Intuitionistic L-Fuzzy M-Subgroups

P.Pandiammal
Department of Maths,
PSNA CET,
Dindigul-624622.

R.Natarajan
HOD, Department of Maths, Alagappa University,
Karaikudi-630 004.

N.Palaniappan
Department of Maths, Alagappa University, Karaikudi-630 004.

ABSTRACT
This paper contains some definitions and results in intuitionistic L-fuzzy M-subgroups, which are required in the sequel. Some properties of homomorphism and anti-homomorphism of intuitionistic L-fuzzy M-subgroups are also established.

Keywords
L-fuzzy set, L-fuzzy M-subgroup, Homomorphism, Anti-homomorphism, Anti L-fuzzy M-subgroup, intuitionistic L-fuzzy M-subgroups, strongest intuitionistic L-fuzzy relation.

1. INTRODUCTION:
The notion of fuzzy sets was introduced by **L.A. Zadeh** [9]. Fuzzy set theory has been developed in many directions by many researchers and has evoked great interest among mathematicians working in different fields of mathematics, such as topological spaces, functional analysis, loop, group, ring, near ring, vector spaces, automation. In 1971, **Rosenfield** [1] introduced the concept of fuzzy subgroup. Motivated by this, many mathematicians started to review various concepts and theorems of abstract algebra in the broader frame work of fuzzy settings. In [2], **Biswas** introduced the concept of anti-fuzzy subgroups of groups. **Palaniappan. N** and **Muthuraj,** [6] defined the homomorphism, anti-homomorphism of a fuzzy and an anti-fuzzy subgroups. **Pandiammal. P, Natarajan. R** and **Palaniappan. N,** [8] defined the homomorphism, anti-homomorphism of an anti L-fuzzy M-subgroup. In this paper we define a new algebraic structure of intuitionistic L-fuzzy M-subgroup of M-groups and study some their related properties.

2. PRELIMINARIES:
2.1 Definition: Let G be a M-group. A L-fuzzy subset A of G is said to be **anti L-fuzzy M-subgroup** (ALFMSG) of G if its satisfies the following axioms:

(i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$,
(ii) $\mu_A(x^{-1}) \leq \mu_A(x)$, for all x and y in G.

2.2 Definition: Let $(G, \cdot)$ be a M-group. An intuitionistic L-fuzzy subset A of G is said to be an **intuitionistic L-fuzzy M-subgroup (ILFMSG)** of G if the following conditions are satisfied:

(i) $\mu_A(mxy) \geq \mu_A(x) \wedge \mu_A(y)$,
(ii) $\mu_A(x^{-1}) \geq \mu_A(x)$,
(iii) $\nu_A(mxy) \leq \nu_A(x) \vee \nu_A(y)$,
(iv) $\nu_A(x^{-1}) \leq \nu_A(x)$, for all x and y in G.

2.3 Definition: Let $(G, \cdot)$ and $(G^{l}, \cdot)$ be any two M-groups. Let $f : G \rightarrow G^{l}$ be any function and A be an intuitionistic L-fuzzy M-subgroup in G, V be an intuitionistic L-fuzzy M-subgroup in $f(G)=G^{l}$, defined by $\mu_V(y) = \sup_{x\in f^{-1}(y)} \mu_A(x)$ and $\nu_V(y) = \inf_{x\in f^{-1}(y)} \nu_A(x)$, for all x in G and y in G^{l}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

2.4 Definition: Let A and B be any two intuitionistic L-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by AxB, is defined as $A\mathrm{x}B = \{ \langle (x, y), \mu_{A\mathrm{x}B}(x, y) , \nu_{A\mathrm{x}B}(x, y) \rangle$ / for all x in G and y in H $\}$, where $\mu_{A\mathrm{x}B}(x, y) = \mu_A(x) \wedge \mu_B(y)$ and $\nu_{A\mathrm{x}B}(x, y) = \nu_A(x) \vee \nu_B(y)$.

2.5 Definition: Let A and B be any two intuitionistic L-fuzzy M-subgroups of a M-group $(G, \cdot)$. Then A and B are said to be **conjugate intuitionistic L-fuzzy M-subgroups** of G if for some g in G, $\mu_A(x) = \mu_B(g^{-1}xg)$ and $\nu_A(x) = \nu_B(g^{-1}xg)$, for every x in G.

2.6 Definition: Let A be an intuitionistic L-fuzzy subset in a set S, the **strongest intuitionistic L-fuzzy relation** on S, that is an intuitionistic L-fuzzy relation on A is V given by $\mu_V(x, y) = \mu_A(x) \wedge \mu_A(y)$ and $\nu_V (x, y) = \nu_A(x) \vee \nu_A(y)$, for all x and y in S.

3 – PROPERTIES OF INTUITIONISTIC L-FUZZY M-SUBGROUPS:

3.1 Theorem: If A is an intuitionistic L-fuzzy M-subgroup of a M-group $(G, \cdot)$, then $\mu_A(x^{-1}) = \mu_A(x)$ and $\nu_A(x^{-1}) = \nu_A(x)$, $\mu_A(x) \leq \mu_A(e)$ and $\nu_A(x) \geq \nu_A(e)$, for x in G, where e is the identity element in G.

Proof: For x in G and e is the identity element in G.
Now, $\mu_A(x) = \mu_A((x^{-1})^{-1}) \geq \mu_A(x^{-1}) \geq \mu_A(x)$.
Therefore, $\mu_A(x^{-1}) = \mu_A(x)$.
And, $\nu_A(x) = \nu_A((x^{-1})^{-1}) \leq \nu_A(x^{-1}) \leq \nu_A(x)$.
Therefore, $\nu_A(x^{-1}) = \nu_A(x)$, for all x in G.
Now, $\mu_A(e) = \mu_A(xx^{-1}) \geq \mu_A(x) \wedge \mu_A(x^{-1}) = \mu_A(x)$.
Therefore, $\mu_A(e) \geq \mu_A(x)$.
And, $\nu_A(e) = \nu_A(xx^{-1}) \leq \nu_A(x) \vee \nu_A(x^{-1}) = \nu_A(x)$. Therefore, $\nu_A(e) \leq \nu_A(x)$, for all x in G.

3.2 Theorem: If A is an intuitionistic L-fuzzy M-subgroup of a M-group (G, ·), then
(i) $\mu_A(xy^{-1}) = \mu_A(e)$ gives $\mu_A(x) = \mu_A(y)$,
(ii) $\nu_A(xy^{-1}) = \nu_A(e)$ gives $\nu_A(x) = \nu_A(y)$, for x & y in G, where e is the identity element in G.
Proof: Let x & y in G and e is the identity element in G.
Now, $\mu_A(x) = \mu_A(xy^{-1}y)$
$\geq \mu_A(xy^{-1}) \wedge \mu_A(y)$
$= \mu_A(e) \wedge \mu_A(y)$
$= \mu_A(y) = \mu_A(yx^{-1}x)$
$\geq \mu_A(yx^{-1}) \wedge \mu_A(x)$
$= \mu_A(e) \wedge \mu_A(x)$
$= \mu_A(x)$.
Therefore,$\mu_A(x) = \mu_A(y)$, for all x and y in G.
And, $\nu_A(x) = \nu_A(xy^{-1}y)$
$\leq \nu_A(xy^{-1}) \vee \nu_A(y)$
$= \nu_A(e) \vee \nu_A(y)$
$= \nu_A(y)$
$= \nu_A(yx^{-1}x)$
$\leq \nu_A(yx^{-1}) \vee \nu_A(x)$
$= \nu_A(e) \vee \nu_A(x)$
$= \nu_A(x)$.
Therefore, $\nu_A(x) = \nu_A(y)$, for all x and y in G.

3.3 Theorem: A is an intuitionistic L-fuzzy M-subgroup of a M-group (G, ·) if and only if $\mu_A(mxy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(mxy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$, for all x and y in G.
Proof: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group (G, ·).
Then, $\mu_A(mxy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1})$
$\geq \mu_A(x) \wedge \mu_A(y$
Therefore, $\mu_A(mxy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$, for all x & y in G.
And, $\nu_A(mxy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1})$
$\leq \nu_A(x) \vee \nu_A(y)$, since A is an ILFMSG of G.
Therefore, $\nu_A(mxy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$, for all x & y in G.
Conversely, if $\mu_A(xy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$,
replace y by x, then, $\mu_A(x) \leq \mu_A(e)$ and $\nu_A(x) \geq \nu_A(e)$, for all x and y in G.
Now, $\mu_A(x^{-1}) = \mu_A(ex^{-1})$
$\geq \mu_A(e) \wedge \mu_A(x) = \mu_A(x)$.
Therefore, $\mu_A(x^{-1}) \geq \mu_A(x)$.
It follows that, $\mu_A(xy) = \mu_A(x(y^{-1})^{-1})$
$\geq \mu_A(x) \wedge \mu_A(y^{-1})$
$\geq \mu_A(x) \wedge \mu_A(y)$.
Therefore, $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$, for all x and y in G.
And, $\nu_A(x^{-1}) = \nu_A(ex^{-1})$
$\leq \nu_A(e) \vee \nu_A(x)$
$= \nu_A(x)$.
Therefore, $\nu_A (x^{-1}) \leq \nu_A(x)$.
Then, $\nu_A(xy) = \nu_A(x(y^{-1})^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1})$
$\leq \nu_A(x) \vee \nu_A(y)$.
Therefore, $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$, for all x and y in G.
Hence A is an intuitionistic L-fuzzy M-subgroup of G.

3.4 Theorem: Let A be an intuitionistic L-fuzzy subset of a group (G, ·). If $\mu_A(e) = 1$ and $\nu_A(e) = 0$ and $\mu_A(mxy^{-1}) \geq \mu_A(x) \wedge \mu_A(y)$ } and $\nu_A(mxy^{-1}) \leq \nu_A(x) \vee \nu_A(y)$, , for all x and y in G, then A is an intuitionistic L-fuzzy M-subgroup of a M-group G.
Proof: Let x & y in G and e is the identity element in G.
Now, $\mu_A(x^{-1}) = \mu_A(ex^{-1}) \geq \mu_A(e) \wedge \mu_A(x)$
$= 1\wedge \mu_A(x) = \mu_A(x)$
Therefore, $\mu_A(x^{-1}) \geq \mu_A(x)$, for all x in G.
And $\nu_A(x^{-1}) = \nu_A(ex^{-1})$
$\leq \nu_A(e) \vee \nu_A(x)$
$= 0 \vee \nu_A(x)$
$= \nu_A(x)$.
Therefore, $\nu_A(x^{-1}) \leq \nu_A(x)$, for all x in G.
Now, $\mu_A(mxy) = \mu_A(x(y^{-1})^{-1})$
$\geq \mu_A(x) \wedge \mu_A(y^{-1})$
$\geq \mu_A(x) \wedge \mu_A(y)$.
Therefore, $\mu_A(mxy) \geq \mu_A(x) \wedge \mu_A(y)$, for all x and y in G.
And, $\nu_A(mxy) = \nu_A(x(y^{-1})^{-1})$
$\leq \nu_A(x) \vee \nu_A(y^{-1})$
$\leq \nu_A(x) \vee \nu_A(y)$.
Therefore, $\nu_A(mxy) \leq \nu_A(x) \vee \nu_A(y)$, for all x and y in G.
Hence A is an intuitionistic L-fuzzy M-subgroup of a M-group G.

3.5 Theorem: If A is an intuitionistic L-fuzzy M-subgroup of a M-group (G, ·), then $H = \{x / x\in G : \mu_A(x) = 1, \nu_A(x) = 0\}$ is either empty or is a M-subgroup of a M-group G.
Proof: If no element satisfies this condition, then H is empty.
If x and y in H, then $\mu_A(mxy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1})$
$\geq {}_A(x) \wedge \mu_A(y) = 1$.
Therefore, $\mu_A(xy^{-1}) = 1$, for all x and y in G.
And, $\nu_A(xy^{-1}) \leq \nu_A(x) \vee \nu_A(y^{-1})$
$= \nu_A(x)\vee\nu_A(y)$, since A is an ILFMSG of G
$= 0 \vee 0 = 0$.
Therefore, $\nu_A(xy^{-1}) = 0$, for all x and y in G. We get mxy^{-1} in H.
Therefore, H is a M-subgroup of a M-group G.
Hence H is either empty or is a M-subgroup of M-group G.

3.6 Theorem: If A is an intuitionistic L-fuzzy M-subgroup of a M-group (G, ·), then H= { $x\in G$: $\mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)$ } is either empty or is a M-subgroup of a M-group G.
Proof: If no element satisfies this condition, then H is empty.
If x and y satisfies this condition, then $\mu_A(x^{-1}) = \mu_A(x) = \mu_A(e)$, $\nu_A(x^{-1}) = \nu_A(x) = \nu_A(e)$, by Theorem 2.1.
Therefore, $\mu_A(x^{-1}) = \mu_A(e)$ and $\nu_A(x^{-1}) = \nu_A(e)$.
Hence x^{-1} in H.
Now, $\mu_A(mxy^{-1}) \geq \mu_A(x) \wedge \mu_A(y^{-1})$
$\geq \mu_A(x) \wedge \mu_A(y)$
$= \mu_A(e) \wedge \mu_A(e) = \mu_A(e)$.
Therefore, $\mu_A(mxy^{-1}) \geq \mu_A(e)$, for all x and y in G---(1).
And, $\mu_A(e) = \mu_A((xy^{-1})(xy^{-1})^{-1})$
$\geq \mu_A(xy^{-1}) \wedge \mu_A((xy^{-1})^{-1})$
$\geq \mu_A(xy^{-1}) \wedge \mu_A(xy^{-1})$
$= \mu_A(xy^{-1})$.
Therefore, $\mu_A(e) \geq \mu_A(xy^{-1})$, for all x and y in G----(2).
From (1) and (2), we get $\mu_A(e) = \mu_A(xy^{-1})$.

Now, $\nu_A(mxy^{-1}) \le \nu_A(x) \vee \nu_A(y^{-1})$
$\le \nu_A(x) \vee \nu_A(y)$
$= \nu_A(e) \vee \nu_A(e) = \nu_A(e)$.
Therefore, $\nu_A(mxy^{-1}) \le \nu_A(e)$, for all x and y in G----(3).
And, $\nu_A(e) = \nu_A((xy^{-1})(xy^{-1})^{-1})$
$\le \nu_A(xy^{-1}) \vee \nu_A((xy^{-1})^{-1})$
$\le \nu_A(xy^{-1}) \vee \nu_A(xy^{-1})$
$= \nu_A(xy^{-1})$.
Therefore, $\nu_A(e) \le \nu_A(xy^{-1})$, for all x and y in G----- (4).
From (3) and (4), we get $\nu_A(e) = \nu_A(xy^{-1})$.
Hence $\mu_A(e) = \mu_A(xy^{-1})$ and $\nu_A(e) = \nu_A(xy^{-1})$.
Therefore, mxy^{-1} in H.
Hence H is either empty or is a M-subgroup of a M-group G.

3.7 Theorem: Let (G, ·) be a M-group. If A is an intuitionistic L-fuzzy M-subgroup of G, then $\mu_A(xy) = \mu_A(x) \wedge \mu_A(y)$ and $\nu_A(xy) = \nu_A(x) \vee \nu_A(y)$ with $\mu_A(x) \ne \mu_A(y)$ and $\nu_A(x) \ne \nu_A(y)$, for each x and y in G.
Proof: Let x and y belongs to G.
Assume that $\mu_A(x) > \mu_A(y)$ and $\nu_A(x) < \nu_A(y)$.
Now, $\mu_A(y) = \mu_A(x^{-1}xy)$
$\ge \mu_A(x^{-1}) \wedge \mu_A(xy)$
$\ge \mu_A(x) \wedge \mu_A(xy)$
$= \mu_A(xy)$
$\ge \mu_A(x) \wedge \mu_A(y)$
$= \mu_A(y)$.
Therefore, $\mu_A(xy) = \mu_A(y) = \mu_A(x) \wedge \mu_A(y)$, for all x and y in G.
And, $\nu_A(y) = \nu_A(x^{-1}xy)$
$\le \nu_A(x^{-1}) \vee \nu_A(xy)$
$\le \nu_A(x) \vee \nu_A(xy) = \nu_A(xy)$
$\le \nu_A(x) \vee \nu_A(y) = \nu_A(y)$.
Therefore, $\nu_A(xy) = \nu_A(y) = \nu_A(x) \vee \nu_A(y)$, for all x and y in G.

3.8 Theorem: If A and B are two anti L-fuzzy M-subgroups of a M-group (G, ·), then their union A∪B is an anti L-fuzzy M-subgroup of G.
Proof: Let x and y belong to G,
A = { ⟨ x, $\mu_A(x)$ ⟩ / x∈G } and
B = { ⟨ x, $\mu_B(x)$ ⟩ / x∈G }. Let C = A∪B and
C = { ⟨ x, $\mu_C(x)$ ⟩ / x∈G }.
(i) $\mu_C(mxy) = \mu_A(mxy) \vee \mu_B(mxy)$
$\le \{ \mu_A(x) \vee \mu_A(y) \} \vee \{ \mu_B(x) \vee \mu_B(y) \}$
$\le \{ \mu_A(x) \vee \mu_B(x) \} \vee \{ \mu_A(y) \vee \mu_B(y) \}$
$= \mu_C(x) \vee \mu_C(y)$.
Therefore, $\mu_C(xy) \le \mu_C(x) \vee \mu_C(y)$, for all x and y in G.
(ii) $\mu_C(x^{-1}) = \mu_A(x^{-1}) \vee \mu_B(x^{-1}) \le \mu_A(x) \vee \mu_B(x) = \mu_C(x)$.
Therefore, $\mu_C(x^{-1}) \le \mu_C(x)$, for all x in G.
Hence A∪B is an anti L-fuzzy M-subgroup of a M-group G.

3.9 Theorem: The union of a family of anti L-fuzzy M-subgroups of a M-group (G, ·) is an anti L-fuzzy M-subgroup of a M-group G.
Proof: Let $\{ A_i \}_{i\in I}$ be a family of anti L-fuzzy M-subgroups of a M-group G and let $A = \cup A_i$. Then for x and y belong to G, we have

(i) $\mu_A(mxy) = \sup_{i\in I} \mu_{A_i}(mxy) \le \sup_{i\in I} \{ \mu_{A_i}(x) \vee \mu_{A_i}(y) \}$
$\le \sup_{i\in I} (\mu_{A_i}(x)) \vee \sup_{i\in I} (\mu_{A_i}(y))$
$= \mu_A(x) \vee \mu_A(y)$.
Therefore, $\mu_A(mxy) \le \mu_A(x) \vee \mu_A(y)$, for all x and y in G.
(ii) $\mu_A(x^{-1}) = \sup_{i\in I} \mu_{A_i}(x^{-1}) \le \sup_{i\in I} \mu_{A_i}(x) = \mu_A(x)$.
Therefore, $\mu_A(x^{-1}) \le \mu_A(x)$, for all x in G. Hence the union of a family of anti L-fuzzy M-subgroups of a M-group G is an anti L-fuzzy M-subgroup of G.

3.10 Theorem: If A is an intuitionistic L-fuzzy M-subgroup of a M-group G, then (i) $\mu_A(xy) = \mu_A(yx)$ if and only if $\mu_A(x) = \mu_A(y^{-1}xy)$, (ii) $\nu_A(xy) = \nu_A(yx)$ if and only if $\nu_A(x) = \nu_A(y^{-1}xy)$, for x and y in G.
Proof: Let x and y be in G.
Assume that $\mu_A(xy) = \mu_A(yx)$, we have
$\mu_A(y^{-1}xy) = \mu_A(y^{-1}yx) = \mu_A(ex) = \mu_A(x)$.
Therefore, $\mu_A(x) = \mu_A(y^{-1}xy)$, for all x and y in G.
Conversely, assume that $\mu_A(x) = \mu_A(y^{-1}xy)$,
we have $\mu_A(xy) = \mu_A(xyxx^{-1}) = \mu_A(yx)$.
Therefore, $\mu_A(xy) = \mu_A(yx)$, for all x and y in G.
Hence (i) is proved
Now, we assume that $\nu_A(xy) = \nu_A(yx)$,
we have $\nu_A(y^{-1}xy) = \nu_A(y^{-1}yx) = \nu_A(ex) = \nu_A(x)$.
Therefore, $\nu_A(x) = \nu_A(y^{-1}xy)$, for all x and y in G.
Conversely, we assume that $\nu_A(x) = \nu_A(y^{-1}xy)$,
we have $\nu_A(xy) = \nu_A(xyxx^{-1}) = \nu_A(yx)$.
Therefore, $\nu_A(xy) = \nu_A(yx)$, for all x and y in G.
Hence (ii) is proved.

3.11 Theorem: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group G. If $\mu_A(x) < \mu_A(y)$ and $\nu_A(x) > \nu_A(y)$, for some x and y in G, then (i) $\mu_A(xy) = \mu_A(x) = \mu_A(yx)$,
(ii) $\nu_A(xy) = \nu_A(x) = \nu_A(yx)$, for all x and y in G .
Proof: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group G.
Also we have $\mu_A(x) < \mu_A(y)$ and $\nu_A(x) > \nu_A(y)$, for some x and y in G,
$\mu_A(xy) \ge \mu_A(x) \wedge \mu_A(y)$ (as A is an ILFMSG of G)
$= \mu_A(x)$; and
$\mu_A(x) = \mu_A(xyy^{-1})$
$\ge \mu_A(xy) \wedge \mu_A(y^{-1})$
$\ge \mu_A(xy) \wedge \mu_A(y)$, as A is an ILFMSG of G
$= \mu_A(xy)$.
Therefore, $\mu_A(xy) = \mu_A(x)$, for all x and y in G.
And, $\mu_A(yx) \ge \mu_A(y) \wedge \mu_A(x)$ (as A is an ILFMSG of G) $= \mu_A(x)$; and
$\mu_A(x) = \mu_A(y^{-1}yx)$
$\ge \mu_A(y^{-1}) \wedge \mu_A(yx)$
$\ge \mu_A(y) \wedge \mu_A(yx)$, as A is an ILFMSG of G
$= \mu_A(yx)$.
Therefore, $\mu_A(yx) = \mu_A(x)$, for all x and y in G.
Hence $\mu_A(xy) = \mu_A(x) = \mu_A(yx)$, for all x and y in G. Thus (i) is proved.

Now, $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$, as A is an ILFMSG of G $= \nu_A(x)$; and

$$\begin{aligned}\nu_A(x) &= \nu_A(x yy^{-1}) \\ &\leq \nu_A(xy) \vee \nu_A(y^{-1}) \\ &\leq \nu_A(xy) \vee \nu_A(y), \text{ as A is an ILFMSG of G} \\ &= \nu_A(xy).\end{aligned}$$

Therefore, $\nu_A(xy) = \nu_A(x)$, for all x and y in G.

And, $\nu_A(yx) \leq \nu_A(y) \vee \nu_A(x)$, as A is an ILFMSG of G $= \nu_A(x)$; and

$$\begin{aligned}\nu_A(x) &= \nu_A(y^{-1}yx) \\ &\leq \nu_A(y^{-1}) \vee \nu_A(yx) \\ &\leq \nu_A(y) \vee \nu_A(yx), \text{ as A is an ILFMSG of G} \\ &= \nu_A(yx).\end{aligned}$$

Therefore, $\nu_A(yx) = \nu_A(x)$, for all x and y in G.

Hence $\nu_A(xy) = \nu_A(x) = \nu_A(yx)$, for all x and y in G. Thus (ii) is proved.

3.12 Theorem: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group G such that Im $\mu_A = \{ \alpha \}$ and Im $\nu_A = \{ \beta \}$, where α and β in L. If $A = B \cup C$, where B and C are intuitionistic L-fuzzy M-subgroups of G, then either $B \subseteq C$ or $C \subseteq B$.

Proof:

Case (i) : Let $A = B \cup C = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in G \}$, $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in G \}$ and $C = \{ \langle x, \mu_C(x), \nu_C(x) \rangle / x \in G \}$.

Assume that $\mu_B(x) > \mu_C(x)$ and $\mu_B(y) < \mu_C(y)$, for some x and y in G.

Then, $\alpha = \mu_A(x) = \mu_{B\cup C}(x) = \mu_B(x) \vee \mu_C(x) = \mu_B(x) > \mu_C(x)$.

Therefore, $\alpha > \mu_C(x)$, for all x in G.

And, $\alpha = \mu_A(y) = \mu_{B\cup C}(y) = \mu_B(y) \vee \mu_C(y) = \mu_C(y) > \mu_B(y)$.

Therefore, $\alpha > \mu_B(y)$, for all y in G.So that, $\mu_C(y) > \mu_C(x)$ and $\mu_B(x) > \mu_B(y)$.

Hence $\mu_B(xy) = \mu_B(y)$ and $\mu_C(xy) = \mu_C(x)$.

But then, $\alpha = \mu_A(xy) = \mu_{B\cup C}(xy) = \mu_B(xy) \vee \mu_C(xy) = \mu_B(y) \vee \mu_C(x) < \alpha$ --------------(1).

Case (ii): Assume that $\nu_B(x) < \nu_C(x)$ and $\nu_B(y) > \nu_C(y)$, for some x and y in G.

Then, $\beta = \nu_A(x) = \nu_{B\cup C}(x) = \nu_B(x) \wedge \nu_C(x) = \nu_B(x) < \nu_C(x)$. Therefore, $\beta < \nu_C(x)$, for all x in G.

And, $\beta = \nu_A(y) = \nu_{B\cup C}(y) = \nu_B(y) \wedge \nu_C(y) = \nu_C(y) < \nu_B(y)$.

Therefore, $\beta < \nu_B(y)$, for all x in G. So that, $\nu_C(y) < \nu_C(x)$ and $\nu_B(x) < \nu_B(y)$.

Hence $\nu_B(xy) = \nu_B(y)$ and $\nu_C(xy) = \nu_C(x)$.

But then, $\beta = \nu_A(xy) = \nu_{B\cup C}(xy) = \nu_B(xy) \wedge \nu_C(xy) = \nu_B(y) \wedge \nu_C(x) > \beta$ --------------(2).

It is a contradiction by (1) and (2).

Therefore, either $B \subseteq C$ or $C \subseteq B$ is true.

3.13 Theorem: If A is an intuitionistic L-fuzzy M-subgroup of a M-group G and if there is a sequence $\{x_n\}$ in G such that $\lim_{n\to\alpha} \{ \mu_A(x_n) \wedge \mu_A(x_n) \} = 1$ and $\lim_{n\to\alpha} \{ \nu_A(x_n) \vee \nu_A(x_n) \} = 0$, then $\mu_A(e) = 1$ and $\nu_A(e) = 0$, where e is the identity element in G.

Proof: Let A be an intuitionistic L-fuzzy M-subgroup of a M-group G with e as its identity element in G and x in G be an arbitrary element. We have x in G implies x^{-1} in G and hence $xx^{-1} = e$.

Then, we have $\mu_A(e) = \mu_A(xx^{-1}) \geq \mu_A(x) \wedge \mu_A(x^{-1}) \geq \mu_A(x) \wedge \mu_A(x)$.

For each n, we have $\mu_A(e) \geq \mu_A(x) \wedge \mu_A(x)$.

Since $\mu_A(e) \geq \lim_{n\to\alpha} \{ \mu_A(x_n) \wedge \mu_A(x_n) \} = 1$.

Therefore $\mu_A(e) = 1$.

And, $\nu_A(e) = \nu_A(xx^{-1}) \leq \nu_A(x) \vee \nu_A(x^{-1}) = \nu_A(x) \vee \nu_A(x)$.

For each n, we have $\nu_A(e) \leq \nu_A(x) \vee \nu_A(x)$ .Since $\nu_A(e) \leq \lim_{n\to\alpha} \nu_A(x_n) \vee \nu_A(x_n) = 0$.

Therefore, $\nu_A(e) = 0$.

3.14 Theorem: If A and B are intuitionistic L-fuzzy M-subgroups of the M-groups G and H, respectively, then AxB is an intuitionistic L-fuzzy M-subgroup of GxH.

Proof: Let A and B be intuitionistic L-fuzzy M-subgroups of the M-groups G and H respectively. Let x_1 and x_2 be in G, y_1 and y_2 be in H.

Then (x_1, y_1) and (x_2, y_2) are in GxH.

Now,

$$\begin{aligned}\mu_{AxB}[m(x_1, y_1)(x_2, y_2)] &= \mu_{AxB}(mx_1x_2, my_1y_2) \\ &= \mu_A(mx_1x_2) \wedge \mu_B(my_1y_2) \\ &\geq \{\mu_A(x_1) \wedge \mu_A(x_2)\} \wedge \{\mu_B(y_1) \wedge \mu_B(y_2) \} \\ &= \{\mu_A(x_1) \wedge \mu_B(y_1) \} \wedge \{\mu_A(x_2) \wedge \mu_B(y_2) \} \\ &= \mu_{AxB}(x_1, y_1) \wedge \mu_{AxB}(x_2, y_2).\end{aligned}$$

Therefore, $\mu_{AxB}[m(x_1, y_1)(x_2, y_2)] \geq \mu_{AxB}(x_1, y_1) \wedge \mu_{AxB}(x_2, y_2)$, for all x_1 and x_2 in G, y_1 and y_2 in H.

And, $$\begin{aligned}\nu_{AxB}[m(x_1, y_1)(x_2, y_2)] &= \nu_{AxB}(mx_1x_2, my_1y_2) \\ &= \nu_A(mx_1x_2) \vee \nu_B(my_1y_2) \\ &\leq \{\nu_A(x_1) \vee \nu_A(x_2) \} \vee \{ \nu_B(y_1) \vee \nu_B(y_2) \} \\ &= \{\nu_A(x_1) \vee \nu_B(y_1) \} \vee \{\nu_A(x_2) \vee \nu_B(y_2) \} \\ &= \nu_{AxB}(x_1, y_1) \vee \nu_{AxB}(x_2, y_2).\end{aligned}$$

Therefore, $\nu_{AxB}[m(x_1, y_1)(x_2, y_2)] \leq \nu_{AxB}(x_1, y_1) \vee \nu_{AxB}(x_2, y_2)$, for all x_1 and x_2 in G, y_1 and y_2 in H. Hence AxB is an intuitionistic L-fuzzy M-subgroup of GxH.

3.15 Theorem: Let an intuitionistic L-fuzzy M-subgroup A of a M-group G be conjugate to an intuitionistic L-fuzzy M-subgroup M of G and an intuitionistic L-fuzzy M-subgroup B of a M-group H be conjugate to an intuitionistic L-fuzzy M-subgroup N of H. Then an intuitionistic L-fuzzy M-subgroup AxB of a M-group GxH is conjugate to an intuitionistic L-fuzzy M-subgroup MxN of GxH.

Proof: Let A and B be intuitionistic L-fuzzy M-subgroups of the M-groups G and H respectively. Let x, x^{-1} and f be in G and y, y^{-1} and g be in H.

Then (x, y) ,(x^{-1}, y^{-1}) and (f, g) are in GxH.

Now,
$$\begin{aligned}\mu_{AxB}(f, g) &= \mu_A(f) \wedge \mu_B(g) \\ &= \mu_M(xf x^{-1}) \wedge \mu_N(yg y^{-1}) \\ &= \mu_{MxN}(xf x^{-1}, yg y^{-1}) \\ &= \mu_{MxN}[(x, y)(f, g)(x^{-1}, y^{-1})] \\ &= \mu_{MxN}[(x, y) (f, g)(x, y)^{-1}].\end{aligned}$$

Therefore, $\mu_{AxB}(f, g) = \mu_{MxN}[(x, y) (f, g)(x, y)^{-1}]$, for all x, x^{-1} and f in G and y, y^{-1} and g in H.

And,
$$\begin{aligned}\nu_{AxB}(f, g) &= \nu_A(f) \vee \nu_B(g) \\ &= \nu_M(xf x^{-1}) \vee \nu_N(yg y^{-1}) \\ &= \nu_{MxN}(xf x^{-1}, yg y^{-1}) \\ &= \nu_{MxN}[(x, y)(f, g)(x^{-1}, y^{-1})]\end{aligned}$$

$= \nu_{MxN}[(x, y) (f, g)(x, y)^{-1}]$.

Therefore, $\nu_{AxB} (f, g) = \nu_{MxN}[(x, y) (f, g)(x, y)^{-1}]$, for all x, x^{-1} and f in G and y, y^{-1} and g in H.

Hence an intuitionistic L-fuzzy M-subgroup AxB of GxH is conjugate to an intuitionistic L-fuzzy M-subgroup MxN of GxH.

3.16 Theorem: Let A and B be intuitionistic L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and e^{l} are the identity element of G and H, respectively. If AxB is an intuitionistic L-fuzzy M-subgroup of GxH, then at least one of the following two statements must hold.

(i) $\mu_B(e^l) \geq \mu_A(x)$ and $\nu_B(e^l) \leq \nu_A(x)$, for all x in G,

(ii) $\mu_A(e) \geq \mu_B(y)$ and $\nu_A(e) \leq \nu_B(y)$, for all y in H.

Proof: Let AxB is an intuitionistic L-fuzzy M-subgroup of GxH. By contraposition, suppose that none of the statements (i) and (ii) holds.

Then we can find a in G and b in H such that $\mu_A(a) > \mu_B(e^l)$, $\nu_A(a) < \nu_B(e^l)$ and $\mu_B(b) > \mu_A(e)$, $\nu_B(b) < \nu_A(e)$.

We have, $\mu_{AxB} (a, b) = \mu_A(a) \wedge \mu_B(b)$
$> \mu_A(e) \wedge \mu_B(e^l) = \mu_{AxB} (e, e^l)$.

And, $\nu_{AxB} (a, b) = \nu_A(a) \vee \nu_B(b)$
$< \nu_A(e) \vee \nu_B(e^l) = \nu_{AxB} (e, e^l)$.

Thus AxB is not an intuitionistic L-fuzzy M-subgroup of GxH.

Hence either $\mu_B(e^l) \geq \mu_A(x)$ and $\nu_B(e^l) \leq \nu_A(x)$, for all x in G or $\mu_A(e) \geq \mu_B(y)$ and $\nu_A(e) \leq \nu_B(y)$, for all y in H.

3.17 Theorem: Let A be an intuitionistic L-fuzzy subset of a M-group G and V be the strongest intuitionistic L-fuzzy relation of G. Then A is an intuitionistic L-fuzzy M-subgroup of G if and only if V is an intuitionistic L-fuzzy M-subgroup of GxG.

Proof: Suppose that A is an intuitionistic L-fuzzy M-subgroup of G.

Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in GxG.

We have, $\mu_V (x– y) = \mu_V [(x_1, x_2) – (y_1, y_2)]$
$= \mu_V (x_1-y_1 , x_2-y_2)$
$= \mu_A(x_1-y_1) \wedge \mu_A(x_2-y_2)$
$\geq \{\mu_A(x_1) \wedge \mu_A(y_1)\}\wedge \{\mu_A(x_2)\wedge\mu_A(y_2) \}$
$= \{\mu_A(x_1) \wedge \mu_A(x_2) \}\wedge\{ \mu_A(y_1 \wedge\mu_A(y_2) \}$
$= \mu_V (x_1, x_2) \wedge \mu_V (y_1, y_2)$
$= \mu_V (x) \wedge\mu_V (y)$.

Therefore, $\mu_V (x – y) \geq \mu_V (x) \wedge\mu_V (y)$, for all x and y in GxG.

Also we have, $\nu_V (x – y) = \nu_V [(x_1, x_2) – (y_1, y_2)]$
$= \nu_V(x_1 – y_1 , x_2 – y_2)$
$= \nu_A(x_1– y_1) \vee\nu_A(x_2 – y_2)$
$\leq \{\nu_A(x_1) \vee \nu_A(y_1) \} \vee \{ \nu_A(x_2) \vee \nu_A(y_2) \}$
$= \{\nu_A(x_1) \vee \nu_A(x_2) \} \vee \{ \nu_A(y_1) \vee \nu_A(y_2) \}$
$= \nu_V (x_1, x_2) \vee \nu_V (y_1, y_2)$
$= \nu_V (x) \vee \nu_V (y)$.

Therefore, $\nu_V (x – y) \leq \nu_V (x) \vee \nu_V (y)$, for all x and y in GxG.

This proves that V is an intuitionistic L-fuzzy M-subgroup of GxG.

Conversely, assume that V is an intuitionistic L-fuzzy M-subgroup of GxG, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in GxG, we have

$\min \{\mu_A(x_1– y_1) , \mu_A(x_2 – y_2)\}= \mu_V(x_1– y_1 , x_2 – y_2)$
$= \mu_V [(x_1, x_2) – (y_1, y_2)]$
$= \mu_V (x – y) \geq \mu_V (x) \wedge\mu_V (y)$
$= \mu_V (x_1, x_2) \wedge \mu_V (y_1, y_2)$
$=\{\mu_A(x_1)\wedge\mu_A(x_2)\}\wedge\{\mu_A(y_1) \wedge \mu_A(y_2) \}$.

If we put $x_2 = y_2 = 0$,

We get, $\mu_A(x_1– y_1) \geq \mu_A(x_1) \wedge \mu_A(y_1)$, for all x_1 and y_1 in G.

Also we have,

$\nu_A(x_1 – y_1) \vee \nu_A(x_2 – y_2) = \nu_V (x_1 – y_1 , x_2 – y_2)$
$= \nu_V [(x_1, x_2) – (y_1, y_2)]$
$= \nu_V (x – y) \leq \nu_V (x) \vee\nu_V (y)$
$= \nu_V (x_1, x_2) \vee \nu_V (y_1, y_2)$
$=\{\nu_A(x_1)\vee\nu_A(x_2)\}\vee\{\nu_A(y_1)\vee \nu_A(y_2) \}$.

If we put $x_2 = y_2 = 0$,

We get, $\nu_A (x_1 – y_1) \leq \nu_A(x_1) \vee \nu_A(y_1)$, for all x_1 and y_1 in G.

Hence A is an intuitionistic L-fuzzy M-subgroup of a M-group G.

4 – INTUITIONISTIC L-FUZZY M-SUBGROUPS OF A M-GROUP UNDER HOMOMORPHISM AND ANTI-HOMOMORPHISM:

4.1 Theorem: Let (G, ·) and (G^l, ·) be any two M-groups. The homomorphic image (pre-image) of an intuitionistic L-fuzzy M-subgroup of G is an intuitionistic L-fuzzy M-subgroup of G^l.

Proof: Let (G, ·) and (G^l, ·) be any two groups and $f : G\rightarrow G^l$ be a homomorphism.

That is f(xy) = f(x)f(y), f(mx) = mf(x), for all x and y in G and m in M.

Let V=f(A), where A is an intuitionistic L-fuzzy M-subgroup of a M-group G.

We have to prove that V is an intuitionistic L-fuzzy M-subgroup of G^l.

Now, for f(x) and f(y) in G^l,

we have

$\mu_V(mf(x)f(y))= \mu_V(f(mxy))$,(as f is a homomorphism)
$\geq\mu_A(mxy)$
$\geq\mu_A(x)\wedge\mu_A(y)$,as A is an ILFMSG of G

which implies that $\mu_V(mf(x)f(y)) \geq \mu_V(f(x)) \wedge \mu_V(f(y))$, for all x and y in G.

For f(x) in G^l, we have,

$\mu_V([f(x)]^{-1}) = \mu_V(f(x^{-1}))$, as f is a homomorphism
$\geq \mu_A(x^{-1})$
$\geq \mu_A(x)$, as A is an ILFMSG of G,

which implies that $\mu_V([f(x)]^{-1}) \geq \mu_V(f(x))$, for all x in G.

$\nu_V(mf(x)f(y)) = \nu_V(f(mxy))$, as f is a homomorphism
$\leq \nu_A(mxy)$
$\leq \nu_A(x) \vee \nu_A(y)$, as A is an ILFMSG of G,

which implies that $\nu_V(f(x)f(y)) \leq \nu_V(f(x)) \vee \nu_V(f(y))$, for all x and y in G.

$\nu_V([f(x)]^{-1}) = \nu_V(f(x^{-1}))$, as f is a homomorphism
$\leq \nu_A(x^{-1})$
$\leq \nu_A(x)$, as A is an ILFMSG of G,

which implies that $\nu_V([f(x)]^{-1}) \leq \nu_V(f(x))$, for all x in G.

Hence V is an intuitionistic L-fuzzy M-subgroup of a M-group G^l.

4.2 Theorem: Let (G, ·) and (G^l, ·) be any two M-groups. The anti-homomorphic image(pre-image) of an intuitionistic L-fuzzy M-subgroup of G is an intuitionistic L-fuzzy M-subgroup of G^l.

Proof: Let (G, ·) and (G^{l}, ·) be any two M-groups and f : G → G^{l} be an anti-homomorphism. That is f(xy) = f(y)f(x), f(mx) = m f(x), for all x and y in G and m in M. Let V = f(A), where A is an intuitionistic L-fuzzy M-subgroup of a M-group G.
We have to prove that V is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}.
Now, let f(x) and f(y)∈G^{l}, we have
$\mu_V(mf(x)f(y)) = \mu_V(f(myx))$, as f is an anti-homomorphism
$\geq \mu_A(myx)$
$\geq \mu_A(x) \wedge \mu_A(y)$, as A is an ILFMSG of G,
which implies that $\mu_V(mf(x)f(y)) \geq \mu_V(f(x)) \wedge \mu_V(f(y))$, for all x and y in G.
For x in G,
$\mu_V([f(x)]^{-1}) = \mu_V(f(x^{-1}))$, as f is an anti-homomorphism
$\geq \mu_A(x^{-1}) \geq \mu_A(x)$, as A is an ILFMSG of G,
which implies that $\mu_V([f(x)]^{-1}) \geq \mu_V(f(x))$, for all x in G.
And,
$\nu_V(mf(x)f(y)) = \nu_V(f(myx))$, as f is an anti-homomorphism
$\leq \nu_A(myx)$
$\leq \nu_A(x) \vee \nu_A(y)$, as A is an ILFMSG of G,
which implies that $\nu_V(f(x)f(y)) \leq \nu_V(f(x)) \vee \nu_V(f(y))$, for all x and y in G.
Also,
$\nu_V([f(x)]^{-1}) = \nu_V(f(x^{-1}))$, as f is an anti-homomorphism
$\leq \nu_A(x^{-1}) \leq \nu_A(x)$, as A is an ILFMSG of G,
which implies that $\nu_V([f(x)]^{-1}) \leq \nu_V(f(x))$, for all x in G.
Hence V is an intuitionistic L-fuzzy M-subgroup of a M-group G^{l}.

5. CONCLUSION

In this paper, we define a new algebraic structure of Intuitionistic L-fuzzy M-subgroups of M-groups and Homomorphism and anti-homomorphism of Intuitionistic L-fuzzy M-subgroups of M-groups, we wish to define Intuitionistic L-fuzzy Normal M-subgroups and Level subset of Intuitionistic L-fuzzy M-subgroups of M-groups and other some M-groups are in progress.

6. REFERENCES

[1] Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications 35, (1971) 512-517.

[2] R. Biswas, Fuzzy subgroups and anti-fuzzy subgroups, Fuzzy Sets and Systems 35(1990), 121-124.

[3] J.A. Goguen, L-fuzzy sets, J. Math. Anal. Appl.18(1967),145-179.

[4] Mohamed Asaad, Groups and Fuzzy Subgroups, Fuzzy Sets and Systems 39(1991)323-328.

[5] Prabir Bhattacharya, Fuzzy subgroups: Some characterizations, J. Math. Anal. Appl. 128 (1981) 241-252.

[6] N.Palaniappan, R.Muthuraj, The homomorphism, Anti-homomorphism of a fuzzy and an anti-fuzzy group, Varahmihir Journal of mathematical Sciences, 4 (2)(2004) 387-399.

[7] N.Palaniappan , S. Naganathan, & K. Arjunan, A Study on Intuitionistic L-Fuzzy Subgroups, Applied mathematical Sciences, 3 (53) (2009) 2619-2624.

[8] Pandiammal. P, Natarajan. R, and Palaniappan. N , Anti L-fuzzy M-subgroups, Antarctica J. Math., Vol. 7, number 6, 683-691, (2010).

[9] L.A. Zadeh, Fuzzy sets, Information and control, 8, (1965) 338-353.