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Anti Q-Fuzzy Group and Its Lower Level Subgroups

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ABSTRACT
In this paper, we define the algebraic structures of anti Q-fuzzy subgroup and some related properties are investigated. The purpose of this study is to implement the fuzzy set theory and group theory in anti-Q fuzzy subgroups. Characterizations of lower level subsets of an anti-Q fuzzy subgroup of a group are given.

Keywords

Fuzzy set , Q-fuzzy set, fuzzy subgroup , Q-fuzzy subgroup, anti-Q fuzzy subgroups.

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1. INTRODUCTION

K.H.Kim introduce the concept of intuitionistic Q-fuzzy semi prime ideals in semi groups and Osman kazanci, sultan yamark and serife yilmaz introduce the concept of intuitionistic Q-fuzzy R-subgroups of near rings and F.H. Rho, K.H.Kim, J.G Lu introduce the concept of intuitionistic Q-fuzzy subalgebras of BCK / BCI– algebras. A.Solairaju and R.Nagarajan introduce and define a new algebraic structure of Q-fuzzy groups. In this paper we define a new algebraic structure of anti Q-fuzzy subgroups and study some their related properties.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

2.1 Definition

Let S be any non empty set. A fuzzy subset A of S is a function $A: S \rightarrow [0,1]$.

2.2 Definition

Let G be a group. A fuzzy subset A of G is called a fuzzy subgroup if for $x, y \in G$,

(i) $A(xy) \geq \min \{ A(x) , A(y)\}$,

(ii) $A(x^{-1}) = A(x)$.

2.3 Definition

Let G be a group. A fuzzy subset A of G is called an anti fuzzy subgroup if for $x, y \in G$,

(i) $A(xy) \leq \max \{ A(x) , A(y)\}$,

(ii) $A(x^{-1}) = A(x)$.

2.4 Definition

Let Q and G be any two sets. A mapping $A : G \times Q \rightarrow [0,1]$ is called a Q–fuzzy set in G.

2.5 Definition

A Q-fuzzy set 'A' is called Q-fuzzy group of a group G if for $x, y \in G$, $q \in Q$,

(i) $A(xy , q) \geq \min \{A(x,q), A(y,q)\}$

(ii) $A(x^{-1}, q) = A (x,q)$.

2.6 Definition

A Q-fuzzy set 'A' is called an anti Q-fuzzy group of a group G if for $x, y \in G$, $q \in Q$,

(i) $A(xy , q) \leq \max \{A(x,q), A(y,q)\}$

(ii) $A(x^{-1}, q) = A (x,q)$.

3. Properties of anti Q-fuzzy subgroups

In this section, we discuss some of the properties of anti Q-fuzzy subgroups.

3.1 Theorem

Let 'A' be an anti Q-fuzzy subgroup of a group G then

(i) $A(x, q) \geq A(e, q)$ for all $x \in G$, $q \in Q$ and e is the identity element of G.

(ii) The subset $H = \{x \in G \,/\, A(x, q) = A(e, q)\}$ is a subgroup of G.

Proof

(i) Let $x \in G$ and $q \in Q$.

$$\begin{aligned} A(x, q) &= \max\{A(x, q), A(x, q)\} \\ &= \max\{A(x, q), A(x^{-1}, q)\} \\ &\geq A(xx^{-1}, q) \\ &= A(e, q). \end{aligned}$$

$$A(x, q) \geq A(e, q).$$

(ii) Let $H = \{x \in G \,/\, A(x, q) = A(e, q)\}$.

Clearly H is non-empty as $e \in H$. Let $x, y \in H$.

Then, $A(x, q) = A(y, q) = A(e, q)$.

$$\begin{aligned} A(xy^{-1}, q) &\leq \max\{A(x, q), A(y^{-1}, q)\} \\ &= \max\{A(x, q), A(y, q)\} \\ &= \max\{A(e, q), A(e, q)\} \\ &= A(e, q). \end{aligned}$$

That is, $A(xy^{-1}, q) \leq A(e, q)$ and obviously

$$A(xy^{-1}, q) \geq A(e, q).$$

Hence, $A(xy^{-1}, q) = A(e, q)$ and $xy^{-1} \in H$.

Clearly, H is a subgroup of G.

3.2 Theorem

If 'A' is a Q-fuzzy subgroup of G, iff A^C is an anti Q-fuzzy subgroup of G.

Proof

Suppose A is a Q-fuzzy subgroup of G. Then for all x, y $\in$ G and $q \in Q$,

$$A(xy, q) \geq \min\{A(x, q), A(y, q)\}$$

$$\Leftrightarrow \quad 1 - A^c(xy, q) \geq \min\{(1 - A^c(x, q)), (1 - A^c(y, q))\}$$

$$\Leftrightarrow \quad A^c(xy, q) \leq 1 - \min\{(1 - A^c(x, q)), (1 - A^c(y, q))\}$$

$$\Leftrightarrow \quad A^c(xy, q) \leq \max\{A^c(x, q), A^c(y, q)\}.$$

We have, $A(x, q) = A(x^{-1}, q)$ for all x in G and $q \in Q$,

$$\Leftrightarrow \quad 1 - A^c(x, q) = 1 - A^c(x^{-1}, q).$$

Therefore, $A^c(x, q) = A^c(x^{-1}, q)$.

Hence A^c is an anti Q-fuzzy subgroup of G.

3.3 Theorem

Let A be any anti Q-fuzzy subgroup of a group G with identity e. Then $A(xy^{-1}, q) = A(e, q) \Rightarrow A(x, q) = A(y, q)$ for all x ,y in G and $q \in Q$.

Proof

Given A is an anti Q-fuzzy subgroup of G and $A(xy^{-1}, q) = A(e, q)$.

Then for all x , y in G and $q \in Q$,

$$\begin{aligned} A(x, q) &= A(x(y^{-1}y), q) \\ &= A((xy^{-1})y, q) \\ &\leq \max\{A(xy^{-1}, q), A(y, q)\} \\ &= \max\{A(e, q), A(y, q)\} \\ &= A(y, q). \end{aligned}$$

That is, $A(x, q) \leq A(y, q)$.

Now, $A(y, q) = A(y^{-1}, q)$, since A is an anti Q-fuzzy subgroup of G.

$$\begin{aligned} &= A(ey^{-1}, q) \\ &= A((x^{-1}x)y^{-1}, q) \\ &= A(x^{-1}(xy^{-1}), q) \end{aligned}$$

$\leq$ max $\{ A(x^{-1}, q), A(xy^{-1}, q)\}$

$=$ max $\{A(x, q), A(e, q)\}$

$=$ $A(x, q)$.

(i.e.) $A(y, q) \leq A(x, q)$.

Hence, $A(x, q) = A(y, q)$.

3.4 Theorem

A is an anti Q-fuzzy subgroup of a group G if and only if $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$, for all x, y in G and $q \in Q$.

Proof

Let A be an anti Q-fuzzy subgroup of a group G. Then for all x ,y in G and $q \in Q$,

$$A(xy, q) \leq \max\{A(x, q), A(y, q)\}$$

and
$$A(x, q) = A(x^{-1}, q).$$

Now, $A(xy^{-1}, q) \leq \max\{A(x, q), A(y^{-1}, q)\}$.

$= \max\{A(x, q), A(y, q)\}$

$\Leftrightarrow A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

4. Properties of Lower level subsets of an anti Q-fuzzy subgroup

In this section, we introduce the concept of lower level subset of an anti Q-fuzzy subgroup and discuss some of its properties.

4.1 Definition

Let A be an anti Q-fuzzy group of a group G. For any $t \in [0,1]$, we define the lower level subset of A is the set, $L(A; t) = \{ x \in G / A(x, q) \leq t \}$.

4.1 Theorem

Let A be an anti Q-fuzzy subgroup of a group G. Then for $t \in [0,1]$ such that $t \geq A(e, q)$, $L(A; t)$ is a subgroup of G.

Proof

For all $x, y \in L(A; t)$, we have,

$$A(x, q) \leq t ; \quad A(y, q) \leq t.$$

Now, $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

$A(xy^{-1}, q) \leq \max\{t, t\}$.

$A(xy^{-1}, q) \leq t$.

$$xy^{-1} \in L(A; t).$$

Hence $L(A; t)$ is a subgroup of G.

4.2 Theorem

Let G be a group and A be a Q-fuzzy subset of G such that $L(A; t)$ is a subgroup of G. For $t \in [0,1]$ such that $t \geq A(e)$, A is an anti Q-fuzzy subgroup of G.

Proof

Let x, y in G and $A(x) = t_1$ and $A(y) = t_2$.

Suppose $t_1 < t_2$, then $x, y \in L(A; t_2)$.

As $L(A; t_2)$ is a subgroup of G, $xy^{-1} \in L(A; t_2)$.

Hence, $A(xy^{-1}, q) \leq t_2 = \max\{t_1, t_2\}$

$\leq \max\{A(x, q), A(y, q)\}$

That is, $A(xy^{-1}, q) \leq \max\{A(x, q), A(y, q)\}$.

Hence A is an anti Q-fuzzy subgroup of G.

4.2 Definition

Let A be an anti Q-fuzzy subgroup of a group G. The subgroups L (A ; t) for $t \in [0,1]$ and $t \geq A(e)$, are called lower level subgroups of A.

4.3 Theorem

Let A be an anti Q-fuzzy subgroup of a group G. If two lower level subgroups L (A ; t_1), L (A ; t_2), for, $t_1, t_2 \in [0,1]$ and $t_1, t_2 \geq A(e, q)$ with $t_1 < t_2$ of A are equal then there is no x in G such that $t_1 < A(x, q) \leq t_2$.

Proof

Let L (A ; t_1) = L (A ; t_2).

Suppose there exists a $x \in G$ such that $t_1 < A(x, q) \leq t_2$ then L (A ; t_1) $\subseteq$ L (A ; t_2).

Then $x \in$ L (A ; t_2) , but $x \notin$ L (A ; t_1), which contradicts the assumption that, L (A ; t_1) = L (A ; t_2).Hence there is no x in G such that $t_1 < A(x, q) \leq t_2$.

Conversely, suppose that there is no x in G such that $t_1 < A(x, q) \leq t_2$.

Then, by definition, L (A ; t_1) $\subseteq$ L (A ; t_2).

Let $x \in$ L (A ; t_2) and there is no x in G such that $t_1 < A(x, q) \leq t_2$.

Hence $x \in$ L (A ; t_1) and L (A ; t_2) $\subseteq$ L (A ; t_1).

Hence L (A ; t_1) = L (A ; t_2).

4.4 Theorem

A Q-fuzzy subset A of G is an anti Q-fuzzy subgroup of a group G if and only if the lower level subsets L (A ; t), $t \in$ Image A, are subgroups of G.

Proof It is clear.

4.5 Theorem

Any subgroup H of a group G can be realized as a lower level subgroup of some anti Q-fuzzy subgroup of G.

Proof:

Let A be a Q-fuzzy subset and $x \in G$ and $q \in Q$.

Define,

$$A(x, q) = \begin{cases} 0 & \text{if } x \in H \\ t & \text{if } x \notin H \text{, where } t \in (0,1]. \end{cases}$$

We shall prove that A is an anti Q-fuzzy subgroup of G.

Let $x, y \in G$ and $q \in Q$.

i. Suppose $x, y \in H$, then $xy \in H$ and $xy^{-1} \in H$.

$A(x, q) = 0$, $A(y, q) = 0$, A and $A(xy^{-1}, q) = 0$.

Hence $A(xy^{-1}, q) \leq \max \{ A(x, q), A(y, q) \}$.

ii. Suppose $x \in H$ and $y \notin H$, then $xy \notin H$ and $xy^{-1} \notin H$.

$A(x, q) = 0$, $A(y, q) = t$ and $A(xy^{-1}, q) = t$.

Hence $A(xy^{-1}, q) \leq \max \{ A(x, q), A(y, q) \}$.

iii. Suppose $x, y \notin H$, then $xy^{-1} \in H$ or $xy^{-1} \notin H$.

$A(x, q) = t$, $A(y, q) = t$ and $A(xy^{-1}, q) = 0$ or t.

Hence $A(xy^{-1}, q) \leq \max \{ A(x, q), A(y, q) \}$.

Thus in all cases, A is an anti Q-fuzzy subgroup of G.

For this anti fuzzy subgroup, L (A ; t) = H.

Remark

As a consequence of the Theorem 4.3, the lower level subgroups of an anti Q-fuzzy subgroup A of a group G form a chain. Since

$A(e, q) \leq A(x, q)$ for all x in G and $q \in Q$, therefore $L(A; t_0)$, where $A(e, q) = t_0$ is the smallest and we have the chain :

$\{e\} \subset L(A; t_0) \subset L(A; t_1) \subset L(A; t_2) \subset \ldots \subset L(A; t_n) = G$, where $t_0 < t_1 < t_2 < \ldots\ldots < t_n$.

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