac{bt_2^2}{2} \right]$$

$$t_1 \leq t \leq t_2 \quad \dots (38)$$

$$I_3(t) = \frac{1}{\delta} \left\{ a + b \left( T + \frac{1}{\delta} \right) \right\} \log \left( \frac{1 + \delta(T-t)}{1 + \delta(T-t_2)} \right) + \frac{b}{\delta^2} (t - t_2)$$

$$t_2 \leq t \leq \mu \quad \dots (39)$$

$$I_4(t) = \frac{(a+b\mu)}{\delta} \log \left( \frac{1 + \delta(T-t)}{1 + \delta(T-\mu)} \right) + \frac{1}{\delta} \left\{ a + b \left( T + \frac{1}{\delta} \right) \right\} \log \left( \frac{1 + \delta(T-\mu)}{1 + \delta(T-t_2)} \right) + \frac{b}{\delta^2} (\mu - t_2)$$

$$t_2 \leq t \leq T \quad \dots (40)$$

Since  $I_1(t_1) = I_2(t_1)$

$$\Rightarrow e^{gt_1^h} \left\{ -a \left( t_1 + \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{gt_1^{h+1}}{h+1} \right) - b \left( \frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{\beta+2} - \frac{gt_1^{h+2}}{h+2} \right) + W \right\} + (a+bt_1) \frac{\alpha t_1^{\beta+1}}{\beta+1} - \frac{b\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}$$

$$+ at_1 + \frac{bt_1^2}{2} - (a+bt_2) \frac{\alpha t_2^{\beta+1}}{\beta+1} + \frac{b\alpha t_2^{\beta+2}}{(\beta+1)(\beta+2)} - at_2 - \frac{bt_2^2}{2} = 0 \quad \dots (41)$$

Now, the costs contributing the total cost of the system in case 1 are given as follows:

1) Present worth ordering and purchase cost :

$$PC = c_o + c_p W \quad \dots (42)$$

2) Present worth holding cost :

$$HC = c_h \left[ \int_0^{t_1} I_1(t) e^{-rt} dt + \int_{t_1}^{t_2} I_2(t) e^{-r(t_1+t)} dt \right] \quad \dots (43)$$

3) Present worth amelioration cost :

$$AC = c_a \int_0^{t_1} ght^{h-1} I_1(t) e^{-rt} dt \quad \dots (44)$$

4) Present worth deterioration cost :

$$DC = c_d \left[ \int_0^{t_1} \alpha \beta t^{\beta-1} I_1(t) e^{-rt} dt + \int_{t_1}^{t_2} \alpha \beta t^{\beta-1} I_2(t) e^{-r(t_1+t)} dt \right] \quad \dots (45)$$

5) Present worth shortage cost :

$$SC = c_s \left[ \int_{t_2}^{\mu} (-I_3(t)) e^{-r(\mu+t)} dt + \int_{t_2}^T (-I_4(t)) e^{-r(t_2+t)} dt \right] \quad \dots (46)$$

6) Present worth lost sales cost :

$$LC = c_l \left[ \int_{t_2}^{\mu} \left( 1 - \frac{1}{1 + \delta(T-t)} \right) (a+bt) e^{-r(t_2+t)} dt + \int_{\mu}^T \left( 1 - \frac{1}{1 + \delta(T-t)} \right) (a+b\mu) e^{-r(\mu+t)} dt \right] \quad \dots (47)$$

Our problem is to minimize

$$TC_2(\mu, t_1, t_2, W) = PC + HC + AC + DC + SC + LC \quad \dots (48)$$

subject to  $t_1 < t_2 < \mu < T$

and constraint given by equ.(41).

$$\mu, t_1, t_2, W \geq 0$$

#### 4. GENETIC ALGORITHM

Genetic Algorithm was developed by Holland and his colleagues in the 1960s and 1970s. Genetic Algorithms are inspired by the evolutionist theory explaining the origin of species. In nature, weak and unfit species within their

environment are faced with extinction by natural selection. The strong ones have greater opportunity to pass their genes to future generations via reproduction. In the long run, species carrying the correct combination in their genes become dominant in their population. Sometimes, during the slow

process of evolution, random changes may occur in genes. If these changes provide additional advantages in the challenge for survival, new species evolve from the old ones. Unsuccessful changes are eliminated by natural selection.

GA terminology, a solution vector  $x \in X$  is called an individual or a chromosome. Chromosomes are made of discrete units called genes. Each gene controls one or more features of the chromosome. In the original implementation of GA by Holland, genes are assumed to be binary digits. In later implementations, more varied gene types have been introduced. Normally, a chromosome corresponds to a unique solution  $x$  in the solution space. This requires a mapping mechanism between the solution space and the chromosomes. This mapping is called an encoding. In fact, GA works on the encoding of a problem, not on the problem itself.

GA operates with a collection of chromosomes, called a population. The population is normally randomly initialized. As the search evolves, the population includes fitter and fitter solutions, and eventually it converges, meaning that it is dominated by a single solution. Holland also presented a proof of convergence (the schema theorem) to the global optimum where chromosomes are binary vectors.

GA use two operators to generate new solutions from existing ones: crossover and mutation. The crossover operator is the most important operator of GA. In crossover, generally two chromosomes, called parents, are combined together to form new chromosomes, called offspring. The parents are selected among existing chromosomes in the population with preference towards fitness so that offspring is expected to inherit good genes which make the parents fitter. By iteratively applying the crossover operator, genes of good chromosomes are expected to appear more frequently in the population, eventually leading to convergence to an overall good solution.

The mutation operator introduces random changes into characteristics of chromosomes. Mutation is generally applied at the gene level. In typical GA implementations, the mutation rate (probability of changing the properties of a gene) is very small and depends on the length of the chromosome. Therefore, the new chromosome produced by mutation will not be very different from the original one. Mutation plays a critical role in GA. As discussed earlier, crossover leads the population to converge by making the chromosomes in the population alike. Mutation reintroduces genetic diversity back into the population and assists the search escape from local optima. Reproduction involves selection of chromosomes for the next generation. In the most general case, the fitness of an individual determines the probability of its survival for the next generation.

A more complete discussion of GAs including extensions to the general algorithm and related topics can be found in books by Davis (1991), Holland (1975), Michalewicz (1994) and Goldberg (1989).

## 5. NUMERICAL EXAMPLE

In this section, we solved a numerical example of the proposed model using the above-described GA. for running GA, we set the population size equal to 20, elite count equal to 2, crossover fraction equal to 0.8 with Gaussian mutation function and the stopping criteria include the maximal iterations of each cycle is 50.

### 5.1 Case1 when $t_1 > \mu$

We consider the values of the parameters in appropriate units such that  $a=2$ ,  $b=1$ ,  $\alpha = 0.03$ ,  $\beta=2.2$ ,  $g=0.4$ ,  $h=0.5$ ,  $\delta = 0.06$ ,  $r=0.06$ ,  $c_o = 5$ ,  $c_p = 1$ ,  $c_h = 0.5$ ,  $c_a = 1.5$ ,  $c_d = 5$ ,  $c_s = 1$ ,  $c_l = 2$ ,  $T = 1$ . Optimal value of  $t_1=0.84711$ ,  $\mu=0.07518$ ,  $t_2=0.9396$ ,  $W=1.47204$ . The minimized total cost  $TC_1=7.46853055$  is calculated from equ. (16).

### 5.1.1 Sensitivity Analysis

Table 1.Sensitivity analysis based on example when case 1 with different parameters.

	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
A+50%	6.00666e-05	0.97155	0.97605	2.18359	2.18359	8.289317	10.98
+25%	0.00977	0.16351	0.9661	2.0975	2.0975	8.131126	8.87
-25%	0.00031	0.96893	0.96895	1.08484	1.08484	6.638492	-11.11
-50%	0.00039	0.0991	0.91587	0.81748	0.81748	6.444843	-13.71
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
B+50%	4.03821e-06	0.97943	0.97948	1.45952	1.45952	6.839553	-8.42
+25%	0.4719	0.00065	0.96134	1.52818	1.52818	7.188598	-3.75
-25%	0.07966	0.78936	0.93675	1.46567	1.46567	7.519368	0.68
-50%	0.00293	0.13819	0.98264	1.72107	1.72107	7.566274	1.31
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
A+50%	3.90592e-06	4.46194e-06	0.95555	1.93378	1.93378	7.138736	-4.42
+25%	0.00652	0.22931	0.95663	1.6232	1.6232	7.251037	-2.91
-25%	0.33331	0.88103	0.97069	1.68547	1.68547	8.034298	7.58
-50%	0.24108	0.80933	0.87126	1.4174	1.4174	8.087327	8.29
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$\beta$ +50%	0.02344	0.93993	0.93996	1.42082	1.42082	7.219319	-3.34
+25%	0.01735	0.07074	0.94361	1.7273	1.7273	7.456597	-0.16
-25%	0.21625	0.31466	0.95327	1.73276	1.73276	7.552647	1.13
-50%	0.45366	0.90647	0.95162	1.75147	1.75147	7.787430	4.27



	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
g+50%	3.02792e-05	0.99344	0.99354		1.21323	6.500563	-12.96
+25%	0.01414	0.84566	0.92738		1.29845	7.154485	-4.20
-25%	0.0002	0.16977	0.91043		1.63309	7.549776	1.09
-50%	0.2439	0.49396	0.93073		1.85084	7.905908	5.86
h+50%	0.00011	0.97965	0.97967		1.53598	6.905063	-7.54
+25%	0.01316	0.35634	0.95371		1.61284	7.248990	-2.94
-25%	0.28562	0.71165	0.92374		1.55901	7.544445	1.02
-50%	0.37454	0.87149	0.94055		1.56625	7.577209	1.46
$\delta$ +50%	0.44958	0.90221	0.974		1.7781	7.667156	2.66
+25%	0.01205	0.51896	0.94424		1.49733	7.611114	1.91
-25%	2.28638e-08	0.97083	0.97083		1.44855	7.185103	-3.80
-50%	1.39129e-08	1.39129e-08	0.95972		1.93578	7.090317	-5.06
r+50%	0.50334	0.79236	0.92743		1.75674	7.858291	5.22
+25%	0.02183	0.79174	0.82668		1.28015	7.703888	3.15
-25%	0.00739	0.33988	0.92434		1.51709	7.317177	-2.03
-50%	4.5438e-06	0.98539	0.98541		1.46701	7.143026	-4.36
$c_o$ +50%	0	0.96782	0.96794		1.444 9	9.685661	29.69
+25%	0.0063	0.85289	0.95859		1.45114	8.585960	14.96
-25%	0.00098	0.24474	0.96567		1.62248	6.364058	-14.79
-50%	1.55578e-07	7.18733e-05	0.95649		1.92271	4.603603	-38.36
$c_p$ +50%	0.21167	0.90572	0.96967		1.60035	8.487937	13.65
+25%	0.0215	0.97365	0.97491		1.46932	7.592718	1.66
-25%	0.49731	0.91255	0.99094		1.84358	7.340344	-1.72
-50%	0.02247	0.05619	0.9401		1.74636	6.581384	-11.88
$c_h$ +50%	0.55898	0.57701	0.96509		1.9233	8.314752	11.33
+25%	0.21772	0.25494	0.94707		1.74881	7.565647	1.300
-25%	0.00147	0.95036	0.95231		1.42629	7.147677	-4.30
-50%	0.00864	0.77702	0.97352		1.4859	6.938663	-7.09
$c_s$ +50%	0.14844	0.59337	0.87545		1.46359	7.568550	1.34
+25%	0.42853	0.95924	0.97628		1.757	7.559448	1.22
-25%	6.27729e-05	0.97036	0.97754		1.45795	7.196516	-3.64
-50%	1.32919e-08	1.32919e-08	0.95975		1.93586	7.090379	-5.06
$c_d$ +50%	0.32072	0.60551	0.97339		1.74188	7.645105	2.36
+25%	0.0433	0.77153	0.83073		1.30159	7.579308	1.48
-25%	0	3.35834e-09	0.96081		1.93808	7.070836	-5.32
-50%	0	0	0.96189		1.94034	7.051183	-5.59
$c_i$ +50%	0.71312	0.80305	0.97073		2.01847	8.511481	13.96
+25%	0.3503	0.84224	0.97325		1.71287	7.524059	0.74
-25%	0.00027	0.96311	0.96313		1.43897	7.177566	-3.90
-50%	9.94633e-08	0.95032	0.95033		1.42249	7.161515	-4.11
$c_a$ +50%	0.09693	0.38251	0.94755		1.60535	8.278469	10.84
+25%	0.03209	0.17487	0.9279		1.62255	7.828476	4.82
-25%	0.00914	0.02502	0.929		1.76695	7.270120	-2.66
-50%	0.00091	0.97647	0.97838		1.454	6.842726	-8.38

### 5.2 Case 2 when $t_1 < \mu$

We consider the values of the parameters in appropriate units such that  $a=10, b=2, \alpha=0.02, \beta=3.2, g=0.4, h=0.5, \delta=0.16, r=0.06, c_o=5, c_p=1, c_h=3, c_s=1, c_d=5, c_i=2, c_a=3, T=1$ . Optimal value of  $t_1$ , i.e.,  $t_1^*=0.002, \mu=0.05766, t_2=0.08977, W=0.89509$ . The minimized total cost  $TC_2=3.110885$  is calculated from equ.(32).

5.2.1 Sensitivity Analysis

Table 2 .Sensitivity analysis based on example when case 2 with different parameters.

$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
a+50%	0.05032	0.04913	0.05752	0.78263	1.387925	-55.39
+25%	0.09172	0.00015	0.09816	1.24725	2.780025	-10.64
-25%	0.14632	0.01132	0.17671	1.33928	4.884574	57.02
-50%	0.10635	0.02009	0.21628	1.07501	5.395289	73.43
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
b+50%	0.01568	0.00433	0.01579	0.15456	1.932227	-37.89
+25%	0.05061	0.0004	0.05111	0.51665	2.447445	-21.33
-25%	0.18334	0.00797	0.23634	2.36682	6.120893	96.76
-50%	0.25219	1.75656e-08	0.30949	3.2049	7.640731	145.61
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$\alpha$ +50%	0.0385	0.0006	0.04224	0.42296	2.321306	-25.38
+25%	0.06397	0.00048	0.0641	0.64756	2.657555	-14.57
-25%	0.22057	0	0.23515	2.50397	5.965735	91.77
-50%	0.21578	0.00063	0.25534	2.68308	6.467604	107.90
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$\beta$ +50%	0.0097	0.00485	0.01143	0.11104	1.875583	-39.71
+25%	0.01357	8.99607e-06	0.01409	0.14131	1.907281	-38.69
-25%	0.13858	0.00018	0.22582	2.32784	5.744558	84.66
-50%	0.13753	0.05381	0.22841	2.14865	6.094459	95.91
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
g+50%	0.1835	0.00106	0.18605	1.92445	4.962780	59.53
+25%	0.14666	0.00016	0.15165	1.57261	4.209791	35.32
-25%	0.0476	0.00647	0.07945	0.78473	2.944793	-5.34
-50%	0.06418	0.06354	0.07202	0.69529	2.810594	-9.65
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
h+50%	0.16241	7.53765e-05	0.1703	1.78413	4.561850	46.64
+25%	0.12107	8.82149e-06	0.12133	1.25697	3.618809	16.33
-25%	0.04976	0.00048	0.05095	0.5053	2.465796	-20.74
-50%	0.00399	2.85496e-06	0.00561	0.05529	1.793881	-42.34
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$\delta$ +50%	0.13457	0.00312	0.21979	2.22471	8.573736	175.60
+25%	0.18229	0.00112	0.2438	2.52618	7.805720	150.92
-25%	0.0575	0.04547	0.05777	0.53207	0.847603	-72.75
-50%	0.13688	7.75801e-07	0.13721	1.42795	0.125831	-95.96
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
r+50%	0.06052	0.00257	0.06071	0.60552	0.291371	-90.63
+25%	0.10148	0.09103	0.10156	0.9121	2.092323	-32.74
-25%	0.04119	0.01069	0.04912	0.47564	4.128780	32.72
-50%	0.02596	0.00113	0.02854	0.28326	4.860219	56.23
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_o$ +50%	0.24473	0.03881	0.27045	2.67121	9.684907	211.32
+25%	0.05626	0.01722	0.0636	0.61104	3.944951	26.81
-25%	0.11112	0.01333	0.12679	1.24779	2.579238	-17.09
-50%	0.05134	0.00356	0.06876	0.68064	0.260522	-91.63
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_p$ +50%	0.09046	0.0009	0.18427	1.86175	5.830851	87.43
+25%	0.09466	4.17832e-06	0.09513	0.97749	3.402246	9.37
-25%	0.06988	7.45336e-07	0.07133	0.7279	2.580624	-17.05
-50%	0.0533	0.0243	0.06174	0.58544	2.374890	-23.66
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_h$ +50%	0.0901	6.79773e-05	0.25434	2.58897	6.898401	121.75
+25%	0.12022	0.00196	0.12131	1.23454	3.719978	19.58
-25%	0.07659	4.0168e-05	0.0771	0.78663	2.835932	-8.84
-50%	0.04295	0.00156	0.0617	0.61427	2.608485	-16.15

$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
$c_s +50\%$	0.09236	0.05854		0.10798	0.99612	1.803250	-42.03
+25%	0.08271	0.01522		0.08335	0.81115	2.146414	-31.00
-25%	0.03461	0.01074		0.03465	0.33469	3.159460	1.56
-50%	0.0772	0.01336		0.08218	0.80089	4.773249	53.44
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
$c_d +50\%$	0.20871	0.15781		0.21498	1.92602	5.805854	-55.39
+25%	0.14424	0.02775		0.17884	1.73677	4.946861	59.02
-25%	0.0777	0.06839		0.08634	1.78344	3.087571	-0.75
-50%	0.05905	0.00351		0.05909	1.58706	2.594540	-16.60
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
$c_1 +50\%$	0.20539	0.00309		0.24114	2.49648	6.383779	105.21
+25%	0.11426	0.10576		0.11605	1.03625	3.747681	20.47
-25%	0.05053	0.01751		0.05142	0.49275	2.349360	-24.48
-50%	0.05561	0.00015		0.05699	0.57655	2.262828	-27.26
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
$c_a +50\%$	0.13359	0.00137		0.16688	1.70578	4.580843	47.25
+25%	0.10618	0.00094		0.15469	1.57162	4.308193	38.49
-25%	0.08101	0.02322		0.08407	0.80668	3.017143	-3.01
-50%	0.04898	0.00079		0.06058	0.60727	2.600193	-16.42

### 5.3 Case 3 when $t_1 < t_2 < \mu$

We consider the values of the parameters in appropriate units such that  $a=10, b=0.5, \alpha=0.04, \beta=2.2, g=0.4, h=0.5, \delta=0.16, r=0.6, c_0=5, c_p=1, c_h=3, c_s=1, c_d=5, c_r=2, c_a=3, T=1$ . Optimal value of  $t_1, i.e., t_1^*=0.06315, \mu=1, t_2=0.14019, W=1.29054$ . The minimized total cost  $TC_3=1.205013$  is calculated from equ.(48).

#### 5.3.1. Sensitivity Analysis:

Table 3.Sensitivity analysis based on example when case 3with different parameters.

$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
a+50%	1	4.56439e-08		0.19287	2.90307	2.522166	109.31
+25%	1	0.01167		0.17841	2.14603	2.236127	85.57
-25%	1	0.00092		0.09184	0.6825	0.126330	-89.52
-50%	0.9995	0.00057		0.00801	0.0397	-0.997823	-182.81
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
b+50%	1	0.04091		0.07777	0.72915	-2.404196	-299.52
+25%	1	0.00093		0.084	0.8321	-1.567898	-230.11
-25%	0.99983	0.01623		0.14734	1.40685	1.915507	58.96
-50%	1	0.03053		0.14734	1.38308	2.504718	107.86
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
$\alpha+50\%$	1	0.04561		0.07492	0.70055	-1.136903	-194.35
+25%	1	0.12413		0.13227	1.19949	0.952947	-20.99
-25%	1	0.00069		0.34518	3.4482	7.564523	527.75
-50%	0.99986	4.15431e-14		0.53191	5.39746	13.169198	992.87
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
$\beta+50\%$	1	0.23886		0.27459	2.38726	5.905424	390.07
+25%	1	0.00054		0.1926	1.91759	2.740872	127.46
-25%	1	0.01135		0.03668	0.35337	-2.558105	-312.29
-50%	1	0.00093		0.00694	0.06858	-3.667661	-404.37
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
g+50%	1	1.58187e-05		0.00107	0.01065	-3.886639	-422.54
+25%	1	2.43071e-05		0.08564	0.85614	-0.856076	-171.04
-25%	0.99976	8.11067e-10		0.24193	2.43515	4.286349	255.71
-50%	1	4.52692e-05		0.34688	3.49821	7.560138	527.39
$\mu$	$t_1$	$t_2$	w	TC	PCI(%)		
h+50%	1	0.06837		0.33334	3.20416	7.393521	513.56
+25%	1	0.00019		0.18805	1.88634	2.567359	113.06
-25%	1	0.12084		0.12522	1.08929	0.772998	-35.85
-50%	1	0.00046		0.06628	0.62628	-1.458916	-221.07

	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$\delta+50\%$	1	0.21799	0.35558		3.08021	8.315032	590.04
+25%	0.99976	0.06047	0.2117		1.94663	3.451691	186.44
-25%	1	0.00646	0.08803		0.85505	-0.408207	-133.88
-50%	1	0.00796	0.04622		0.44743	-2.478417	-305.68
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$r+50\%$	1	0.12992	0.19466		1.74368	2.676524	122.12
+25%	1	0.01452	0.15899		1.52362	1.494389	24.01
-25%	1	0.0031	0.01883		0.18443	-3.058731	-353.83
-50%	1	0.01405	0.02902		0.27907	-4.788978	-497.42
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_o+50\%$	1	0.00069	0.13755		1.36606	3.429755	184.62
+25%	0.99984	0.00012	0.12708		1.26929	1.813235	50.47
-25%	0.99994	0.04989	0.11204		1.04052	-1.049521	-187.1
-50%	1	0.0049	0.00547		0.05365	-6.221350	-616.29
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_p+50\%$	1	0.00088	0.27738		2.76205	6.844612	468.01
+25%	1	0.04761	0.19141		1.77514	3.388805	181.23
-25%	1	0.04234	0.0589		0.55352	-1.862842	-254.59
-50%	1	0.00387	0.02477		0.24207	-3.125544	-359.38
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_h+50\%$	1	6.92159e-05	0.25121		2.52094	5.048351	318.95
+25%	1	0.00369	0.16321		1.6001	1.924328	59.69
-25%	1	0.03397	0.14126		1.32453	1.130643	-6.17
-50%	1	0.03053	0.04937		0.46729	-2.090747	-273.5
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_s+50\%$	1	0.04605	0.20872		1.93776	5.156855	327.95
+25%	1	2.98054e-08	0.15461		1.55225	2.397591	98.97
-25%	1	0.03759	0.13883		1.29797	0.158733	-86.83
-50%	1	0.00043	0.05152		0.51161	-4.403550	-465.44
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_d+50\%$	1	0.00042	0.2634		2.63125	5.007950	315.59
+25%	1	0.00149	0.17816		1.76269	2.293831	90.36
-25%	1	0.0044	0.10049		0.98154	-0.298230	-124.75
-50%	0.99974	0.00233	0.0603		0.59256	-1.71999	-242.74
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_1+50\%$	1	0.32389	0.45513		3.84264	10.275865	752.76
+25%	0.99992	0.03061	0.20243		1.90416	1.291785	7.20
-25%	0.99997	0.00077	0.00634		0.0627	-0.200685	-116.65
-50%	1	0.0013	0.00481		0.04745	3.260886	170.61
	$\mu$	$t_1$	$t_2$	w	TC	PCI(%)	
$c_a+50\%$	1	0.0006	0.21907		2.19264	3.596365	198.45
+25%	1	0.00034	0.15185		1.51333	1.406826	16.75
-25%	1	0.02316	0.06855		0.65044	-1.411178	-217.11
-50%	0.99999	8.79471e-05	0.00013		0.00127	-3.921597	-425.44

## 6. CONCLUSION

This work is an attempt for analyzing an order level inventory model for ameliorating items. The analysis helps us formulate a generic model for further work. Hence, all the efforts have been carefully directed towards the possible futuristic enhancements of the model. The demand rate has been generically selected to represent any function of time till the stabilization instant arrives (general ramp type demand rate), and the backlogging rate has been generically chosen to represent any non-increasing function of the waiting time, up to the next replenishment. The inventory model has been analyzed for the scenarios of replenishment policy, starting with no shortages. The optimal replenishment policy for the model is derived for the above mentioned inventory system. A numerical example is also presented to illustrate this model.

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