Anti Q-Fuzzy BG –Ideals in BG – Algebra

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Abstract
In this paper, we introduce the notion of anti Q-fuzzy BG-ideals of BG-algebras, lower level cuts of a fuzzy set, lower level BG-ideal and prove some results on these. we show that a Q-fuzzy subset of a BG-algebra is a Q-fuzzy BG-ideal if and only if the complement of this Q-fuzzy subset is an anti Q-fuzzy BG-ideal.

Keywords:
BG-algebra, sub BG-algebra, BG-ideal, fuzzy BG-ideal, Anti fuzzy BG-ideal , Q- fuzzy BG-ideal, Anti Q-fuzzy BG-ideal

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1.Introduction

The concept of fuzzy sets was introduced by Zadeh. Since then these ideas have been applied to other algebraic structures such as semigroups, groups, rings, modules, vector spaces and topologies. In 1991 O.G.Xi applied the concept of fuzzy sets to BCK-algebras which are introduced by Y. Imai and K. Iseki BCK – algebras generalize ,on the other hand ,the notion of the algebra sets with the set subtraction as the only fundamental non-nully operation and ,on the other hand, the notion of the implication algebra. J. Neggers and H.S.Kim introduced a new notion, called B – algebra. C.B.Kim and H.S.Kim introduced the notion of the BG – algebra which is a generalization of B – algebra.. R.Biswas introduced the concept of anti fuzzy subgroups of groups. Modifying his idea, in this paper ,we apply the idea to BG-algebras. We introduce the notion of anti Q-fuzzy ideals of BG – algebras, lower level cuts of a Q-fuzzy set, and prove some results on these. In this paper, we classify the anti Q-fuzzy BG – ideals in BG – Algebra.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1

A nonempty set X with a constant 0 and a binary operation ‘ $*$ ‘ is called a BG – Algebra if it satisfies the following axioms.

1. $x * x = 0$,
2. $x * 0 = x$,
3. $(x * y) * (0 * y) = x$, for all $x,y \in X$.

Example 2.1

Let X = { 0,a,b } be the set with the following table.

*	0	a	b
0	0	a	b
a	a	0	a
b	b	b	0

Then $(X , * , 0)$ is a BG – Algebra.

We can define a relation (partial ordering) $x \leq y$ if and only if $x * y = 0$.

Proposition 2.1 In any BG-algebra X ,the following hold:

1) $x * y \leq 0$,

2) $(x * y) * (x * y) \leq x * y$

3) $x * (x * (x * y)) = x * y$

4) $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$

Definition 2.2

Let S be a non empty subset of a BG -algebra X , then S is called a subalgebra of X if $x * y \in S$, for all $x ,y \in S$.

Definition 2.3

Let X be a BG-algebra and I be a subset of X ,then I is called a BG- right ideal of X if it satisfies following conditions:

1. $0 \in I$,
2. $x * y \in I$ and $y \in I \Rightarrow x \in I$,
3. $x \in I$ and $y \in X \Rightarrow x * y \in I$, $I \times X \subseteq I$.

Definition 2.4

Let X be a BG-algebra and I be a subset of X ,then I is called a BG- left ideal of X if it satisfies following conditions:

1. $0 \in I$,
2. $x * y \in I$ and $y \in I \Rightarrow x \in I$,
3. $y \in I$ and $x \in X \Rightarrow y * x \in I$, $I \times X \subseteq I$.

Definition 2.5

Let X be a BG-algebra and I be a subset of X ,then I is called a BG- ideal of X if it satisfies following conditions:

1. $0 \in I$,
2. $x * y \in I$ and $y \in I \Rightarrow x \in I$,
3. $x \in I$ and $y \in X \Rightarrow x * y \in I$ and $y * x \in I$, $I \times X \subseteq I$.

Definition 2.6

Let X be a non-empty set .A fuzzy sub set μ of the set X is a mapping $\mu : X \to [0,1]$.

Definition 2.6

Let Q and G be any two sets. A mapping $A : G \times Q \to [0,1]$ is called a Q–fuzzy set in G.

Definition 2.7

Let μ be a Q-fuzzy set in a set X. For $t \in [0,1]$, the set $\mu_t = \{ x \in X / \mu(x, q) \geq t$ for all $q \in Q \}$ is called a level subset of μ .

Definition 2.8

If μ be a Q-fuzzy set in set X . Then the complement denoted by μ^c is the Q-fuzzy subset of X given by $\mu^c(x, q) = 1 - \mu(x, q)$, for all $x \in X$ and $q \in Q$.

Definition 2.9

Let μ be a Q-fuzzy set in BG – algebra. Then μ is called a Q-fuzzy sub algebra of X if

$\mu(x * y, q) \geq \min\{ \mu(x, q), \mu(y, q)\}$, for all $x, y \in X$ and $q \in Q$.

Definition 2.10

A Q-fuzzy set μ in X is called Q-fuzzy BG – Ideal of X if it satisfies the following inequalities. For all $x, y \in X$ and $q \in Q$,

i. $\mu(0, q) \geq \mu(x, q)$,
ii. $\mu(x, q) \geq \min\{ \mu(x * y, q), \mu(y, q)\}$,
iii. $\mu(x * y, q) \geq \min\{ \mu(x, q), \mu(y, q)\}$.

3. Anti fuzzy ideals

Definition 3.1

A Q-fuzzy set μ of a BG-algebra X is called an anti Q-fuzzy sub algebra of X if

$\mu(x * y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$, for all $x, y \in X$ and $q \in Q$.

Definition 3.2

A Q-fuzzy set μ of a BG-algebra X is called an anti Q-fuzzy BG-ideal of X. if for all $x, y \in X$ and $q \in Q$,

i.. $\mu(0, q) \leq \mu(x, q)$,
ii. $\mu(x, q) \leq \max\{\mu(x * y, q), \mu(y, q)\}$,
iii. $\mu(x * y, q) \leq \max\{ \mu(x, q), \mu(y, q)\}$.

Example 3.1

Let X = { 0,a,b,c } be the set with the following table.

*	0	a	b	c
0	0	a	b	c
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Let $t_0, t_1, t_2 \in [0,1]$ be such that $t_0 < t_1 < t_2$. Define a Q-fuzzy set $\mu : X \times Q \to [0,1]$ by $\mu(0, q) = t_0$, $\mu(a, q) = t_1 = \mu(b, q)$ and $\mu(c, q) = t_2$, routine calculations μ is an anti Q-fuzzy sub algebra of X , and it is an anti Q-fuzzy BG-ideal of X and $q \in Q$.

Theorem 3.1

Every anti Q-fuzzy BG-ideal of a BG-algebra X is order preserving.

Proof

Let μ be an anti Q-fuzzy BG-ideal of a BG-algebra X and let $x, y \in X$ and $q \in Q$ be such that $x \leq y$ then $x * y = 0$.

Now, $\mu(x, q) \leq \max\{ \mu(x * y, q), \mu(y, q)\}$

$= \max \{ \mu(0, q), \mu(y, q) \}$

$= \mu(y, q)$.

$\mu(x, q) \leq \mu(y, q)$.

Theorem 3.2

Let X be a BG-algebra , Let μ is an anti Q-fuzzy BG-ideal of X if and only if μ is an anti Q-fuzzy sub algebra of X.

Proof

By definition , every anti Q-fuzzy BG-ideal of a BG-algebra X is an anti Q-fuzzy sub algebra of X.

Conversely, let μ be an anti Q-fuzzy sub algebra of X.

To prove : μ is an anti Q-fuzzy BG-ideal of X.

For all $x, y \in X$ and $q \in Q$,

$$\mu(x, q) = \mu(((x * y) * (0 * y)), q)$$
$$\leq \max \{ \mu(x * y, q), \mu(0 * y, q) \}$$
$$\leq \max\{ \mu(x * y, q), \max \{ \mu(0, q), \mu(y, q) \}\}$$
$$= \max \{ \mu(x * y, q), \mu(y, q) \}$$

i.e $\mu(x, q) \leq \max \{ \mu(x * y, q), \mu(y, q) \}$.

Hence μ is an anti Q-fuzzy BG-ideal of X.

Theorem 3.3

Let μ is an anti Q-fuzzy BG-ideal of a BG-algebra X. If the inequality $x * y \leq z$ holds in X, then $\mu(x, q) \leq \max \{\mu(y, q), \mu(z, q)\}$, for all $x,y,z \in X$ and $q \in Q$.

Proof

Assume the inequality $x * y \leq z$ holds in X then

$$\mu(x * y, q) \leq \max \{ \mu(x * y * z, q), \mu(z, q) \}$$
$$= \max \{ \mu(0, q), \mu(z, q) \}$$
$$= \mu(z, q)$$

it follows that $\mu(x, q) \leq \max \{ \mu(x * y, q), \mu(y, q) \}$

$$\mu(x, q) \leq \max \{ \mu(z, q), \mu(y, q) \}.$$

Hence the result.

Theorem 3.4

A Q-fuzzy subset μ of a BG-algebra X is a Q-fuzzy BG-ideal of X if and only if its complement μ^c is an anti Q-fuzzy BG-ideal of X.

Proof

Let μ be a Q-fuzzy BG-ideal of X and let $x, y \in X$ and $q \in Q$ then,

i) $\mu^c(0, q) = 1-\mu(0, q) \leq 1-\mu(x, q) = \mu^c(x, q)$.

ii) $\mu^c(x, q) = 1-\mu(x, q)$

$$\leq 1-\min \{ \mu(x * y, q), \mu(y, q) \}$$
$$= 1-\min \{1-\mu^c(x * y, q), 1-\mu^c(y, q)\}$$
$$= \max\{ \mu^c(x * y, q), \mu^c(y, q) \}.$$

That is, $\mu^c(x, q) \leq \max\{ \mu^c(x * y, q), \mu^c(y, q) \}$.

iii) $\mu^c(x * y, q) = 1-\mu(x * y, q)$

$$\leq 1-\min \{ \mu(x, q), \mu(y, q) \}$$
$$\leq 1-\min \{1-\mu^c(x, q), 1-\mu^c(y, q)\}$$
$$= \max \{\mu^c(x, q), \mu^c(y, q)\}$$

That is, $\mu^c(x * y, q) \leq \max \{\mu^c(x, q), \mu^c(y, q)\}$.

Thus, μ^c is an anti Q-fuzzy ideal of X .The converse also can be proved similarly.

Definition 3.3

Let μ be a Q-fuzzy subset of a BG-algebra X. For $t \in [0,1]$, the set $\mu^t = \{ x \in X / \mu(x, q) \leq t \}$ is called a lower level cut of μ . Cleary, $\mu^1 = X$ and $\mu_t \cup \mu^t = X$ for $t \in [0,1]$.

If $t_1 < t_2$ then $\mu^{t1} \subseteq \mu^{t2}$.

Theorem 3.5

Let μ be a Q-fuzzy subset of a BG-algebra X. If μ is an anti Q-fuzzy BG – ideal of X, then the lower level cut μ^t is a BG – ideal of X for all $t \in [0, 1]$, $t \geq \mu(0, q)$.

Proof:

Let μ be an anti Q-fuzzy BG – ideal of X. Then

For all $x, y \in X$ and $q \in Q$,

i. $\mu(0, q) \leq \mu(x, q)$,

ii. $\mu(x, q) \leq \max\{ \mu(x * y, q), \mu(y, q)\}$,

iii. $\mu(x * y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$.

To prove that μ^t is a BG – ideal of X

We know that $\mu^t = \{x \in X / \mu(x, q) \leq t\}$

Let $x, y \in \mu^t$ and μ is an anti Q-fuzzy BG – ideal of X .

Since $\mu(0, q) \leq \mu(x, q) \leq t$ implies $0 \in \mu^t$, for all $t \in [0, 1]$.

Let $x * y \in \mu^t$ and $y \in \mu^t$

Therefore, $\mu(x * y, q) \leq t$ and $\mu(y, q) \leq t$.

Now $\mu(x, q) \leq \max\{\mu(x * y, q), \mu(y, q)\}$

$\leq \max\{t, t\}$

$\leq t$.

Hence $\mu(x, q) \leq t$.

That is , $x * y \in \mu^t$ and $y \in \mu^t$ implies $x \in \mu^t$.

(i) Let $x \in \mu^t$, $y \in X$.

Choose y in X such that $\mu(y, q) \leq t$.

Since $x \in \mu^t$ implies $\mu(x, q) \leq t$.

We know that $\mu(x * y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$

$\leq \max\{t, t\}$

$\leq t$.

That is, $\mu(x * y, q) \leq t$ implies $x * y \in \mu^t$

(ii) Let $y \in \mu^t$, $x \in X$.

Choose x in X such that $\mu(x, q) \leq t$.

Since $y \in \mu^t$ implies $\mu(y, q) \leq t$.

We know that $\mu(y * x, q) \leq \max\{\mu(y, q), \mu(x, q)\}$

$\leq \max\{t, t\}$

$\leq t$.

That is, $\mu(y * x, q) \leq t$ implies $y * x \in \mu^t$

Hence μ^t is a BG – ideal of X.

Theorem 3.6

Let μ be a Q-fuzzy subset of a BG-algebra X If for each $t \in [0, 1]$, $t \geq \mu(0, q)$ the lower level cut μ^t is a BG – ideal of X, then μ is an anti Q-fuzzy BG – ideal of X.

Proof

Since μ^t is a BG - ideal of X .

i) $0 \in \mu^t$

ii) $x * y \in \mu^t$ and $y \in \mu^t$ implies $x \in \mu^t$,

iii) $x \in \mu^t$ and $y \in X$ implies $x * y \in \mu^t$.

To prove that μ^t is an anti Q-fuzzy BG- ideal of X.

For all $x, y \in X$ and $q \in Q$,

i) Let $x, y \in \mu^t$ then $\mu(x, q) \leq t$ and $\mu(y, q) \leq t$.

Let $\mu(x, q) = t_1$ and $\mu(y, q) = t_2$,

Without loss of generality, let $t_1 \leq t_2$.

Then $x \in \mu^{t2}$.

Now $x \in \mu^{t2}$ and $y \in X$ implies $x * y \in \mu^{t2}$.

That is , $\mu(x * y, q) \leq t_2$

$= \max\{t_1, t_2\}$

$= \max\{\mu(x, q), \mu(y, q)\}$.

Therefore, $\mu(x * y, q) \leq \max\{\mu(x, q), \mu(y, q)\}$.

ii) Let $\mu(0, q) = \mu(x*x, q)$

$\leq \max\{\mu(x, q), \mu(x, q)\}$

$\leq \mu(x, q)$.

Therefore, $\mu(0, q) \leq \mu(x, q)$,

iii) Let $\mu(x, q) = \mu((x * y) * (0 * y), q)$

$\leq \max\{\mu(x * y, q), \mu(0 * y, q)\}$ (by (i))

$\leq \max\{\mu(x * y, q), \max\{\mu(0, q), \mu(y, q)\}\}$

$\leq \max\{\mu(x * y, q), \mu(y, q)\}$ (by (ii)).

Therefore $\mu(x, q) \leq \max\{\mu(x * y, q), \mu(y, q)\}$.

Hence μ is an anti Q-fuzzy BG – ideal of X.

Theorem 3.7

Let μ be an anti Q-fuzzy BG-ideal of a BG-algebra X . Two lower level cuts μ^{t1} and μ^{t2} (with $t_1 < t_2$) are equal if and only if there is no $x \in X$ such that $t_1 < \mu(x, q) \leq t_2$.

Proof

Let $\mu^{t1} = \mu^{t2}$

Suppose that there exists $x \in X$ such that $t_1 < \mu(x, q) \leq t_2$.

$\mu^{t1} \subseteq \mu^{t2}$ then $x \in \mu^{t2}$ but $x \notin \mu^{t1}$ which contradicts the assumption $\mu^{t1} = \mu^{t2}$.

Hence there is no $x \in X$ such that $t_1 < \mu(x, q) \leq t_2$.,

Conversely

Let there is no $x \in X$ such that $t_1 < \mu(x, q) \leq t_2$ then

$\mu^{t2} \subseteq \mu^{t1}$ as $t_1 < t_2$ implies $\mu^{t1} \subseteq \mu^{t2}$,

hence $\mu^{t1} = \mu^{t2}$.

Now we consider the fuzzification of a lower t-level cut.

Definition 3.4

Let μ be a Q -fuzzy subset of a BG-algebra X .the fuzzification μ^t $t \in [0,1]$,is the fuzzy subset $A_\mu{}^t$ of X defined by

$$A_\mu{}^t(x, q) = \begin{cases} \mu(x,q), x \in \mu^t \\ 0 \quad otherwise \end{cases}$$

Remark: It can be easily verified that $A_\mu \leq \mu$.

That is, $A_\mu(x, q) \leq \mu(x, q)$ for all $x \in X$ and $q \in Q$ and $(A_\mu)^t = \mu^t$.

Theorem 3.8

If μ is an anti Q-fuzzy BG-ideal of a BG-algebra X, then A_μ is also an anti Q-fuzzy BG-ideal of X where $t \in [0, 1]$, $t \geq \mu(0, q)$.

Proof

Let μ be an anti Q-fuzzy BG-ideal of a BG-algebra X then by Theorem 3.5, the lower level cut μ^t is a BG – ideal of X for all $t \in [0, 1]$, $t \geq \mu(0, q)$.

$(A_\mu)^t = \mu^t$ is a BG – ideal of X for all $t \in [0, 1]$, $t \geq \mu(0, q)$.

By Theorem 3.6, A_μ is an anti Q-fuzzy BG-ideal of X, where $t \in [0, 1]$, $t \geq \mu(0, q)$.

Hence Proved.

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