# Product Intuitionistic Fuzzy Graph 

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#### Abstract

In this paper, we introduce product intuitionistic fuzzy graphs and prove several results which are analogous to intuitionistic fuzzy graphs. We conclude by giving properties for a product partial intuitionistic fuzzy sub graph.


Key words: Intuitionistic Fuzzy Graphs, Product Intuitionistic fuzzy graphs.

## 1. INTRODUCTION

The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld [1] introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov[2]. Dr. V. Ramaswamy and Poornima .B introduce the concept of product fuzzy graph. In this paper we develop the concept of Product Intuitionistic fuzzy graphs of intuitionistic fuzzy graphs. Further investigate properties Product Intuitionistic fuzzy graphs.

## 2. DEFINITION AND MAIN RESULTS

Definition 2.1 An intuitionistic fuzzy graph (IFG) is of the form $\mathrm{G}=(\mathrm{V}, \mathrm{E})$, where $\mathrm{V}=\left\{\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots, \mathrm{v}_{\mathrm{n}}\right\}$ such that $\mu_{1}: \mathrm{V} \rightarrow[0,1]$, $\gamma_{1}: V \rightarrow[0,1]$ denote the degree of membership and nonmember ship of the element vi $\in \mathrm{V}$ respectively and $0 \leq \mu_{1}\left(\mathrm{v}_{\mathrm{i}}\right)+\gamma_{1}\left(\mathrm{v}_{\mathrm{i}}\right) \leq 1$ for every $\mathrm{v}_{\mathrm{i}} \in \mathrm{V}$,(i $\left.=1,2, \ldots \mathrm{n}\right)$ (ii) $\mathrm{E} \subseteq \mathrm{V} \times \mathrm{V}$ where $\mu_{2}: \mathrm{V} \times \mathrm{V} \rightarrow[0,1]$ and $\gamma_{2}: \mathrm{V} \times \mathrm{V} \rightarrow[0,1]$ are such that $\mu_{2}\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right) \leq \mu_{1}\left(\mathrm{v}_{\mathrm{i}}\right) \wedge \mu_{1}\left(\mathrm{v}_{\mathrm{j}}\right) \gamma_{2}\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right) \leq \gamma_{1}\left(\mathrm{v}_{\mathrm{i}}\right) \wedge \gamma_{1}\left(\mathrm{v}_{\mathrm{j}}\right)$ and $0 \leq \mu_{2}\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right)+\gamma_{2}\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right) \leq 1$.

Definition 2.2 Let $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ be an intuitionistic fuzzy graph. If $\mu_{2}(\mathrm{x}, \mathrm{y}) \leq \mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y})$ and $\gamma_{2}(\mathrm{x}, \mathrm{y}) \leq \gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y})$ the intuition fuzzy graph is called product partial intuitionistic fuzzy sub graph of G.

Remark: If $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ is a product intuitionistic fuzzy graph then since $\mu_{1}(x)$ and $\mu_{1}(y)$, are less than or equal to 1 , it follows that $\mu_{2}\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right) \leq \mu_{1}\left(\mathrm{v}_{\mathrm{i}}\right) \times \mu_{1}(\mathrm{vj}) \leq \mu_{1}\left(\mathrm{v}_{\mathrm{i}}\right) \wedge \mu_{1}\left(\mathrm{v}_{\mathrm{j}}\right)$ and $\gamma_{2}\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right) \leq$ $\gamma_{1}\left(\mathrm{v}_{\mathrm{i}}\right) \times \gamma_{1}\left(\mathrm{v}_{\mathrm{j}}\right) \leq \gamma_{1}\left(\mathrm{v}_{\mathrm{i}}\right) \wedge \gamma_{1}\left(\mathrm{v}_{\mathrm{j}}\right)$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{V}$. Thus every product intuitionistic fuzzy graph is an intuitionistic fuzzy graph.

Definition 2.3 A product Intuitionistic fuzzy graph $G=(V, E)$ is said to be complete if $\mu_{2}(x, y)=\mu_{1}(x) \times \mu_{1}(y)$ and $\gamma_{2}(\mathrm{x}, \mathrm{y})=\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y})$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{V}$.

Proposition 2.1 Let $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ be a complete product intuitionistic fuzzy graph where $\mu_{1}$ and $\gamma_{1}$ are normal. Then $\mu_{2}{ }^{n}(x, y)=\mu_{2}(x, y)$ and $\gamma_{2}{ }^{n}(x, y)=\gamma_{2}(x, y)$ for all $x, y \in V$ and for all positive integer n for $\mathrm{n} \geq 2$

$$
\begin{aligned}
& \mu_{2}{ }^{n}(x, y)=v_{z \in V}\left\{\mu_{2}{ }^{n-1}(x, y) \times \mu_{2}(x, y)\right\} \\
& \gamma_{2}{ }^{n}(x, y)=v_{z \in V}\left\{\gamma_{2}{ }^{n-1}(x, y) \times \gamma_{2}(x, y)\right\}
\end{aligned}
$$

Proof: We prove by method of induction .Let $\mathrm{n}=2$ then for all $x, y \in V$, we have

$$
\begin{aligned}
& \mu_{2}^{2}(\mathrm{x}, \mathrm{y})=\vee_{\mathrm{z} \in \mathrm{~V}}\left\{\mu_{2}(\mathrm{x}, \mathrm{z}) \times \mu_{2}(\mathrm{z}, \mathrm{y})\right\} \\
& \mu_{2}{ }^{2}(\mathrm{x}, \mathrm{y})=\vee_{\mathrm{z} \in \mathrm{~V}}\left\{\left[\mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{z})\right] \times\left[\mu_{1}(\mathrm{z}) \times \mu_{1}(\mathrm{y})\right]\right\} \\
& \mu_{2}^{2}(\mathrm{x}, \mathrm{y})=\vee_{\mathrm{z} \in \mathrm{~V}}\left\{\mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y}) \times \mu_{1}(\mathrm{z})^{2}\right\} \\
& \text { Since } \mu_{1}(\mathrm{z})^{2} \leq 1 \text { for all } \mathrm{z}\left[\therefore \mu_{1}(\mathrm{z}) \leq 1\right] \\
& \mu_{2}^{2}(\mathrm{x}, \mathrm{y})=\mathrm{V}_{\mathrm{z} \in \mathrm{~V}}\left\{\mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y})\right\} \\
& \mu_{2}^{2}(\mathrm{x}, \mathrm{y})=\mu_{2}(\mathrm{x}, \mathrm{y}) \rightarrow(1) \\
& \text { and } \quad \gamma_{2}^{2}(\mathrm{x}, \mathrm{y})=\mathrm{V}_{\mathrm{z} \in \mathrm{~V}}\left\{\gamma_{2}(\mathrm{x}, \mathrm{z}) \times \gamma_{2}(\mathrm{z}, \mathrm{y})\right\} \\
& \gamma_{2}^{2}(\mathrm{x}, \mathrm{y})=\mathrm{V}_{\mathrm{z} \in \mathrm{~V}}\left\{\left[\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{z}) \times\left[\gamma_{1}(\mathrm{z}) \times \gamma_{1}(\mathrm{y})\right]\right\}\right. \\
& \gamma_{2}^{2}(\mathrm{x}, \mathrm{y})=\mathrm{V}_{\mathrm{zV}}\left\{\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y}) \times \gamma_{1}(\mathrm{z})^{2}\right\} \\
& \text { since } \gamma_{1}(\mathrm{z})^{2} \leq 1 \text { for all } \mathrm{z}\left[\therefore \gamma_{1}(\mathrm{z}) \leq 1\right] \\
& \gamma_{2}^{2}(\mathrm{x}, \mathrm{y})=\mathrm{V}_{\mathrm{z} \in \mathrm{~V}}\left\{\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y})\right\} \\
& \gamma_{2}^{2}(\mathrm{x}, \mathrm{y})=\gamma_{2}(\mathrm{x}, \mathrm{y}) \rightarrow(2)
\end{aligned}
$$

If $\mu_{1}$ and $\gamma_{1}$ normal, then $\mu_{1}(t)=1$ and $\gamma_{1}(t)=1$ for some $t$. Then

$$
\begin{aligned}
\mu_{2}^{2}(\mathrm{x}, \mathrm{y}) & =v_{\mathrm{z} \in \mathrm{~V}}\left\{\mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y}) \times \mu_{1}(\mathrm{z})^{2}\right\} \\
& \geq \mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y}) \times \mu_{1}(\mathrm{t})^{2} \\
& =\mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y}) \quad\left[\therefore \mu_{1}(\mathrm{t})^{2}=1\right] \\
\mu_{2}^{2}(\mathrm{x}, \mathrm{y}) & \geq \mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y})
\end{aligned}
$$

$\mu_{2}(x, y) \geq \mu_{2}(x, y) \rightarrow(3)\left[\right.$ since $\mu_{2}^{2}(x, y)=\mu_{1}(x) \times \mu_{1}(y) \quad G$ is complete]
from (1) and (3) we get $\mu_{2}(x, y)=\mu_{2}(x, y) \rightarrow$ (4)
If $\gamma_{1}$ normal, $\gamma_{1}(t)=1$ for some $t$. Then

$$
\begin{aligned}
\gamma_{2}^{2}(\mathrm{x}, \mathrm{y})= & \vee_{\mathrm{z} \in \mathrm{~V}}\left\{\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y}) \times \gamma_{1}(\mathrm{z})^{2}\right\} \\
& \geq \gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y}) \times \gamma_{1}(\mathrm{t})^{2} \\
& =\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y}) \quad\left[\therefore \mu_{1}(\mathrm{t})^{2}=1\right] \\
\gamma_{2}^{2}(\mathrm{x}, \mathrm{y}) \geq & \gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y}) \quad \\
\gamma_{2}(\mathrm{x}, \mathrm{y}) \geq \gamma_{2}(\mathrm{x}, \mathrm{y}) & \rightarrow(5) \quad\left[\text { since } \gamma_{2}^{2}(\mathrm{x}, \mathrm{y})=\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y}), \mathrm{G}\right. \text { is }
\end{aligned}
$$ complete]

from (2) and (5) we get $\gamma_{2}(x, y)=\gamma_{2}(x, y) \rightarrow$ (6)
Now assuming that $\mu_{2}{ }^{k}(x, y)=\mu_{2}(x, y)$ and $\gamma_{2}{ }^{k}(x, y)=\gamma_{2}(x, y)$ we will prove $\mu_{2}{ }^{\mathrm{k}+1}(\mathrm{x}, \mathrm{y})=\mu_{2}(\mathrm{x}, \mathrm{y})$ and $\gamma_{2}{ }^{\mathrm{k}+1}(\mathrm{x}, \mathrm{y})=\gamma_{2}(\mathrm{x}, \mathrm{y})$ we have
$\mu_{2}{ }^{\mathrm{k}+1}(\mathrm{x}, \mathrm{y})=\mathrm{V}_{\mathrm{z} \in \mathrm{V}}\left\{\mu_{2}{ }^{\mathrm{k}}(\mathrm{x}, \mathrm{z}) \times \mu_{2}(\mathrm{z}, \mathrm{y})\right\}$
$\mu_{2}^{2}(x, y)=V_{z \in V}\left\{\left[\mu_{2}(x, z) \times \mu_{2}(z, y)\right]\right\}=\mu_{2}{ }^{2}(x, y)$
$\mu_{2}{ }^{\mathrm{k}+1}(\mathrm{x}, \mathrm{y})=\mu_{2}(\mathrm{x}, \mathrm{y})$ using (5)
Similarly we get $\gamma_{2}{ }^{\mathrm{k}+1}(\mathrm{x}, \mathrm{y})=\gamma_{2}(\mathrm{x}, \mathrm{y})$
Hence proved

Definition 2.4 The complement of product intuitionistic fuzzy graph $G=(V, E)$ is $G^{c}=\left(V^{c}, E^{c}\right)$ where $V^{c}=\left(\mu^{c}{ }_{1}, \gamma_{1}^{c}\right)$ and $\mathrm{E}^{\mathrm{c}}=\left(\mu^{\mathrm{c}}{ }_{2}, \gamma^{\mathrm{c}}{ }_{2}\right)$ here $\mu_{1}^{\mathrm{c}}=\gamma_{1}^{\mathrm{c}}, \mu_{1}^{\mathrm{c}}{ }_{1}=\gamma_{1}^{\mathrm{c}}$ and
$\mu^{\mathrm{c}}{ }_{2}(\mathrm{x}, \mathrm{y})=\mu_{1}(\mathrm{x}) \times \mu_{1}(\mathrm{y})-\mu_{2}(\mathrm{x}, \mathrm{y})$,
$\gamma^{c}{ }_{2}(\mathrm{x}, \mathrm{y})=\gamma_{1}(\mathrm{x}) \times \gamma_{1}(\mathrm{y})-\gamma_{2}(\mathrm{x}, \mathrm{y})$.
Remark: The complement of a $\mathrm{G}^{\mathrm{c}}$ is G .
Definition2.5 Let $G_{1}=\left(V_{1}, E_{1}\right)$ and $G_{2}=\left(V_{2}, \mathrm{E}_{2}\right)$ be a product intuitionistic fuzzy graph. Here $\mathrm{V}_{1}=\left(\mu_{11}, \gamma_{11}\right), \mathrm{E}_{1}=\left(\mu_{12}, \gamma_{12}\right)$, $V_{2}=\left(\mu_{21}, \gamma_{21}\right)$ and $E_{2}=\left(\mu_{22}, \gamma_{22}\right)$. Let $X^{\prime}$ denotes the set of all arcs joining the vertices $V_{1}$ and $V_{2}$. We further assume that $\mathrm{V}_{1} \cap \mathrm{~V}_{2}=\phi$. Then the join of $\mathrm{G}_{1}$ and $\mathrm{G}_{2}$ is defined as $\left(V_{1}+V_{2}, E_{1}+E_{2}\right)$ where $V_{1}+V_{2}=\left(\mu_{11}+\mu_{21}, \gamma_{11}+\gamma_{21}\right)$ and
$V_{1}+V_{2}=\left(\mu_{12}+\mu_{22}, \gamma_{12}+\gamma_{22}\right)$ here
$\left(\mu_{11}+\mu_{21}\right)=\mu_{11}(u)$ if $u \in V_{1}$
$=\mu_{21}(u)$ if $u \in V_{2}$
$\left(\gamma_{11}+\gamma_{21}\right)=\gamma_{11}(u)$ if $u \in V_{1}$
$=\gamma_{21}(u)$ if $u \in V_{2}$
$\left(\mu_{12}+\mu_{22}\right)(u, v)=\mu_{12}(u, v)$ if $(u, v) \in E_{1}$
$=\mu_{22}(u, v)$ if $(u, v) \in E_{2}$
$=\mu_{11}(u) \times \mu_{21}(v)$ if $(u, v) \in X$,
$\left(\gamma_{12}+\gamma_{22}\right)(u, v)=\gamma_{12}(u, v)$ if $(u, v) \in E_{1}$
$=\gamma_{22}(u, v)$ if $(u, v) \in E_{2}$
$=\gamma_{11}(u) \times \gamma_{21}(v)$ if $(u, v) \in X^{\prime}$.
Proposition $2.2 G_{1}+G_{2}$ is a product intuitionistic fuzzy sub graph of $G=(V, E)$ where and $E=E_{1} \cup E_{2} \cup X^{\prime}$

Proof: We have to prove that

$$
\begin{equation*}
\left(\mu_{12}+\mu_{22}\right)(u, v) \leq\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v) \rightarrow \tag{i}
\end{equation*}
$$

for all $(u, v) \in V$
and $\left(\gamma_{12}+\gamma_{22}\right)(\mathrm{u}, \mathrm{v}) \leq\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{u}) \times\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{v}) \rightarrow$
for all $(u, v) \in V$
Case 1: If $(u, v) \in X_{1}$, then $u, v \in V_{1}$ so that $\left(\mu_{12}+\mu_{22}\right)(u, v)=\mu_{12}(u, v) \rightarrow($ iii $)$
And $\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v)=\mu_{11}(u) \times \mu_{11}(v) \rightarrow \quad$ (iv)
from (iii) and (iv) we get (i)
$\left(\gamma_{12}+\gamma_{22}\right)(\mathrm{u}, \mathrm{v})=\gamma_{12}(\mathrm{u}, \mathrm{v}) \rightarrow$
and $\left(\gamma_{11}+\gamma_{21}\right)(u) \times\left(\gamma_{11}+\gamma_{21}\right)(v)=\gamma_{11}(u) \times \gamma_{11}(v) \rightarrow$
from (v) and (vi) we get (ii)
Therefore we get $G_{1}+G_{2}$ is a product intuitionistic fuzzy sub graph of G.
Similarly we can prove $(u, v) \in X_{1}$.
Case 2: If $(u, v) \in X^{\prime}$ then $u \in V_{1}$ and $v \in V_{2}$. Now $\left(\mu_{12}+\mu_{22}\right)(u, v) \leq\left(\mu_{11} \quad(u) \times \mu_{21}\right.$ (v) whereas $\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v) \quad=\mu_{11}(u) \times \mu_{21}(v)$ therefore we get $\left(\mu_{12}+\mu_{22}\right)(u, v)=\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v) \rightarrow \quad$ (vii) Similarly we get
$\left(\gamma_{12}+\gamma_{22}\right)(\mathrm{u}, \mathrm{v})=\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{u}) \times\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{v}) \rightarrow \quad$ (viii)
From (vii) and (viii) $G_{1}+G_{2}$ is a product intuitionistic fuzzy sub graph of G.Hence proved.
Proposition $2.3 G_{1}+G_{2}$ is complete if and only if $G_{1}$ and $G_{2}$ are both complete.

Proof: First we are assuming that $\mathrm{G}_{1}$ and $\mathrm{G}_{2}$ are both complete. We will prove that $\mathrm{G}_{1}+\mathrm{G}_{2}$ is complete.
Case 1: If $(u, v) \in X_{1}$ therefore $u, v \in V_{1}$.we get $\left(\mu_{12}+\mu_{22}\right)(u, v)=\mu_{12}(u, v)=\mu_{11}(u) \times \mu_{11}(v)$ [since $G_{1}$ is complete]

$$
\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v)=\mu_{11}(u) \times \mu_{11}(v)
$$

and $\quad\left(\gamma_{12}+\gamma_{22}\right)(u, v)=\gamma_{12}(u, v)=\gamma_{11}(u) \times \gamma_{11}(v)\left[\right.$ since $G_{1}$ is complete]

$$
\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{u}) \times\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{v})=\gamma_{11}(\mathrm{u}) \times \gamma_{11}(\mathrm{v})
$$

Therefore $\mathrm{G}_{1}+\mathrm{G}_{2}$ is complete.
Similarly we argue ( $u, v) \in X_{2}$ therefore $u, v \in V_{2}$.
Case 2: Suppose $(u, v) \in X^{\prime}$. Then $u \in V_{1}$ and $v \in V_{2}$,
we get $\left(\mu_{12}+\mu_{22}\right)(u, v)=\mu_{11}(u) \times \mu_{21}(v)$ whereas

$$
\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v)=\mu_{11}(u) \times \mu_{21}(v)
$$

Therefore, $\left(\mu_{12}+\mu_{22}\right)(u, v)=\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}(u) \times \mu_{21}\right)(v)$
And $\left(\gamma_{12}+\gamma_{22}\right)(u, v)=\gamma_{11}(u) \times \gamma_{21}(v)$ whereas

$$
\left(\gamma_{11}+\gamma_{21}\right)(u) \times\left(\gamma_{11}+\gamma_{21}\right)(v)=\gamma_{11}(u) \times \gamma_{21}(v)
$$

Therefore, $\left(\gamma_{12}+\gamma_{22}\right)(\mathrm{u}, \mathrm{v})=\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{u}) \times\left(\gamma_{11}+\gamma_{21}\right)(\mathrm{v})$
Therefore $\mathrm{G}_{1}+\mathrm{G}_{2}$ is complete.
Conversely assume that $G_{1}+G_{2}$ is complete, we will prove $G_{1}$, $\mathrm{G}_{2}$ are complete. First we
$\mathrm{G}_{1}$ complete, we have prove that for al $(\mathrm{u}, \mathrm{v}) \in \mathrm{E}_{1}$, $\mu_{12}(u, v)=\mu_{11}(u) \times \mu_{11}(v)$ and $\quad \gamma_{12}(u, v)=\gamma_{11}(u) \times \gamma_{11}(v)$. $\mathrm{G}_{1}+\mathrm{G}_{2}$ is complete therefore
$\left(\mu_{12}+\mu_{22}\right)(u, v)=\mu_{12}(u, v) \rightarrow \quad$ (i) $\quad\left[\right.$ since $\left.(u, v) \in E_{1}\right]$ and $\left(\gamma_{12}+\gamma_{22}\right)(u, v)=\gamma_{12}(u, v) \rightarrow$ (ii) [since $(u, v) \in E_{2}$ ] whereas $\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v)=\mu_{11}(u) \times \mu_{11}(v) \rightarrow$ (iii) and
$\left(\gamma_{11}+\gamma_{21}\right)(u) \times\left(\gamma_{11}+\gamma_{21}\right)(v)=\gamma_{11}(u) \times \gamma_{11}(v) \rightarrow(i v)$
We know that $\left(\mu_{12}+\mu_{22}\right)(u, v)=\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v)$
$\left(\gamma_{12}+\gamma_{22}\right)(u, v)=\left(\gamma_{11}+\gamma_{21}\right)(u) \times\left(\gamma_{11}+\gamma_{21}\right)(v)$
using (i), (ii), (iii),(iv) we get $\mu_{12}(u, v)=\mu_{11}(u) \times \mu_{11}$ (v) and $\gamma_{12}(u, v)=\gamma_{11}(u) \times \gamma_{11}(v)$. Therefore $G_{1}$ is complete. Similarly we prove $\mathrm{G}_{2}$ is complete. Hence proved.

Proposition 2.4 Let $G_{1}$ and $G_{2}$ be product partial intuitionistic fuzzy sub graph, then
$\left(\mu_{11}+\mu_{21}, \mu_{12}+\mu_{22}\right)^{c}=\left(\mu_{11}{ }^{c} \cup \mu_{21}{ }^{c}, \mu_{12}{ }^{c} \cup \mu_{22}{ }^{c}\right)$
$\left(\gamma_{11}+\gamma_{21}, \gamma_{12}+\gamma_{22}\right)^{c}=\left(\gamma_{11}{ }^{c} \cup \gamma_{21}{ }^{c}, \gamma_{12}{ }^{c} \cup \gamma_{22}{ }^{c}\right)$
Proof: If $u \in V_{1}$ then $\left(\mu_{11}+\mu_{21}\right)^{c}=\left(\mu_{11}+\mu_{21}\right)(u)=\mu_{11}(u)$ and $\max \left(\mu_{11}{ }^{\mathrm{c}}(\mathrm{u}), \mu_{21}{ }^{\mathrm{c}}(\mathrm{u})=\max \left(\mu_{11}(\mathrm{u}), \mu_{21}(\mathrm{u})\right)=\mu_{11}(\mathrm{u})\right.$

$$
\Rightarrow\left(\mu_{11}+\mu_{21}\right) \quad \mathrm{c}(\mathrm{u})=\left(\mu_{11}{ }^{\mathrm{c}} \cup \mu_{21}{ }^{\mathrm{c}}\right)(\mathrm{u})
$$

Similarly $u \in V_{2}$.
Suppose $(u, v) \in X_{1}$, then $u, v \in V_{1}$ and
$\left(\mu_{12}+\mu_{22}\right)^{c}(u, v)=\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v)-\left(\mu_{12}+\mu_{22}\right)$ ( $u, \mathrm{v}$ )

$$
\begin{aligned}
& =\mu_{11}(u) \times \mu_{11}(v)-\mu_{12}(u, v) \\
& =\mu_{12}{ }^{c}(u, v)
\end{aligned}
$$

$\operatorname{Max}\left(\mu_{12}{ }^{\mathrm{c}}(\mathrm{u}, \mathrm{v}), \mu_{21}{ }^{\mathrm{c}}(\mathrm{u}, \mathrm{v})\right)=\mu_{12}{ }^{\mathrm{c}}(\mathrm{u}, \mathrm{v})$

$$
\Rightarrow\left(\mu_{12}+\mu_{22}\right)^{c}(u, v)=\left(\mu_{12}{ }^{c} \cup \mu_{22}{ }^{c}\right)
$$

Similarly $(u, v) \in X_{2}$.
Suppose ( $u, v$ ) $\in X^{\prime}$. Then $u \in V_{1}$ and $v \in V_{2}$ therefore
$\left(\mu_{12}+\mu_{22}\right)^{c}(u, v)=\left(\mu_{11}+\mu_{21}\right)(u) \times\left(\mu_{11}+\mu_{21}\right)(v)-\left(\mu_{12}+\mu_{22}\right)$ ( $\mathrm{u}, \mathrm{v}$ )
$=\mu_{11}(u) \times \mu_{21}(v)-\left(\mu_{11}(u) \times \mu_{21}(v)\right)=0$
$\operatorname{Max}\left(\mu_{12}{ }^{\mathrm{c}}, \mu_{22}{ }^{\mathrm{c}}\right)=\operatorname{Max}\left(\mu_{12}{ }^{\mathrm{c}}(\mathrm{u}, \mathrm{v}), \mu_{22}{ }^{\mathrm{c}}(\mathrm{u}, \mathrm{v})\right)=0$ [since $u \in V_{1}$ and $v \in V_{2}$ ]
This implies $\left(\mu_{11}+\mu_{21}, \mu_{12}+\mu_{22}\right)^{c}=\left(\mu_{11}{ }^{c} \cup \mu_{21}{ }^{c}, \mu_{12}{ }^{c} \cup \mu_{22}{ }^{c}\right)$
$\left(\gamma_{11}+\gamma_{21}, \gamma_{12}+\gamma_{22}\right)^{\mathrm{c}}=\left(\gamma_{11}{ }^{c} \cup \gamma_{21}{ }^{c}, \gamma_{12}{ }^{c} \cup \gamma_{22}{ }^{c}\right)$
Hence proved.
Proposition 2.5 Let $G_{1}$ and $G_{2}$ be product partial intuitionistic fuzzy sub graph, then

$$
\begin{aligned}
& \left(\left(\mu_{11} \cup \mu_{21}\right)^{c},\left(\mu_{12} \cup \mu_{22}\right)^{c}\right)=\left(\mu_{11}{ }^{c}+\mu_{21}{ }^{c}, \mu_{12}{ }^{c}+\mu_{22}{ }^{c}\right) \\
& \left(\left(\gamma_{11} \cup \gamma_{21}\right)^{c},\left(\gamma_{12} \cup \gamma_{22}\right)^{c}\right)=\left(\gamma_{11}{ }^{c}+\gamma_{21}{ }^{c}, \gamma_{12}{ }^{c}+\gamma_{22}{ }^{c}\right)
\end{aligned}
$$

## Proof:

Case 1: If $u \in V_{1}$, then $\left(\mu_{11} \cup \mu_{21}\right)^{c}(u)=\left(\mu_{11} \cup \mu_{21}\right)(u)=\mu_{11}(u)$ $\left(\mu_{11}{ }^{\mathrm{c}}+\mu_{21}{ }^{\mathrm{c}}\right)(\mathrm{u})=\max \left(\mu_{11}(\mathrm{u}), \mu_{21}(\mathrm{u})\right)^{\mathrm{c}}=\max \left(\mu_{11}{ }^{\mathrm{c}}(\mathrm{u}), \mu_{21}{ }^{\mathrm{c}}(\mathrm{v})\right)=$ $\mu_{11}(\mathrm{u})$
This implies $\left(\left(\mu_{11} \cup \mu_{21}\right)^{\mathfrak{c}}=\left(\mu_{11}{ }^{\mathrm{c}}+\mu_{21}{ }^{\mathrm{c}}\right)\right.$. similarly we can prove $\mathrm{u} \in \mathrm{V}_{2}$.
Case 2: If $(u, v) \in X_{1}$, then $u, v \in V_{1}$, therefore
$\left(\mu_{12} \cup \mu_{22}\right)^{c}(u, v)=\left(\mu_{11} \cup \mu_{21}\right)(u) \times\left(\mu_{11} \cup \mu_{21}\right)(v)-\left(\mu_{12} \cup\right.$ $\left.\mu_{22}\right)(\mathrm{u}, \mathrm{v})$

$$
\begin{aligned}
& =\mu_{11}(\mathrm{u}) \times \mu_{11}(\mathrm{v})-\mu_{12}(\mathrm{u}, \mathrm{v}) \\
& =\mu_{12}{ }^{c}(\mathrm{u}, \mathrm{v}) .
\end{aligned}
$$

Case 3: If $(u, v) \in X_{2}$, then $u, v \in V_{2}$, therefore
$\left(\mu_{12} \cup \mu_{22}\right)^{c}(u, v)=\left(\mu_{11} \cup \mu_{21}\right)(u) \times\left(\mu_{11} \cup \mu_{21}\right)(v)-\left(\mu_{12} \cup\right.$ $\left.\mu_{22}\right)(\mathrm{u}, \mathrm{v})$

$$
\begin{aligned}
& =\mu_{21}(\mathrm{u}) \times \mu_{21}(\mathrm{v})-\mu_{22}(\mathrm{u}, \mathrm{v}) \\
& =\mu_{22}{ }^{c}(\mathrm{u}, \mathrm{v}) .
\end{aligned}
$$

Case 4: If $(u, v) \in X$, then $u \in V_{1}$ and $v \in V_{2,}$, therefore $\left(\mu_{12} \cup \mu_{22}\right)^{c}(u, v)=\left(\mu_{11} \cup \mu_{21}\right)(u) \times\left(\mu_{11} \cup \mu_{21}\right)(v)-\left(\mu_{12} \cup\right.$ $\left.\mu_{22}\right)(\mathrm{u}, \mathrm{v})$

$$
\begin{aligned}
& =\mu_{11}(\mathrm{u}) \times \mu_{21}(\mathrm{v}) \\
& \quad\left[\text { since } \mu_{12}(u, v)=\mu_{22}(u, v)=0\right] \\
& =\mu_{11}{ }^{\mathrm{c}}(\mathrm{u}) \times \mu_{21}{ }^{\mathrm{c}}(\mathrm{v}) \\
& =\mu_{12}{ }^{\mathrm{c}}+\mu_{22}{ }^{c}
\end{aligned}
$$

Hence proved.
Proposition 2.6 Let $G_{1}$ and $G_{2}$ be product partial intuitionistic fuzzy sub graph, then $\mathrm{G} 1 \times \mathrm{G}_{2}$ be product partial intuitionistic fuzzy sub graph

Proof: $u_{1}, v_{1} \in V_{1}$ and $u_{2}, v_{2} \in V_{2}$, we have
$\left(\mu_{12} \times \mu_{22}\right)\left(\left(u 1, u_{2}\right),\left(v_{1}, v_{2}\right)=\mu_{12}\left(u_{1}, v_{1}\right) \times \mu_{22}\left(u_{2}, v_{2}\right)\right.$

$$
\begin{aligned}
& \leq\left[\mu_{11}\left(u_{1}\right) \times \mu_{11}\left(v_{1)}\right] \times\left[\mu _ { 2 1 } \left(u_{2)} \times \mu_{21}\left(v_{2)}\right]\right.\right.\right. \\
& =\left[\mu_{11} u_{1}\right) \times \mu_{21}\left(\mathrm{u}_{2)}\right] \times\left[\mu_{11}\left(\mathrm{v}_{1)} \times \mu_{21}\left(\mathrm{v}_{2}\right)\right]\right. \\
& =\left(\mu_{11} \times \mu_{21}\right)\left(\mathrm{u}_{1}, \mathrm{u}_{2}\right) \times\left(\mu_{11} \times \mu_{21}\right)\left(\mathrm{v}_{1}, \mathrm{v}_{2}\right)
\end{aligned}
$$

Therefore $\left(\mu_{12} \times \mu_{22}\right)\left(\left(u_{1}, u_{2}\right),\left(v_{1}, v_{2}\right)\right) \leq\left(\mu_{11} \times \mu_{21}\right)\left(u_{1}, u_{2}\right) \times$
$\left(\mu_{11} \times \mu_{21}\right)\left(v_{1}, v_{2}\right)$
similarly we prove
$\left(\gamma_{12} \times \gamma_{22}\right)\left(\left(u_{1}, u_{2}\right),\left(v_{1}, v_{2}\right) \leq\left(\gamma_{11} \times \gamma_{21}\right)\left(u_{1}, u_{2}\right) \times \gamma_{11} \times \gamma_{21}\right)\left(v_{1}, v_{2}\right)$ Hence proved.

## 3. CONCLUSION

We are able to find the different types of intuitionistic fuzzy graph and its properties. Further we are try to find the engineering applications of the different types of intuitionist fuzzy graph.

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