

# CIR Samples and Channel Taps Based Windowed-DFT Channel Estimation for MIMO-OFDM System

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## ABSTRACT

For high capacity demand for 4G broadband multimedia and internet services with high quality of service (QoS), multiple input multiple output (MIMO) transmission methods are exploited for high speed machine to machine communication. In order to mitigate the fast fading multipath effects, orthogonal frequency division multiplexing (OFDM) is used as modulation technique for MIMO systems. Time-domain channel characteristics are used to make efficient channel state information (CSI) available at the transmitter side. In this paper Windowed-DFT technique is optimized for channel estimation and the performance and complexity are determined. Computer Simulations are used to show the effect of channel filter length and the number of multipath channel taps.

## Keywords

OFDM, MIMO, Windowed-DFT, CIR Samples, LSE, LMMSE

## 1. INTRODUCTION

Due to spectrum unavailability, the demand of increased bandwidth cannot be fulfilled for high data rate wireless systems. In such situations, multiple transmit and receive antennas can be used to obtain transmit diversity and to increase the system capacity, combined with OFDM. MIMO technique can be used either to maximize the power efficiency, to increase capacity or to exploit the channel knowledge at the transmitter to increase the system capacity [1].

Channel can be estimated by using any method: pilot assisted channel estimation, blind channel estimation or semi-blind channel estimation [2]. Though channel estimation by inserting the pilot sequence gives good performance but the spectrum utilization efficiency decreases. Blind estimation is not applicable in real-time wireless communication [3] due to high complexity and slow convergence rate so semi-blind approach is preferred in which pilot signals are superimposed onto the transmitting data sequence.

Both frequency and time domains can be used to estimate the channel. The estimation of pilots in frequency domain can be obtained by using LSE or LMMSE method. LSE is preferred due to less complexity but for better performance LMMSE is used which makes use of SNR and channel statistics. FOR MIMO-OFDM system, LSE is proposed for a system having large number of multipath channel taps, approximately 40, and with channel length of 30 CIR samples. But for less channel taps, LMMSE is preferred [4]. For time domain channel estimation, MST approach can be used for DFT-based and DCT-based channel estimation. For OFDM system of 5-10 channel taps and channel filter length of 2-5 CIR samples, these transform-based estimation approaches are used, as proposed in [5] [6].

The up-sampling caused by the insertion of pilots, the noise effect spreads over the whole CIR samples. This noise can be

suppressed by using a weighting function, like Hanning Window or Hamming Window. In this paper Windowed-DFT approach will be optimized while considering its performance and complexity comparison, based on CIR Samples and channel taps, with DFT and DCT based channel estimation techniques. Performance will be given in terms of Mean Square Error and complexity will be in terms of computational time.

## 2. MIMO-OFDM SYSTEM MODEL

Consider a MIMO-OFDM system having  $N_T$  transmit antennas and  $M_R$  receive antennas with  $k$  subcarriers used for OFDM symbols. The quasi-static channel between  $i^{th}$  transmit antenna and  $j^{th}$  receive antenna can be written as

$$\mathbf{h}_{i,j} = [h_{i,j}[0] \dots h_{i,j}[L-1]]^T$$

Where  $L$  is the number of channel taps. Suppose  $i^{th}$  antenna transmits  $n^{th}$  OFDM symbol over  $k^{th}$  subcarrier, then this transmitted sequence can be written as

$$\mathbf{X}_i(n) = [X_i(0,n) \dots X_i(K-1,n)]_{K \times 1}^T$$

In time-domain the transmitted sequence can be written as  $\mathbf{F}^H \mathbf{X}_i(n)$  where  $\mathbf{F}$  is  $N$ -point DFT matrix, given by [7]

$$\mathbf{F}(x,y) = \frac{1}{\sqrt{N}} e^{-j\pi(x-1)(y-1)/N}$$

After passing through the multipath frequency selective fading channel, the received signal is [8]

$$\mathbf{Y}_i(n) = \sum_{j=1}^{N_T} \mathbf{H}_{i,j} \mathbf{F}^H \mathbf{X}_j(n) + \mathbb{N}_j(n) \\ j = 1, 2, \dots, M_R$$

Where  $\mathbb{N}_j(n)$  is  $M_R \times 1$  unknown noise received at  $j^{th}$  antenna and  $\mathbf{H}_{i,j}$  is  $N_T \times M_R$  channel matrix, obtained by

$$\mathbf{H}_{i,j} = DFT(\mathbf{h}_{i,j})$$

In matrix form, the overall system can be written as

$$\begin{bmatrix} \mathbf{Y}_1(n) \\ \vdots \\ \mathbf{Y}_{M_R}(n) \end{bmatrix}_{M_R \times 1} = \begin{bmatrix} \mathbf{H}_{1,1} & \dots & \mathbf{H}_{N_T,1} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{1,M_R} & \dots & \mathbf{H}_{N_T,M_R} \end{bmatrix}_{M_R \times N_T} \cdot \begin{bmatrix} \mathbf{X}_1(n) \\ \vdots \\ \mathbf{X}_{N_T}(n) \end{bmatrix}_{N_T \times 1} + \begin{bmatrix} \mathbf{N}_1(n) \\ \vdots \\ \mathbf{N}_{M_R}(n) \end{bmatrix}_{M_R \times 1}$$

For coherent detection, the pilot symbols are inserted in the transmitting OFDM data block. For MIMO-OFDM system, 3-Dimensional channel estimation is performed to increase the bandwidth efficiency [7]. In LTE, the positions of the pilot

sequence are shown in Fig.1 . Fig.2 shows a block diagram of necessary steps carried out in MIMO-OFDM system of LTE.

### 3. WINDOWED-DFT CHANNEL ESTIMATION

The use of Most Significant Taps (MST) method for suppression of interference and noise, results in the energy leakage, which degrades the performance of direct DFT-based channel estimation technique, especially for non-integer spaced multipath delays [6]. For alleviation of energy leakage effect, two techniques are proposed: Windowed based DFT channel estimation and second

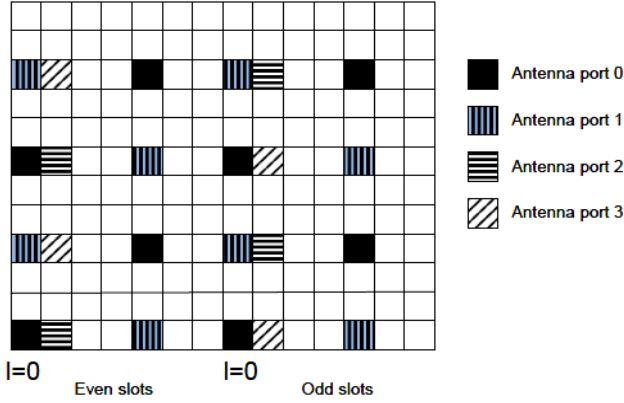


Fig 1: Frame Structure for LTE [7]

approach is the addition of the virtual channel frequency response (VCFR).

Initially the channel is estimated by Least Square (LS) method, which is given by [4]

$$\hat{H}_{r,t}^{LSE}[k] = X_t^{-1}[k] y_r[k]$$

Hanning Window is applied to this estimated channel to reduce the leakage effect, so

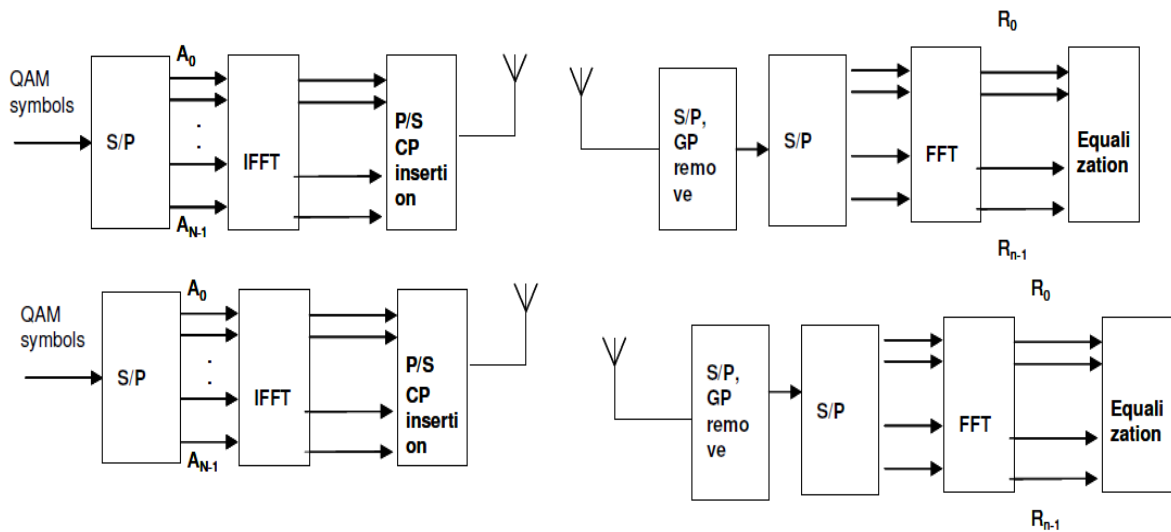


Fig 2: MIMO-OFDM System of LTE [8]

$$\hat{H}_{r,t}^{Windowed}[k] = \hat{H}_{r,t}^{LSE}[k] \cdot d(m)$$

Where

$$d(m) = \left(0.5 + 0.5 \cos \frac{2\pi m}{\Gamma}\right) e^{j\pi \left(\frac{m\Delta}{MT}\right)} \quad m = 1, 2, \dots, N-1$$

Exponential term shows the phase rotation.  $\Delta$  is guard interval length of OFDM and  $T = \frac{T_{carrier}}{N}$

Where  $\frac{1}{T_{carrier}}$  is spacing between OFDM subcarriers.

Now M-point IFFT operation is performed to convert the channel frequency to time-domain channel impulse response, after padding zeros. IFFT operation results in

$$\hat{h}_{r,t}^{Windowed}[i] = \frac{1}{M} \sum_m \hat{H}_{r,t}^{Windowed}[k] e^{\frac{j2\pi mi}{M}} \quad -\frac{M}{2} + 1 \leq i \leq \frac{M}{2}$$

To reduce MSE, a weighting function can also be applied to  $\hat{h}_{r,t}^{Windowed}$  as proposed in [9].

In next step, N-point FFT operation is applied to get frequency domain channel response

$$\hat{H}_{r,t}^{Windowed}[k] = \frac{1}{M} \sum_{i=-\frac{N}{2}+1}^{\frac{N}{2}} \hat{h}_{r,t}^{Windowed}[i] e^{\frac{j2\pi ki}{N}}$$

In last step, windowing effect and phase rotation is removed to get the estimated channel, so

$$\hat{H}_{r,t}[k] = \frac{\hat{H}_{r,t}^{Windowed}[k]}{d'(m)} \quad n = -\frac{N}{2}, \dots, \frac{N}{2}$$

Where

$$d'(m) = \left(0.5 + 0.5 \cos \frac{2\pi m}{\Gamma}\right) e^{j\pi \left(\frac{m\Delta}{NT}\right)}$$

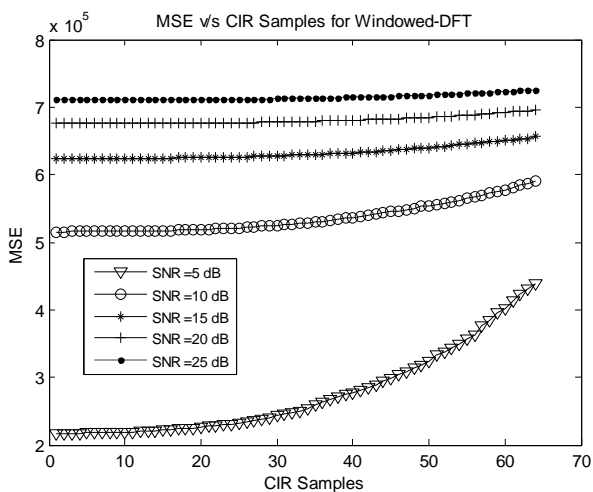
### 4. SIMULATION RESULTS

In this section, the comparison of Windowed-DFT based channel estimation technique is presented in form of Mean Square Error (MSE) and computational time as a function of CIR samples and channel taps. Monte Carlo Simulations are performed for  $2 \times 2$  MIMO system and FFT size is 64 for OFDM with cyclic prefix length of 16. The channel noise is taken of Rayleigh distribution. Maximum number of multipath channel taps are 64 and a filter length of maximum 64 CIR samples is simulated for this MIMO-OFDM system.

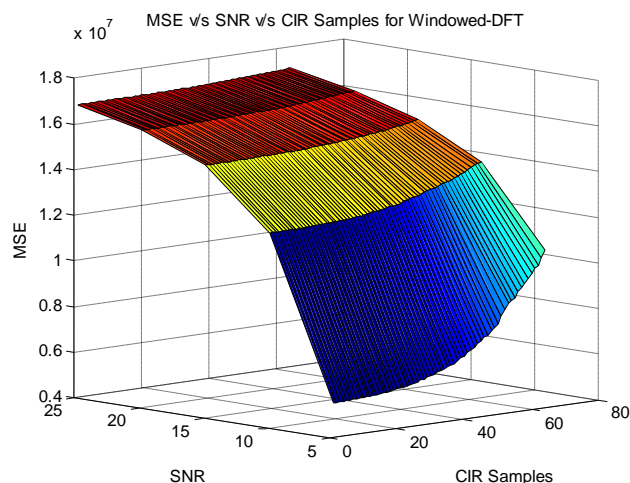
The effect of different channel filter lengths on performance of Windowed-DFT channel estimator is shown in Fig.3. According to this figure, we observe that for low SNR, less number of CIR samples are preferred, approximately of length 0-20, and high value of CIR samples not only degrades the performance but also increase the computational time. As we go on increasing the SNR value, a proportional increase on filter length gives acceptable performance. For example for SNR=25dB, performance remains almost same for all CIR samples. For reliable communication, a channel filter of smaller length is used at low SNR values, as shown in Fig.4.

For high CIR samples, any SNR value can be used because it gives the same performance. The computational time of Windowed-DFT channel estimator for different CIR samples is given in Table.1, as compared to other transform-based channel estimators. For low CIR samples the complexity of Windowed-DFT CE is less than DCT-CE but greater than direct DFT method.

30% increment in filter length increases the computational time by 6.7% while for 100% increment, computational time increases by 30%.



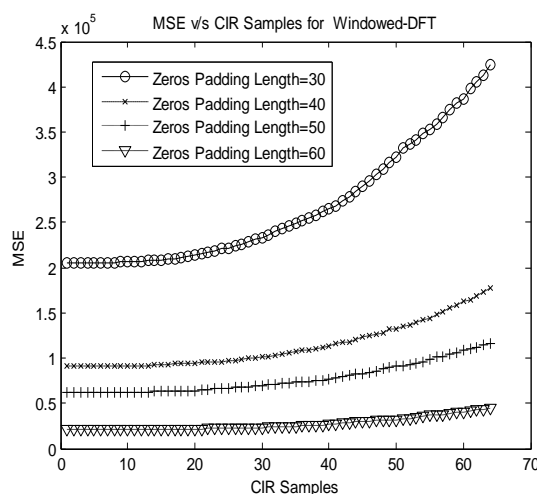
**Fig 3: MSE v/s CIR Samples for Windowed-DFT CE**



**Fig 4: MSE as a function of SNR and CIR Samples**

**TABLE 1. COMPLEXITY COMPARISON**

CIR Samples	5	20	40
Windowed-DFT	0.89	0.015	.02
DFT	.0812	.012	.02148
IDCT/DCT	0.998	.01343	.0184
DCT/IDCT	1.1074	.0153	.0179



**Fig 5: MSE for different Zero Padding Lengths**

To make Windowed-DFT approach, less complex than DFT, a filter length containing higher values of CIR samples is used but in such cases, its complexity will increase than DCT approach.

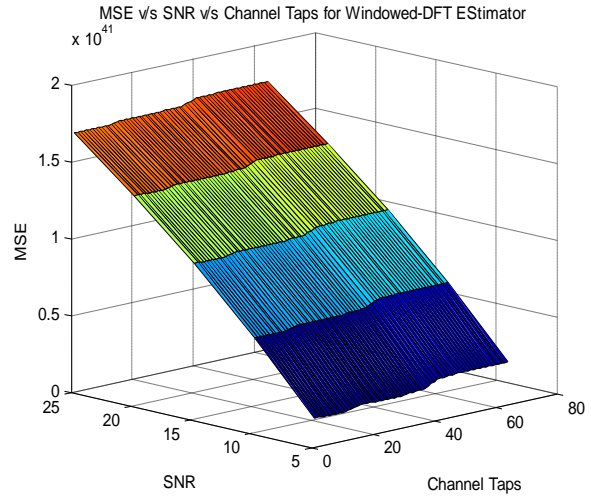
The effect of Zero padding lengths for varying CIR samples is shown in Figure.5. Performance will be better for case of more zeros as in such situation, we consider only high energy CIR samples and less energy components are discarded to remove the noise effect. As we go on padding less number of zeros, the effect of varying CIR samples also becomes significant. For example for more number of zeros, performance does not change by increasing CIR samples but for less number of zeros, the performance shows same behavior for less CIR samples, but as CIR samples increases, the performance degrades and complexity also increases.

**TABLE 2. COMPLEXITY COMPARISON FOR DIFFERENT CHANNEL TAPS**

Channel Taps	3	10	20
Windowed-DFT	.0018	.0018	.0021
DFT	.0251	.0016	.0016
IDCT/DCT	.0021	.002	.0023
DCT/IDCT	.002	.0023	.0022

The performance as a function of channel taps is shown in Fig.6. Performance is better for low SNR for any value of channel taps. For any specific SNR value, the performance degrades slightly for high number of channel taps. The combined effect of SNR and channel taps is given in Fig.7. According to this figure less number of channel taps at low SNR are preferred for better performance. For a specific value of channel taps MSE increase almost linearly by increasing SNR. The comparison of computational time of Windowed-DFT with other methods is given in Table.2. According to Table.2, Windowed-DFT requires more computational time than DFT-CE but its complexity is less than DCT-CE. The effect of increasing number of channel taps on complexity is not considerable so we can use any number of channel taps, from complexity point of view.

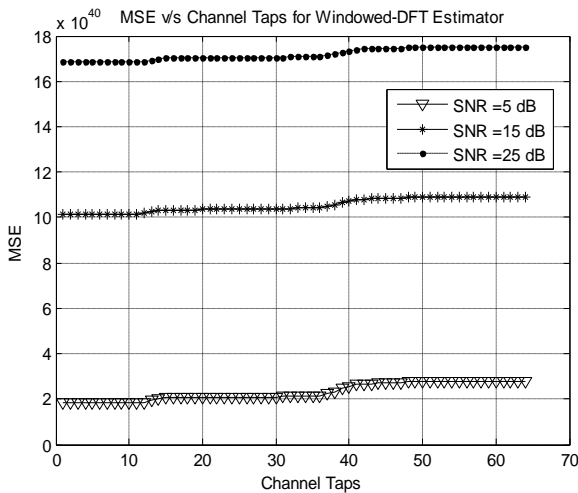
The computational time for 5000 simulations and 1 OFDM symbol is shown in Table.3. By increasing



**Fig 7: MSE vs SNR vs Channel Taps**

channel taps from 5 to 20, complexity increases approximately 126%. But further increment of channel taps from 20 to 64, causes 33% more computational time only.

The effect of zero padding lengths on MSE for different channel taps is demonstrated in Fig.8. Higher the number of zeros padded to CIR, the better will be performance. For any particular zero-length, performance remains same for channel taps less than 10, but as we further increase channel taps, the performance degrades but this effect comes into observation after specific interval length so normally, channel taps less than 10 are proposed, irrespective of the zero-length. The performance comparison of Windowed-DFT for different zero-lengths is given in Fig.9. We note that for less number of zeros, Windowed-DFT performs better than DFT-CE but as we increase zeros then its performance degrades as compared to DFT so to use Windowed-DFT, we prefer less number of zeros to be padded to channel impulse response. The complexity comparison of Windowed-DFT for different channel taps and for different zero lengths is given in Table.4.



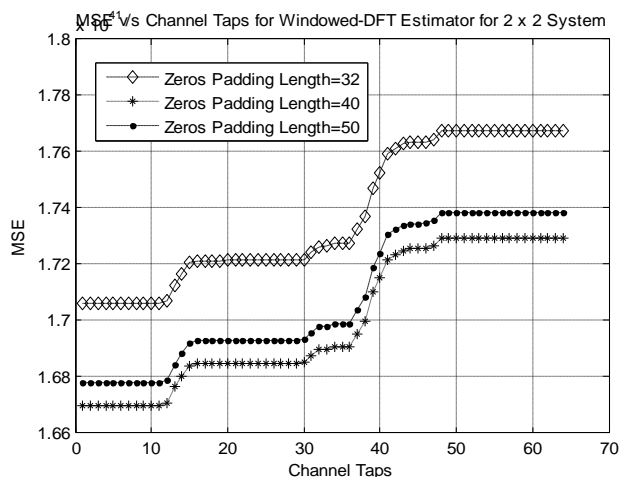
**Fig 6: MSE as a function of Channel Taps**

**TABLE 3. COMPUTATIONAL TIME OF WINDOWED-DFT**

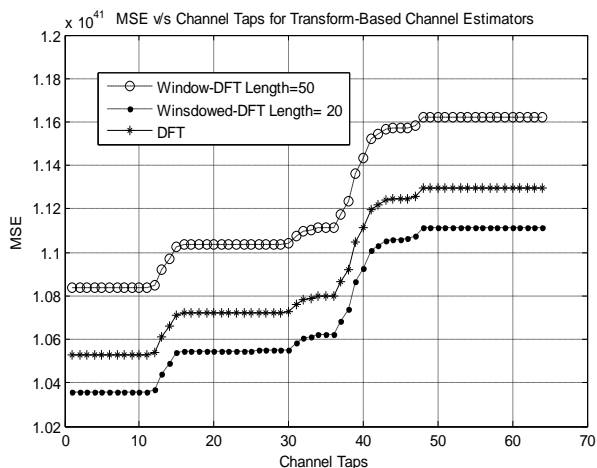
Channel Taps	5000 Simulations (sec)	1 OFDM Symbol (sec)	1 Bit (sec)
5	3.31	0.051	.025
20	7.5	0.12	.06
64	10	0.15625	.078

## 5. CONCLUSION

In this paper, pilot-assisted channel estimation is performed for  $2 \times 2$  MIMO-OFDM system, in which already known pilots are multiplexed along with the transmitting OFDM data. Windowing Function is applied to remove the effect of noise from channel impulse response. For reliable and spectrally efficient transmission, a linear proportional relationship is required between the operating SNR and channel filter length. If channel filter has high length, then any SNR value can be used for that communication system and in such situation another advantage comes in form of reduced complexity than DFT-CE. To make performance more efficient and independent of CIR samples, more zeros are padded to channel impulse response. For low SNR, less number of channel taps are preferred for improved performance and reduced complexity. But the disadvantage of more zeros is only that performance of Windowed-DFT degrades than that of DFT.



**Fig 8: MSE as a function of Channel Taps**



**Fig 9: MSE vs Channel Taps for different Channel Estimators**

**TABLE 4. COMPUTAIONAL TIME FOR DIFFERENT ZERO-PADDINGS**

Channel Taps	Zero-Padding Lengths		
	32	40	50
5	.0021	.0017	.0022
20	.0698	.0015	.0711
64	.098	.0896	.082

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