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Multirate DSP and its technique to reduce the cost of the analog signal conditioning filters

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ABSTRACT
There are many reasons to change the sample rate of a sampled data signal. Here we discuss the two basic operations in a multirate system i.e. decreasing (decimation) and (increasing) the sampling rate of a signal. Also the use of multirate filters at the interfaces of continuous & sampled data which results in a cost reduction of the analog signal conditioning components as well as improvement of signal quality.

1. INTRODUCTION
In single-rate systems, only one sampling rate is used throughout a digital signal processing systems, whereas in multirate systems the sampling rate is changed at least once. Multirate systems have gained popularity since the early 1980s and they are commonly used for audio and video processing, communication systems, and transform analysis to name but a few.

Some one humorously asked the question, "*Resampling!* Does that mean you didn't do it right the first time?" It is not actually, there are many reasons to change the sample rate of a sampled data signal. Applications include conversion of variable rate input data to fixed rate output data in a modulator and the inverse task of converting fixed rate input data to variable rate output data in a demodulator. Another application involves sample rate changes so that filtering can be performed at the Nyquist rate of the signal being processed. In one major application, the multirate filter is used to increase the sample rate of a sampled data signal prior to its delivery for processing by the digital to analog converter involved in transferring the signal between the sampled data world and the continuous world. In the other major application, the multirate filter is used to decrease the sample rate of a sampled data signal after being formed at the output of an analog to digital converter involved in transferring the signal between the continuous world and the sampled data world. [1]

2. DECIMATION
Decimation can be regarded as the discrete-time counterpart of sampling. Whereas in sampling we start with a continuous-time signal $x(t)$ and convert it into a sequence of samples $x[n]$, in

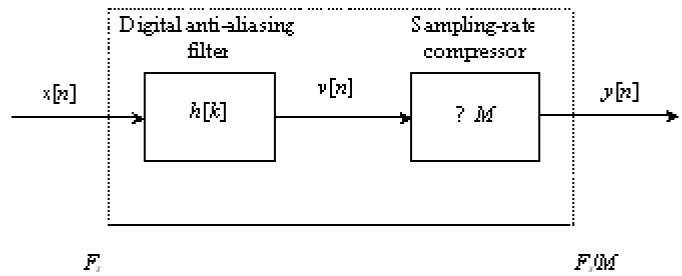

Figure1: Block diagram notation of decimation,by a factor of M

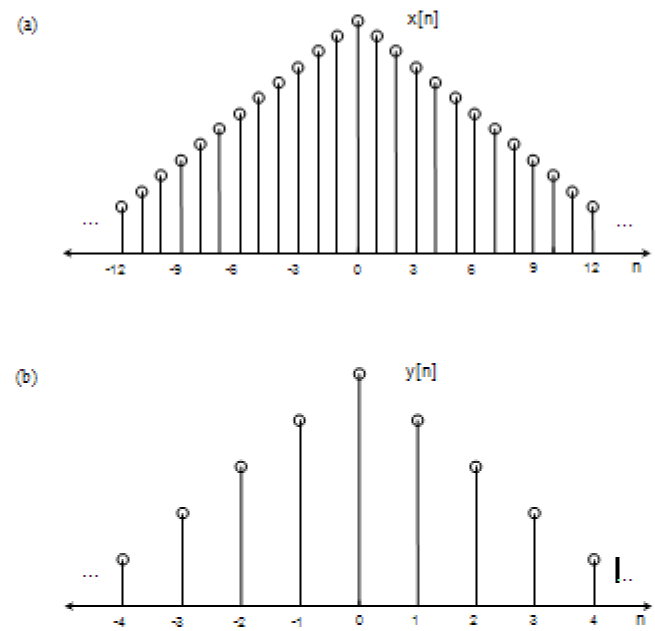

Figure 2: Decimation of a discrete-time signal by a factor of 3

decimation we start with a discrete-time signal $x[n]$ and convert it into another discrete time signal $y[n]$, which consists of *subsamples* of $x[n]$.

Thus, the formal definition of M-fold decimation, or down-sampling, is defined by equation 1. In decimation, the sampling rate is reduced from F_S to F_S/M by discarding $M - 1$ samples for every M samples in the original sequence. [1]

$$y[n] = v[nM] = \sum h[k]x[nM - k] \qquad (1)$$

The block diagram notation of the decimation process is depicted in Figure 1. An anti-aliasing digital filter precedes the downsampler to prevent aliasing from occurring, due to the lower sampling rate. The subject of aliasing in decimated signals is covered in more detail in Section II (A). In Figure 2, it illustrates the concept of 3-fold decimation i.e. $M = 3$. Here, the samples of $x[n]$ corresponding to $n = \dots, -2, 1, 4\dots$ and $n = \dots, -1, 2, 5\dots$ are lost in the decimation process. In general, the samples of $x[n]$ corresponding to $n \neq kM$, where k is an integer, are discarded in M-fold decimation. In Figure 2 (b), it shows samples of the decimated signal $y[n]$ spaced three times wider than the samples of $x[n]$. This is not a coincidence. In real time, the decimated signal appears at a slower rate than that of the original signal by a factor of M. If the sampling frequency of $x[n]$ is F_S, then that of $y[n]$ is F_S/M. [2]

2.1 Frequency Transforms of Decimated Sequences

The analysis of decimation is better understood by assessing the frequency spectrum using the Fourier transform. [2]

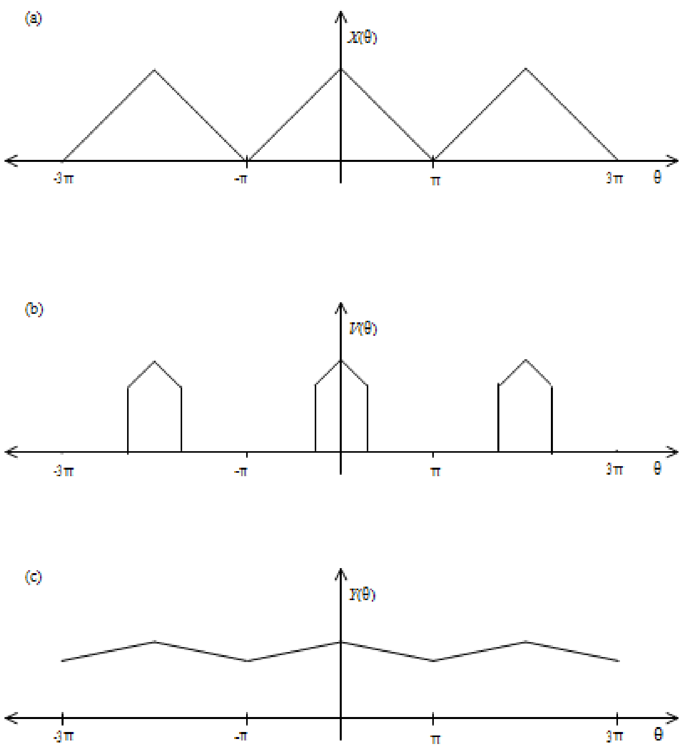

Figure 3: Aliasing caused by decimation; (a) Fourier Transform of the original signal; (b) After decimation Filtering; (c) Fourier transform of the decimated signal.

In Figure 3 (a) it shows the Fourier transform of the original signal. Part (b) shows the signal after lowpass filtering. In Figure 3 (c), it depicts the expanded spectrum after decimation.

The implications of aliasing caused by decimation are very similar to those in the case of sampling a continuous-time signal. In general, if the Fourier transform of a signal, $X(\theta)$, occupies the entire bandwidth from $[-\pi, \pi]$, then the Fourier transform of the decimated signal, $X_{(\downarrow M)}(\theta)$, will be aliased. This is due to the superposition of the M shifted and frequency-scaled transforms. This is illustrated in Figure 3, which shows the aliasing phenomenon for M = 3.

3. INTERPOLATION

Interpolation is the exact opposite of decimation. It is an information preserving operation, in that all samples of $x[n]$ are present in the expanded signal $y[n]$. The mathematical definition of L-fold interpolation is defined by Equation 2 and the block diagram notation is depicted in Figure 4. Interpolation works by inserting $(L–1)$ zero-valued samples for each input sample. The sampling rate therefore increases from F_S to LF_S. With reference to Figure 4, the expansion process is followed by a unique digital low-pass filter called an *anti-imaging filter*. Although the expansion process does not cause aliasing in the interpolated signal, it does however yield undesirable replicas in the signal's frequency spectrum. We shall see how this special filter, in Section III (A), is necessary to remove these replicas from the frequency spectrum [2].

$$Y[n] = L \sum h[k]w[n-k] \qquad (2)$$

In Figure 5 below, it depicts 3-fold interpolation of the signal $x[n]$ i.e. $L = 3$. The insertion of zeros effectively attenuates the signal by L, so the output of the anti-imaging filter must be multiplied by L, to maintain the same signal magnitude.

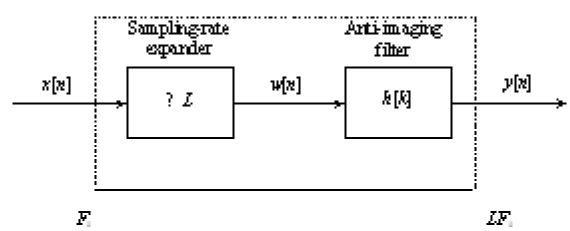

Figure 4: Block diagram notation of interpolation, by a factor of L

3.1 Frequency Transforms of Expanded Sequences

The effect of expansion on a signal in the frequency domain is illustrated in Figure 6 below. Part (a) shows the Fourier transform of the original signal; part (b) illustrates the Fourier transform of the signal with zeros added $W(\theta)$; and part (c) shows the Fourier transform of the signal after the interpolation filter. It is clearly visible that the shape of the Fourier transform is compressed by a factor L in the frequency axis and is also repeated L times in the range of $[-\pi, \pi]$. [1] Despite the compression of the signal in the frequency axis, the shape of the Fourier transform is still preserved, confirming that expansion does not lead to aliasing. These replicas are removed by a digital low-pass filter called an *anti-imaging* filter, as indicated in Figure 4.

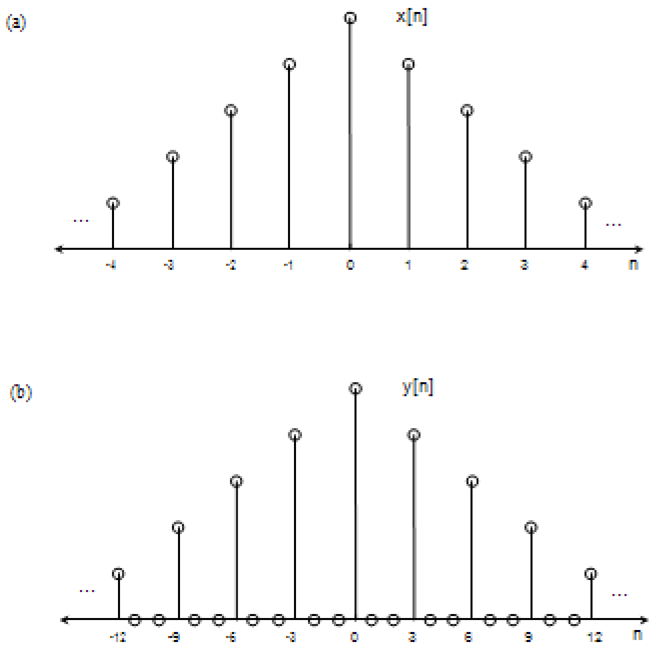

Figure 5: Interpolation of Discrete-time signal by a factor of 3.

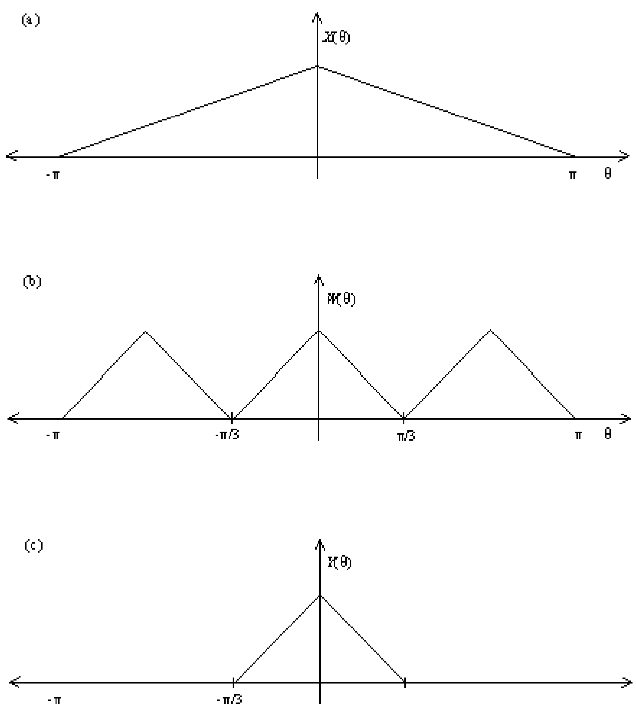

Figure 6: Expansion in the frequency domain of the original signal (a) and the expanded signal (b).

4. SAMPLING RATE CONVERSION

A common use of Multirate signal processing is for sampling-rate conversion. Suppose a digital signal x[n] is sampled at an interval T_1, and we wish to obtain a signal y[n] sampled at an interval T_2. Then the techniques of decimation and interpolation enable this operation, providing the ratio T_1/T_2 is a rational number i.e. L/M. Sampling-rate conversion can be accomplished by L-fold expansion, followed by low-pass filtering and then M-fold decimation, as depicted in Figure 7. It is important to emphasis that the interpolation should be performed first and decimation second, to preserve the desired spectral characteristics of x[n]. Furthermore by cascading the two in this manner, both of the filters can be combined into one single low-pass filter [3].

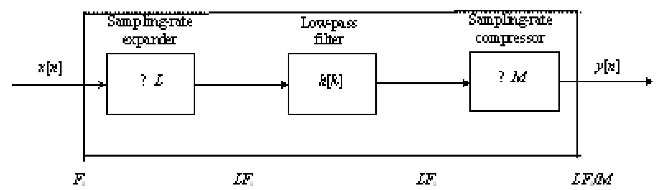

Figure 7: Sampling-rate conversion by expansion, filtering, and decimation.

An example of sampling-rate conversion would take place when data from a CD is transferred onto a DAT. Here the sampling-rate is increased from 44.1 kHz to 48 kHz. To enable this process the non integer factor has to be approximated by a rational number.

$$\frac{L}{M}=\frac{48}{44.1}=\frac{160}{147}=1.08844$$

Hence, the sampling-rate conversion is achieved by interpolating by *L* i.e. from 44.1 kHz to [44.1x160] = 7056 kHz. Then decimating by *M* i.e. from 7056 kHz to [7056/147] = 48 kHz.

5. MULTISTAGE APPROACH

When the sampling-rate changes are large, it is often better to perform the operation in multiple stages, where M_i (L_i), an integer, is the factor for the stage *i*.

$$M = M_1M_2\ldots M_I \quad \text{or } L = L_1L_2\ldots L_I$$

An example of the multistage approach for decimation is shown in figure 8. The multistage approach allows a significant relaxation of the anti-alias and anti-imaging filters, with a consequent reduction in the filter complexity. The optimum number of stages is one that leads to the least computational effort in terms of either the multiplications per second (MPS), or the total storage requirement (TSR).

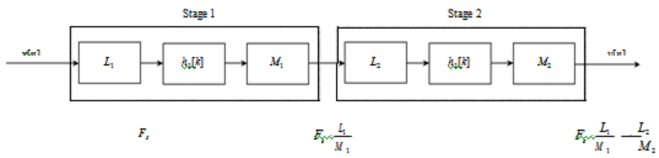

Figure 8: Multistage approach for the decimation process.

6. MULTIRATE DSP IN ANALOG SIGNAL CONDITIONING

6.1 Motivation

A ubiquitous task performed by many communications and multimedia system is that of converting a digital representation of a time waveform into a continuous time representation. This task is performed, for instance, when we convert the digital samples recovered from a compact Disk (CD) to a continuous time low voltage analog waveform. This waveform, in turn, is amplified and converted to sound pressure waves by earphones or loud speakers. Similarly, this task is performed at the output of a digital modulator, which converts digital samples of a modulated waveform to a continuous time low voltage analog waveform [4]. This waveform in turn is presented to an RF up-converter and transmitter for spectral translation and power amplification.

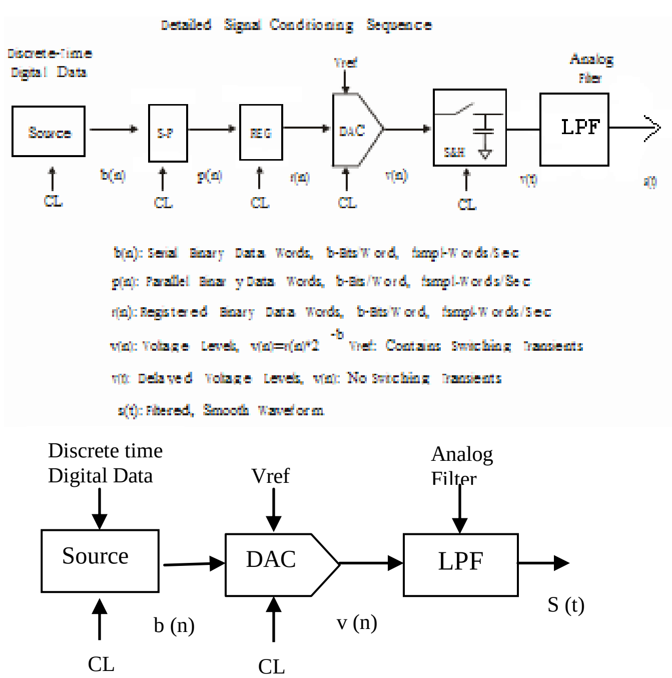

Figure 9: Detailed and Simplified Models of Signal Conditioning Sequence for D-to-A Conversion Process

One final example of this the need for the task is at the output of arbitrary function generators. The function generator pulls digital samples of a desired waveform from deep memory and converts these samples to a continuous time low voltage analog waveform to be applied as an input signal to a system under test.

This process of converting discrete time digital samples of a waveform to a continuous time analog waveform is performed by sequence of three operations. Figure 9 depicts the three operations performed by an analog-to-digital converter (ADC), a sample and hold (S&H), and an analog- smoothing filter. Figure 10 shows the time series that can be observed at each position in the signal conditioning stream, while figure 11 shows the spectral description at the same locations [4].

Examining figure 11, we note that the digital to analog conversion process subjects the reconstructed analog waveform to two forms of distortion. The first is the sin(x)/x spectral distortion due to the frequency response of the zero-order holds (the rectangle time response of the DAC) [9]. The second is the group delay distortion caused by the dominant poles of the analog smoothing (or reconstruction) filter following the DAC. As a reminder, the dominant poles are those close to the **jw** axis in the S- plane, and are near the filter transition band edges. These poles are characterized by large changes in phase over small changes in frequency, which has the effect of delaying signals near the band edge more than those near the band center. [5] This differential time delay over the filter bandwidth distorts the waveform passing through the filter. This distortion is called group delay distortion.

6.2 Multirate filter to correct distortion

When we examine the causes of distortion introduced be the DAC and the analog low pass filter we are guided to the following conclusions. The first is that the amplitude distortion caused by the DAC can be reduced if we have a larger ratio of sample rate to signal bandwidth. The affect of increasing the sample rate relative to the signal bandwidth can be seen in the frequency domain as an increased separation between the spectral replicates.

The spectral distortion caused by the main-lobe of the DAC sin(x)/x response is reduced since signal bandwidth occupies a smaller fraction of the main-lobe width. This effect can be seen in figure 12. The second effect of operating the DAC at higher sample rates (than the minimum Nyquist rate) can be seen in the levels of the residual spectra centered at multiples of the sample rate. [5]

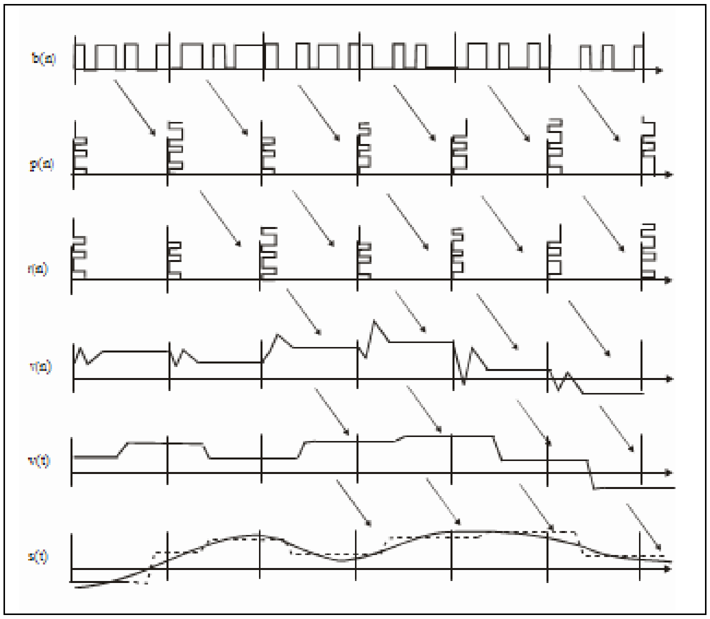

Figure 10 (a): Time Series for Detailed Model of Signal conditioning Sequence for Digital-to-Analog conversion Process.

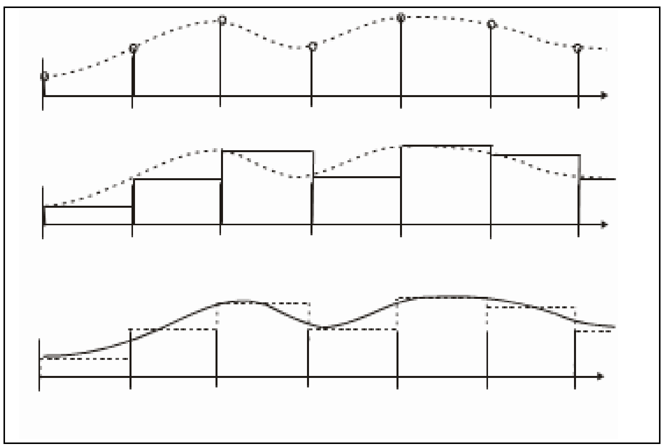

Figure 10 (b): Time Series for Simplified Model of Signal Conditioning Sequence for Digital-to-Analog Conversion Process.

The spectral levels are reduced since spectral replicates stay closer to the spectral zeros of the DAC's sin(x)/x. A third effect can also be attributed to operating the DAC at higher sample rates. Since the distance between the spectral residues is increased, the analog filter required to finish the reconstruction task can now have a wider transition bandwidth. Analog filters with wider transition bandwidth are lower order, hence have fewer components and reduced cost. An additional benefit of the lower order filter is that filters with wider transition bandwidth exhibit smaller amounts of group delay distortion. Hence operating a DAC with over sampled data results in lower cost analog filters, with reduced group delay distortion. [6]

We now have a quandary. We would like to reduce distortion by operating the DAC at rates above the Nyquist rate for the signal being processed but we don't want to store or process data at rates above the Nyquist rate since that results in increased storage or processing speed requirements. We resolve this sticky situation by collecting, storing, and processing sampled input data at the Nyquist rate, but raise the sample rate with a Multirate filter prior to passing the sample values on to the DAC for analog reconstruction.

The up-sampling operation, sometimes called zero packing, raises the input sample rate by inserting zero sampled data values between the input samples. This is demonstrated for a 1-to-4 up-sampler in figure 13, and the process can be visualized as a commutator operating at the desired up-sample rate accessing the input samples and inserted zero-value samples.

A moments thought leads us to the conclusion that the original input data and the zero inserted output data have the same Fourier transform. This is because the inserted zeros cannot contribute to the transform, which is simply a weighted summation of input samples. The zeros do accomplish something however. What they do is raise the sample rate. We know that the spectrum of a sampled data input signal is a periodic function with period (of spectral spacing) of **fs**, the input sample rate. We cannot, in general view the periodic spectrum. We can only view one cycle of the periodic spectrum, an interval called the primary strip of width **fs**, the reciprocal of the time interval between samples [6]. Fortunately the width of the strip and the spacing between spectral copies are the same so we can observe all the information in the spectrum when we perform a discrete Fourier transform (DFT). When we raise the sample rate by zero insertion, we redefine the Nyquist interval, but not the spacing between the spectral copies. Thus if we up-sample 1-to-4, the Nyquist interval widens by a factor of 4 and now spans 4 spectral cycles of the input periodic spectrum. Now that we can see the wider spectral interval, spanning 4 cycles of the input spectrum, we can pass the data through a sampled data filter to reject three of the four copies. The output of that filter now has a wider spacing between spectral copies, a distance that now matches the new sample rate. When the spectral spacing matches the higher spectral width, the output represents samples of the band-limited signal at the higher output rate. The process of up sampling and filtering to realize new samples of the output series is called interpolation. The spectral and time domain representation of this process is shown in figure 14.

7. PRACTICAL REALIZATIONS

The up-sampled digital-to-analog configuration still exhibits amplitude distortion from the DAC and group delay distortion from the analog low-pass smoothing filter. The amount of distortion has been reduced by the use of the up sampler, but it has not been completely eliminated. We note that the source and the amount of distortion caused by the analog signal conditioning are known. Standard design procedures completely eliminate these distortion terms by pre-compensating the digital low-pass filter following the upsampling commutator [6]. The inverse sin(x)/x amplitude is designed into the pass band of the digital filter as is the conjugate of the analog filter's phase distortion. The cascade of the pre-compensated digital filter, DAC, and analog lowpass filter essentially is an ideal filter reconstruction system. The analog signal formed at the output of the cascade process only contains quantization noise related to the number of bits used to represent the digital sampler.

The most common application of the reconstruction technique described here is in the playback systems for the Compact Disk Player. Data is extracted from the CD as left and right channel samples of 16-bits per sample at a data rate of 44.1 kHz [9]. Both channels are upsampled 1-to-4 in a pair of Polyphase filters to obtain output data of 16-bits per sample at a data rate of 176.4 kHz. The up-sampled data is then presented to a DAC and analog low-pass filter. The use of a 1-to-4 higher sample rate permits the analog filters to implement with less severe design constraints and at reduced cost. CD players that perform this function are identified by the term "four times over sampled". [7]

For completeness, we mention that many CD players continue to raise the sample rate by another factor of 16 by a second Multirate filter to obtain output 16-bit output samples at a sample rate of 64 **fs** or 2.8224 MHz This data is highly over sampled, hence highly correlated. This highly correlated data is presented to a digital sigma-delta converter that converts the 16-bit data (with 96 dB dynamic range) to 1-bit data (with the same 96 dB dynamic range over the signal's Nyquist band- width). The 1-bit data is presented to a 1-bit DAC and a very simple analog filter. The reasoning behind this sequence of operations is that it is less expensive to be fast in the digital world than it is to be accurate in the analog world. The 1-bit 2.82 MHz DAC only has to form 2 accurate output levels as opposed to the 16-bit 176.4 kHz DAC that has to form 65,536 accurate output levels. CD players that perform this form of DAC operation are identified by the term "1-bit MASH converter". [8]

In a similar fashion, over sampling analog-to-digital converters that use sigma-delta converters followed by Digital filters with Multirate capabilities have become the standard, cost effective, option of the audio community and of the instrumentation community.

8. APPLICATIONS OF MULTIRATE DSP

Multirate systems are used in a CD player when the music signal is converted from digital into analogue (DAC). Digital data (16-bit words) are read from the disk at a sampling rate of 44.1 kHz. If this data were converted directly into an analogue signal, image frequency bands centered on multiples of the sampling-rate would occur, causing amplifier overload, and distortion in the music signal. To protect against this, a common technique called *oversampling* is often implemented nowadays in all CD players and in most digital processing systems of music signals [8].

Figure 15 illustrates the procedure of converting a digital waveform into an analogue signal in a CD player using x8 oversampling. As an example, Figure 15 (a) illustrates a 20 kHz sinusoidal signal sampled at 44.1 kHz, denoted by $x[n]$. The six samples of the signal represent the waveform over two periods. If the signal $x[n]$ was converted directly into an analogue waveform, it would be very hard to exactly reconstruct the 20 kHz signal from this diagram. Now, Figure 15 (b) shows $x[n]$ with an x8 interpolation, denoted by $y[n]$. Figure 15(c) shows the analogue signal $y(t)$, reconstructed from the digital signal $y[n]$ by passing it through a DAC. Finally, Figure 15(d) shows the waveform of $z(t)$, which is obtained by passing the signal $y(t)$ through an analogue low-pass filter [9].

The effect of oversampling also has some other desirable features. Firstly, it causes the image frequencies to be much higher and therefore easier to filter out. The anti-alias filter specification can therefore be very much relaxed i.e. the cut- off frequency of the filter for the previous example increases from [44.1 / 2] = 22.05 kHz to [44.1x8 / 2] = 176.4 kHz after the interpolation.

One other attractive feature about oversampling is the effect of reducing the *noise power spectral density*, by spreading the noise power over a larger bandwidth. This is illustrated in Figure 16 and mathematical defined below by Equation 3

$$\text{Noise Power Spectral Density} = \frac{\text{Total Power}}{\text{Bandwidth}} \qquad (3)$$

For both sequences, the *total noise power* (shaded area in Figure 16) remains the same. However, as the bandwidth is increased by a factor of x8 because of the interpolation process, it causes the level of the noise power spectral density to decrease by a factor of x8, over the whole range of the bandwidth

As a consequence of the reduction in the noise power spectral density, it means that the level of tolerable noise can be increased by a factor of 8. In terms of the quantization noise power, q^2, it means that it can now be 8 times greater (or the quantization step size, q, can be increased by $\sqrt{8}$). This ultimately means that a reduction in the number of bits for the DAC is possible. [10]

9. CONCLUSION

We have presented a simple overview and common application of one possible Multirate signal processing task. That task is the improvement in signal quality obtained when converting a sampled data signal to an analog signal by a combination of a DAC and analog low-pass filter. In the improved option, a digital resampling filter, commonly implemented as a polyphase filter is used to raise the output sample rate prior to the DAC and low-pass filter operations. The increase in sample rate results in less distortion from the DAC and analog filter. Embedding the appropriate pre-compensating gain and phase in the digital resampling filter further controls the residual distortion.

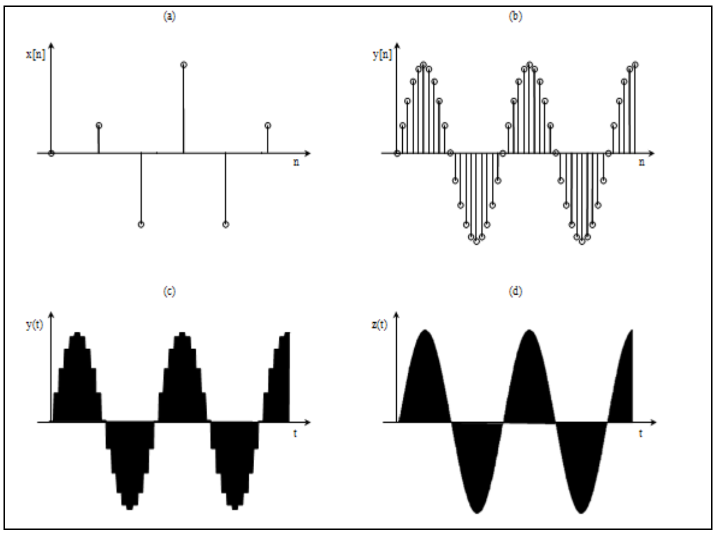

Figure 15: Illustration of oversampling in CD music signal reconstruction

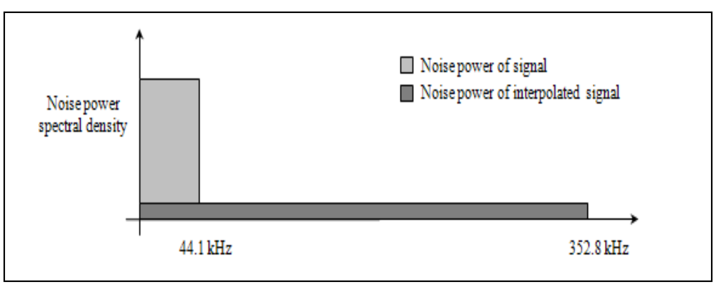

Figure 16: Illustration of noise power spectral density reduction due to oversampling.

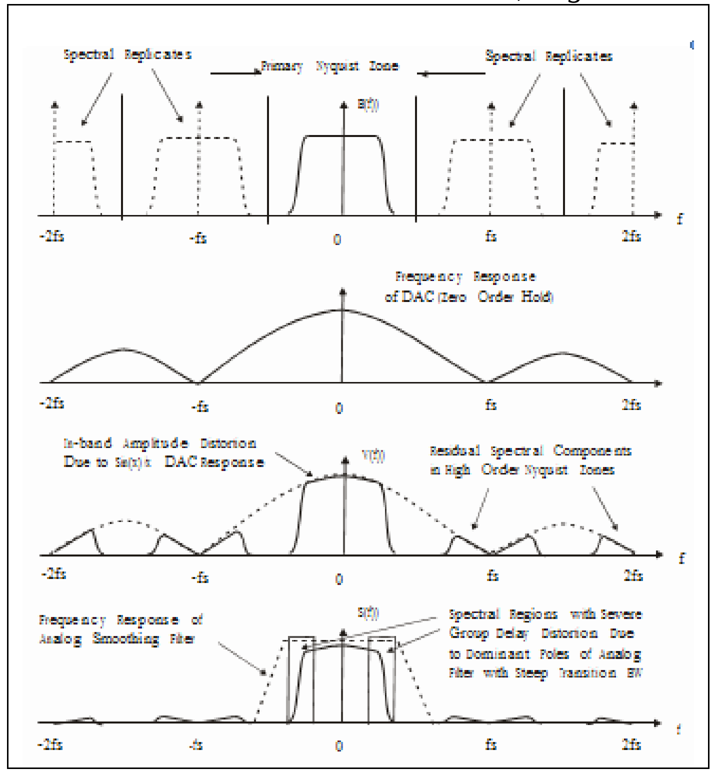

Figure 11: Spectra of various signals for Simplified Model of Signal Conditioning Sequence for Digital-to-Analog Conversion Process

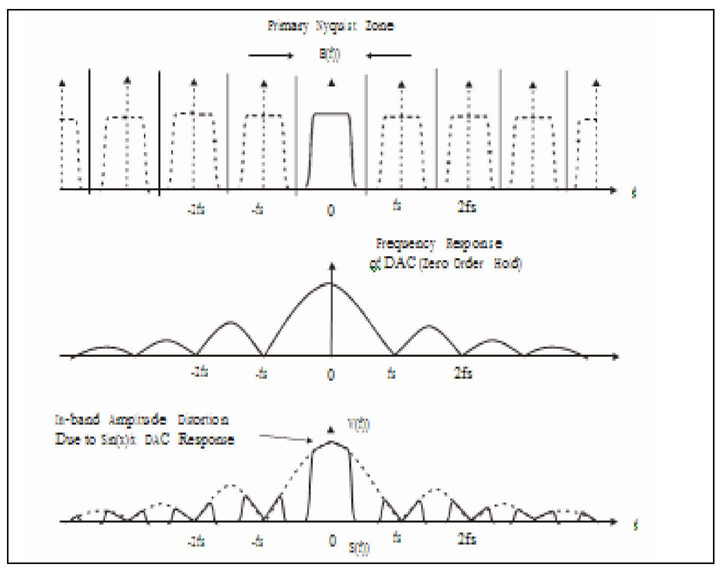

Figure12 (a): Spectral distortion caused by DACs sinx/x at sample rate fs.

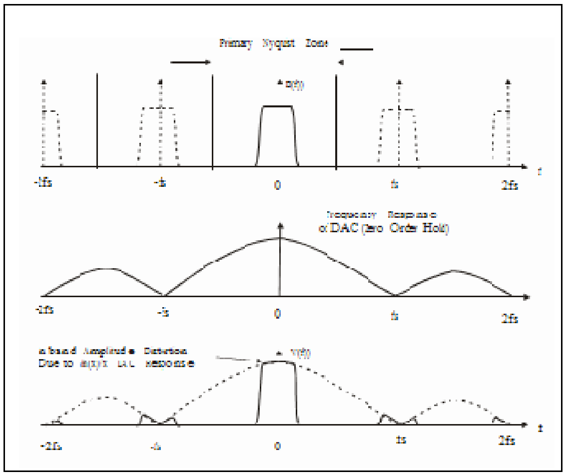

Figure12 (b): Spectral Distortion Caused by DAC's Sin(x)/x at Double sampling rate f_S.

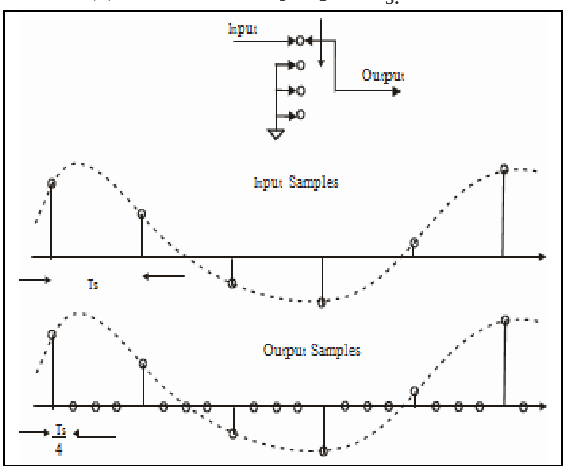

Figure13: 1-to-4 Upsampling or Zero-Packing with Input commutator

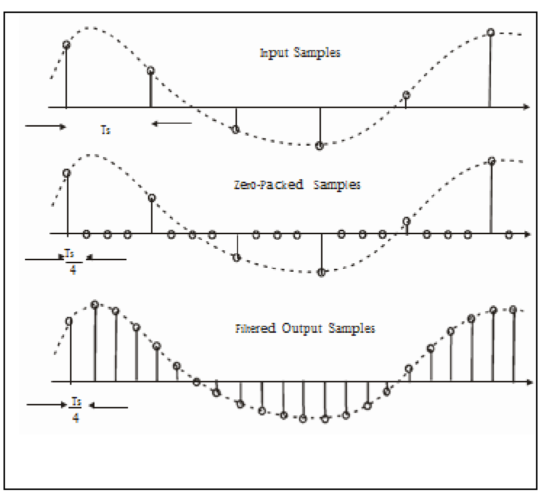

Figure14 (a): Time Series: Input, Zero-Packed, and Filtered Components of 1-to-4 Interpolator

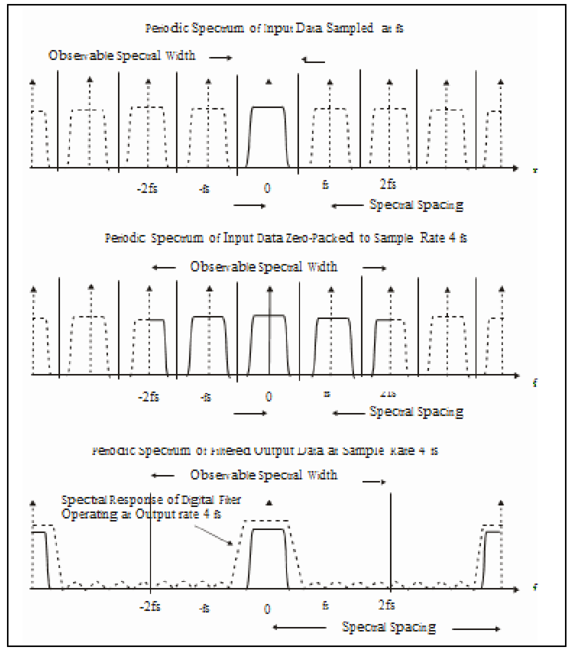

Figure14 (b): Spectra: Input, Zero-Packed & Filtered components of 1–to-4 Interpolator.

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