

# A Comparative Study of the Performance of Wavelet Decomposition of Images using Kekre’s Transform vis-a-vis Haar and Daubechies

Bijith Marakarkandy  
 TCET, Kandivali (E)  
 Mumbai 400101

Dr. B K Mishra  
 TCET ,Kandivali (E)  
 Mumbai 400101

## ABSTRACT

In recent years Image and Signal compression have been receiving a lot of attention by scientists and researchers in order to improve storage and transmission capabilities. In this study we have compared the performance of Kekre’s wavelet with other wavelets viz. Haar and Daubechies 2 with respect to energy in the Low-Low (LL ), Low-High (LH) ,High-Low (HL) and High-High (HH) bands. The energy distribution in each band is an indicator of the performance of the transform for image compression. It is found that the percentage of energy in the low-low band is equal for Kekre’s and Haar wavelet. The daubechies 2 wavelet although offers a slightly higher energy compaction but the tradeoff is that the order of the filter being higher and therefore computational burden increases.

## General Terms

Experimentation, Documentation, Performance, Design.

## Keywords

Haar Wavelet, Kekre’s Wavelet, Daubechies Wavelet.

## 1. INTRODUCTION

Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. Wavelets were developed independently in the fields of mathematics, quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction [1].

The basic concept in wavelet is to use a mother wavelet and then perform analysis using shifted and dilated versions of this mother wavelet [2]

Wavelet Transforms use different window functions to analyse different frequency bands of the signal  $x(t)$ . Different window functions  $\psi(s,b,t)$ ; which are also called daughter wavelets they can be generated by dilation or compression of a mother wavelet  $\psi(t)$ , at different time frame. A scale is the inverse of its corresponding frequency. A continuous type of wavelet transform (CWT) that is applied to the signal  $x(t)$  can be defined as,

$$w(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \Psi(t - b)/a dt \quad (1)$$

Where

$a$  is the dilation factor,

$b$  is the translation factor and

$\psi(t)$  is the mother wavelet.

$1/\sqrt{a}$  is an energy normalization term that makes wavelets of different scale have the same amount of energy.

## 2. METHODOLOGY

We have used three test images viz. lena, rice and one band of a multispectral image of San Francisco bay area for comparison of the performance of Kekre’s transform with Haar and Daubechies wavelets.

### THE KEKRE’S MATRIX [3]

Kekre’s matrix can be generated using the expression

$$K_{xy} = \begin{cases} 1 & ; x \leq y \\ -N + (x-1) & ; x = y + 1 \\ 0 & ; x > y + 1 \end{cases} \quad (2)$$

A 2x2 Kekre’s matrix obtained from the above expression is

$$K(2) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \quad (3)$$

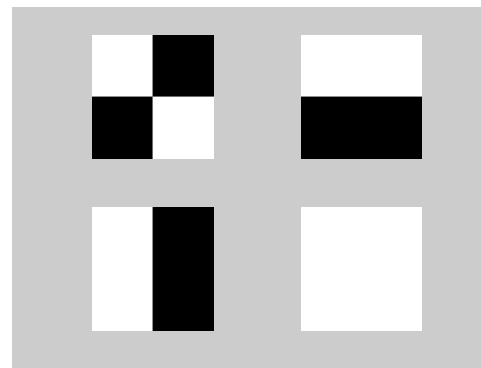


Figure 1. Two dimensional Kekre’s basis functions (N=2)

A 3x3 Kekres matrix obtained from the above expression is

$$\begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \quad (4)$$

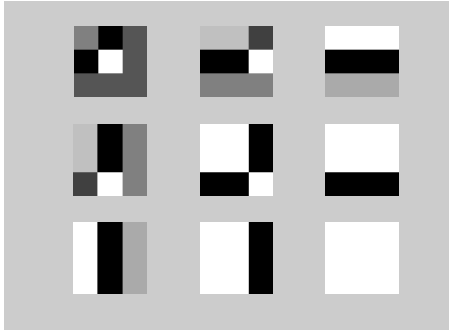


Figure 2 Two dimensional Kekre's basis functions (N=3)

### 3. PROPERTIES OF KEKRE'S WAVELET TRANSFORM

#### 3.1 Orthogonal

The transform matrix K is said to be orthogonal if the following condition is satisfied.

$$[K][K]^T = [D] \quad (5)$$

Where D is the diagonal matrix.

Kekre's Wavelet Transform matrix satisfies this property and hence it is orthogonal.

The diagonal matrix value of Kekre's transform matrix of size NxN can be computed as

$$D(x,y) = \begin{cases} 2 & ; \text{ if } x = y=N \\ N & ; \text{ if } x = y = 1 \\ 0 & ; \text{ if } x \neq y \\ D(x+1,y+1) + 2(N-x+1) & ; \text{ if } x=y=p \text{ and } p \neq 1 \text{ or } N \end{cases} \quad (6)$$

#### Asymmetric

As the Kekre's transform is upper triangular matrix, it is asymmetric.

#### 3.2 Non-Involutorial

An involutorial function is a function that is its own inverse. So involutorial transform is a transform which is inverse transform of itself. Kekre's transform is non involutorial transform.

#### 3.3 Transform on Vector

The Kekre's Wavelet transform on a column vector f is given by

$$F = [KW] f \quad (7)$$

And inverse is given by

$$f = [KW]^T [D]^{-1} F \quad (8)$$

#### 3.4 Transform on 2D Matrix

Kekre's Wavelet transform on 2D matrix f is given by

$$[F] = [KW] [f] [KW]^T \quad (9)$$

Obtaining Inverse: Calculate Diagonal matrix D as,

$$[D] = [KW][KW]^T \quad (10)$$

D1	0	0	0	0	0
0	D2	0	0	0	0
0	0	D3	0	0	0
0	0	0	...	0	0
0	0	0	0	...	0
0	0	0	0	0	DN

Inverse is calculated as

$$[f] = [KW]^T [F_{ij} / D_{ij}] [KW] \quad (11)$$

Where  $D_{ij} = D_i * D_j$  ;  $1 \leq i \leq N$  and  $1 \leq j \leq N$

### 4. DECOMPOSITION USING WAVELETS

The basic idea of subband coding (SBC) is to split up the frequency band of the signal and then to code each subband using a coder and bit rate accurately matched to the statistics of that band [4]. In Subband coding of images each row of the image is convolved with the coefficients of the LPF and HPF and downsampled by two, then we work on the image column wise and again downsample by two. If the original image is of size NxN after first level decomposition we obtain four images of size N/2 x N/2. It is usually found that out of the four sub images the LL subimage has most of the energy and other subimages have energy close to zero. Therefore the LL subimage may be decomposed further and the other subimages are usually not decomposed further.

#### 4.1 Kekre's wavelet [5]

The first row of the Kekre's forward transform matrix is a Low Pass filter. The frequency response of the filter is shown in Fig. 3. This row is used as the impulse response of the low pass decomposition filter.

The impulse response of Kekres analysis filters are

$$h_0[n] = [1, 1] \text{ LPF} \quad (12)$$

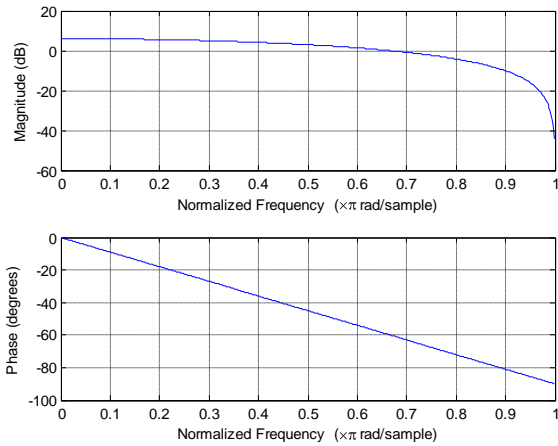
$$h_1[n] = [-1, 1] \text{ HPF} \quad (13)$$

$$|H_0(e^{jw})| = \sqrt{2 + 2\cos w} \quad (14)$$

$$|H_1(e^{jw})| = \sqrt{2 - 2\cos w} \quad (15)$$

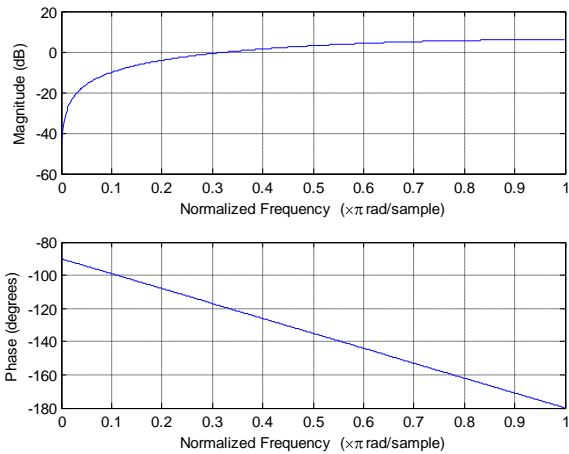
$$|H_0(0)|=2, |H_0(\pi/2)|=\sqrt{2}, |H_0(\pi)|=0 \text{ and } |H_1(0)|=0 \quad (16)$$

$$|H_1(\pi/2)|=\sqrt{2}, |H_1(\pi)|=2 \quad (17)$$

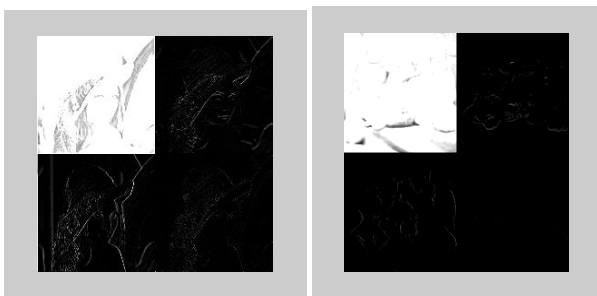


**Figure 3. Frequency Response of Kekres Lowpass Decomposition Filter**

The second row of the Kekres forward transform matrix is a High pass filter and its frequency response is as shown Fig.4 . This row is used as the impulse response of the high pass decomposition filter



**Figure 4. Frequency Response of Kekres Highpass decomposition filter**



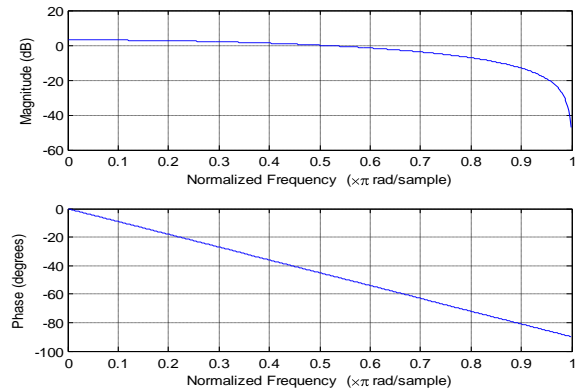
**Figure 5 One level decomposition using Kekres wavelet transform**

### 4.2 Haar Wavelet

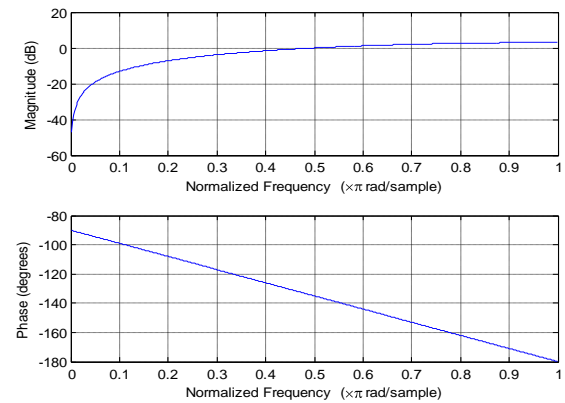
The impulse response of Haar analysis filters are

$$h_0[n] = [ 0.7071 \quad 0.7071 ] \text{ LPF} \quad (18)$$

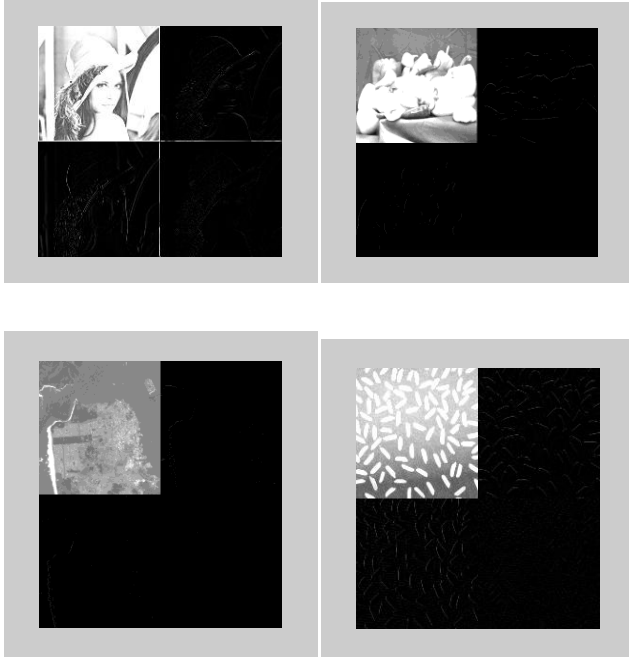
$$h_1[n] = [ -0.7071 \quad 0.7071 ] \text{ HPF} \quad (19)$$



**Figure 6 Frequency response of the LPF used in the Haar analysis filter bank.**



**Figure 7 Frequency response of the HPF used in Haar analysis filter bank**



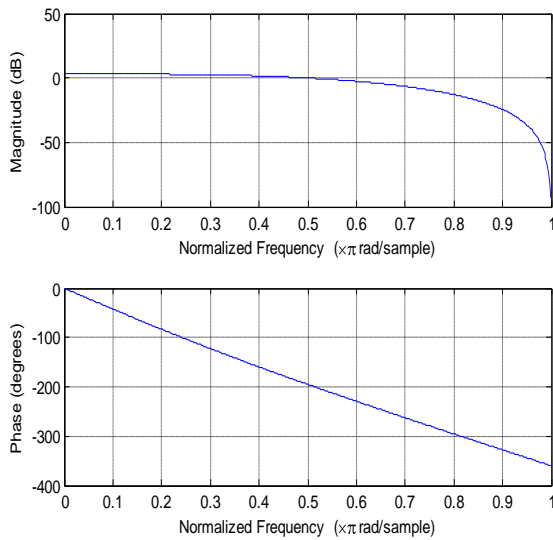
**Figure 8 : One level decomposition using Haar wavelet transform.**

### 4.3 Daubechies Wavelet

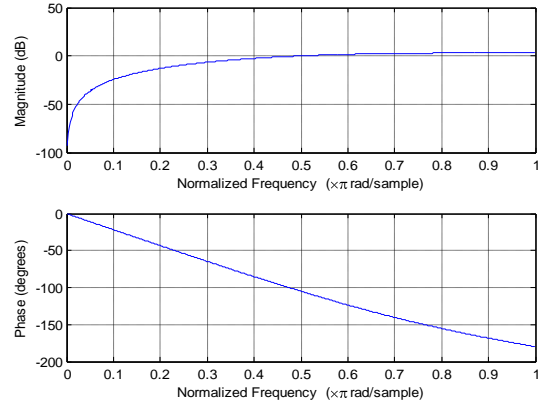
The impulse response of Daubechies 2 analysis filters are

$$h_0[n] = [-0.1294 \quad 0.2241 \quad 0.8365 \quad 0.4830] \text{ LPF} \quad (20)$$

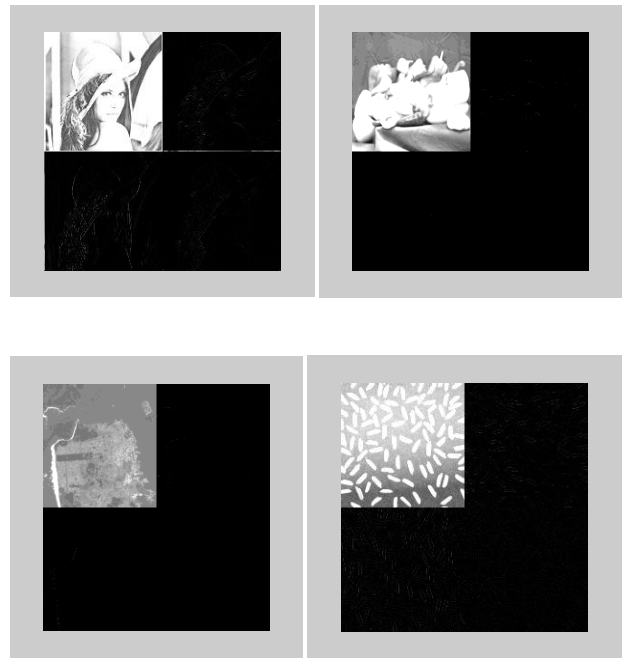
$$h_1[n] = [-0.4830 \quad 0.8365 \quad -0.2241 \quad -0.1294] \text{ HPF} \quad (21)$$



**Figure 9 Frequency response of the LPF used in the Daubechies 2 analysis filter bank**



**Figure 10 Frequency response of the HPF used in the Daubechies 2 analysis filter bank.**



**Figure 11:One level decomposition using Daubechies wavelet transform.**

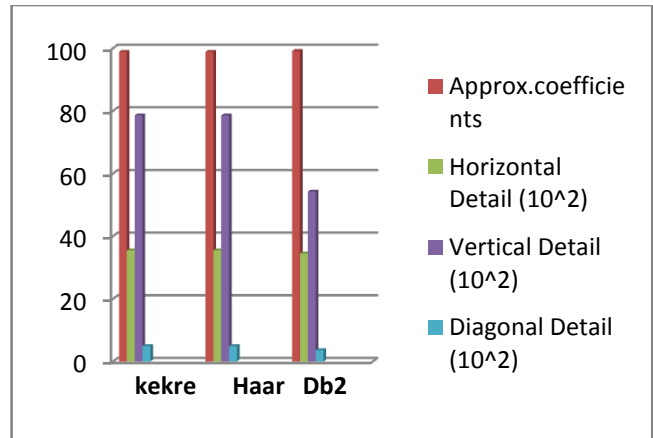
To perform image compression using the transform coding method is to take an image and apply a transform on the image to change its representation. The original image usually has its energy scattered throughout the image . The objective of taking the transform is to pack a lot of energy into as few pixels as possible. Energy compaction capability of the transform is a very useful property which allows allocation of more bits to high energy coefficients which occur in the LL band and fewer bits to low energy coefficients which are quantized and made zero.

When the image is decomposed using wavelets transform is taken most of the energy in the image relocates to the top LL band. Higher order filters are more efficient in decomposition than filters with lower order . However higher the order the number of computation i.e. multiply-add increases.

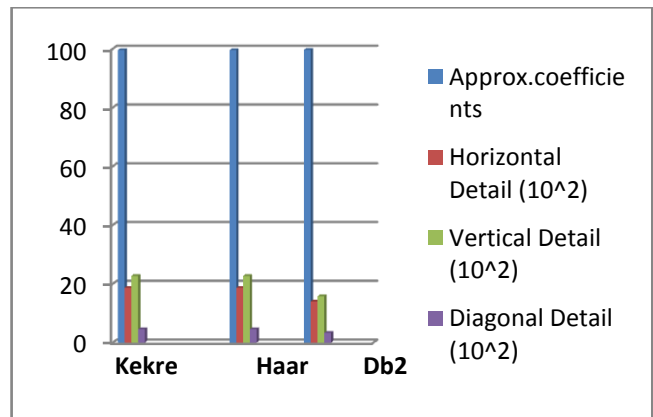
## 5. RESULTS AND DISCUSSION

**Table 1 Comparison of Wavelet Coefficients using Kekre , Haar and Db2 for Different Images.**

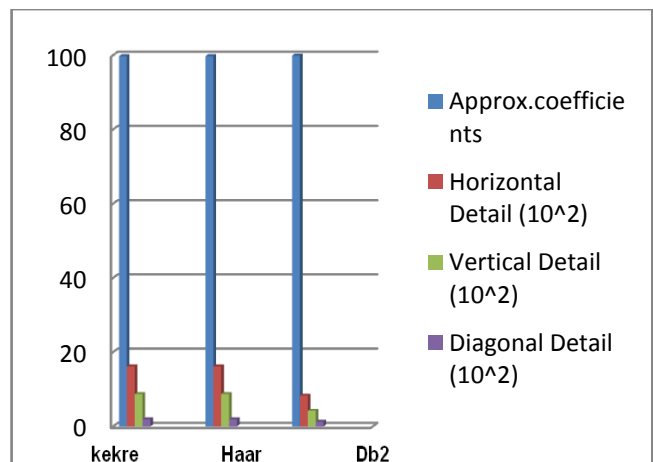
Image Name	Wavelet Name	Approx. coefficients	Horizontal Detail	Vertical Detail	Diagonal Detail
Lena	Kekre	98.8116	0.3544	0.7850	0.0490
	Haar	98.8116	0.3544	0.7850	0.0490
	Db2	99.0780	0.3444	0.5418	0.0358
Peppers	Kekre	99.7323	0.1615	0.0871	0.0191
	Haar	99.7323	0.1615	0.0871	0.0191
	Db2	99.8625	0.0829	0.0420	0.0126
SFO	Kekre	99.5410	0.1863	0.2269	0.0459
	Haar	99.5410	0.1863	0.2269	0.0459
	Db2	99.6693	0.1398	0.1577	0.0332
Rice	Kekre	99.2723	0.2711	0.3681	0.0884
	Haar	99.2723	0.2711	0.3681	0.0884
	Db2	99.5351	0.1516	0.2439	0.0693



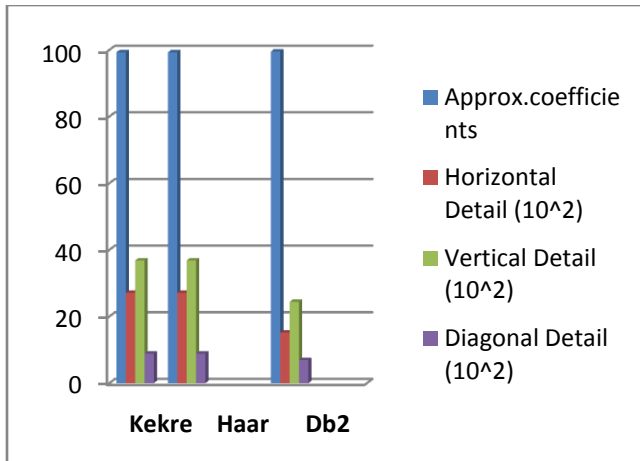
**Figure 12: Wavelet vs. Energy for image Lena**



**Figure 13: Wavelet vs. Energy for image SFO**



**Figure 14: Wavelet vs. Energy for image Peppers**



**Figure 15: Wavelet vs. Energy for image Rice**

From the Table. 1 it is found that maximum energy in all images are in the approximate coefficients, but the amount of energy in the LL band is image dependent. In all images the energy in LL, LH, HL and HH band when kekres transform is used is same as that when Haar transform is used. In case of Daubechies wavelet the energy in the LL band is more compared to Kekres and Haar transform and then energy drastically reduces. The energy in other bands is proportional to the type of edges i.e. Horizontal, Vertical, diagonal present in the respective image.

## 6. CONCLUSION

The performance of Kekre’s wavelet transform is the same as Haar transform with respect to energy compaction ability .The advantage of Kekre’s wavelet over Haar is that the coefficients of the Kekres matrix are integers and computational burden and quantization problems are less in this transform. Another advantage is that Kekres matrix can be generated for any order unlike Haar where the order has to be an integer power of two (i.e. 2, 4, 8,16....)

## 7. REFERENCES

- [1] Amara Grap at: <http://www.amara.com/IEEEwave/IEEEwavelet.html>, accessed on August 9, 2011.
- [2] A.W.Galli, G.T.Heydt, P.F.Rebiero , Exploring the power of wavelet analysis,IEEE computer applicationsin power, October 1996.
- [3] H. B. Kekre, Archan Athawale,Dipali Sadavarti, Algorithm to Generate Kekre’s Wavelet Transform from Kekre’s Transform,IJSET, June 2010.
- [4] Subband coding of images, John W.Woods, Sean D. Oneil, IEEE transactions on acoustic speech and signal processing, vol - assp 34.
- [5] Perfect reconstruction two-channel linear phase quadrature-mirror filter (QMF) bank using Kekre’s transform matrix, B.Marakarkandy, B.K.Mishra, pp 180-184 , ICWET 2011