CMA an Optimum Beamformer for a Smart Antenna System

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ABSTRACT
In this paper constant modulus algorithm (CMA) and least mean square (LMS), kind of blind and nonblind algorithms used for adaptive beamforming are presented. These algorithms are embedded in smart antenna which calculates optimum weight vector that minimizes the total received power except the power coming from desired direction. The efficiency of CMA and LMS algorithms is compared on the basis of gain versus angle and mean square error (MSE) for mobile communication. Simulation results reveal that both algorithms have high resolution for beam formation. However CMA has good performance to minimize MSE as compared to LMS. Therefore, CMA is found more efficient algorithm to implement in the mobile communication environment to enhance service quality and capacity.

General Terms
Adaptive filtering, Adaptive signal processing algorithm

Keywords
Constant Modulus Algorithm (CMA) and Least Mean Square (LMS) Algorithm.

1. INTRODUCTION
Since Radio Frequency (RF) spectrum is limited [1] and its efficient use is only possible by employing smart/adaptive antenna array system to exploit mobile systems capabilities for data and voice communication. The name smart refers to the signal processing capability that forms vital part of the adaptive antenna system which controls the antenna pattern by updating a set of antenna weights. Smart antenna, supported by signal processing capability, points narrow beam towards desired users but at the same time introduces null towards interferers, thus optimizing the service quality and capacity. Consider a smart antenna system with Ne elements equally spaced (d) and user's signal arrives from desired angle Φ_0 as shown in Fig 1 [2]. Adaptive beamforming scheme that is CMA and LMS [2] [3] [4] [5] is used to control weights adaptively to optimize signal to noise ratio (SNR) of the desired signal in look direction Φ_0. CMA is a kind of blind algorithm which doesn't require training signals for its guidance therefore a lot of energy is conserved whereas LMS is a nonblind algorithm requires training signals, known in advance by the receiver, to train the adaptive weights for convergence.

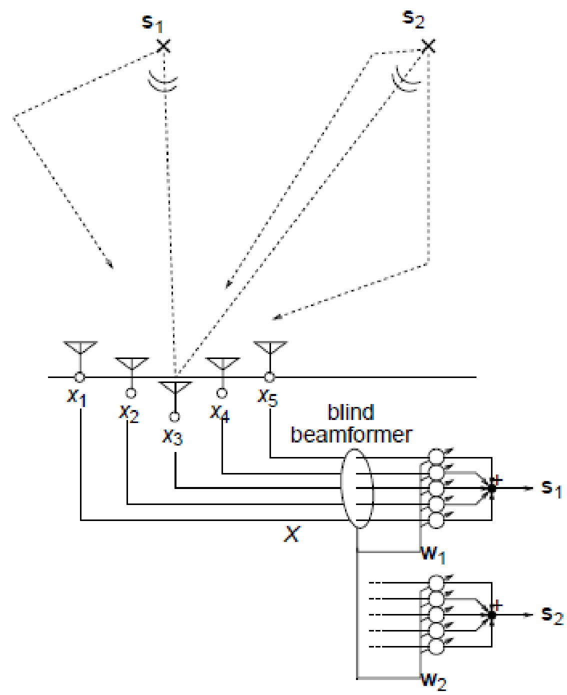

Fig.1. Blind Beamforming Scenario for smart/adaptive antenna array system.

The array factor for elements (Ne) equally spaced (d) linear array is given by

$$AF(\Phi) = \sum_{n=0}^{N-1} A_n . e^{(jn(\frac{2\pi d}{\lambda}\cos\Phi + \alpha))} \qquad (1)$$

where α is the inter element phase shift and is described as:

$$\alpha = \frac{-2\pi d}{\lambda_0}\cos\Phi_0 \qquad (2)$$

and Φ_0 is the desired direction of the beam.

In reality, antennas are not smart; it is the digital signal processing, along with the antenna, which makes the system smart. When smart antenna is deployed in mobile communication using time division multiple access (TDMA) system [6], assigning different time slot to different users, it radiates beam towards desired users only. Each beam becomes a channel, thus avoiding interference in a cell. Because of these, each channel reduces co-channel interference, due to the processing gain of the system. The processing gain (PG) of the TDMA system is described as:

$$PG = 10\log(B / R_b) \quad (3)$$

where B is the TDMA channel bandwidth and R_b is the information rate in bits per second.

Different ideas and its implementations are reported regarding increase in channel capacity and signal quality in [10] [11] [12] [13] [14] [15] [16] [17] [18] [19] [20].

The rest of the paper is organized as follows: Section 2 introduces CMA with simulation results. LMS algorithm with simulation results are presented in section 3. Finally the concluding remarks of this work are provided in section 4.

2. CONSTANT MODULUS ALGORITHM

2.1 Theory

CMA is a blind algorithm, based on the idea, to reduce systems overhead and maintain gain on the signal while minimizing the total output energy. As a result number of bits for transmitting information is increased that leads to enhance capacity. This algorithm seeks for a signal with a constant magnitude i.e. modulus within the received data vector and is only applicable for modulation scheme which uses symbol of equal power includes phase and frequency modulated signals. The received data vector consists of desired signal plus interference and noise. Therefore, it can identify only one signal usually; this is the signal with greatest power [3] [7] [9].

Consider a signal of magnitude α within the received data vector X . The output of smart antenna array is given by

$$y = w^H X \quad (4)$$

The function $f(w)$ with parameters p and q is given by

$$f(w) = E\left[\left||y|^p - |\alpha|^p\right|^q\right] \quad (5)$$

putting the value of y in (5), then we have

$$f(w) = E\left[\left||w^H X|^p - |\alpha|^p\right|^q\right] \quad (6)$$

To minimize the function $f(w)$ for the development of CMA algorithm and setting $\alpha = 1$, $p = 1$ and $q = 2$, is calculated as

$$f(w) = E\left[\left||w^H X|^1 - |1|^1\right|^2\right] \quad (7)$$

$$f(w) = E\left[\left||y| - 1\right|^2\right] \quad (8)$$

Differentiate (8) w.r.t. w ; we get the performance cost function as

$$\nabla f = 2\frac{\partial f}{\partial w^*} = 2\left(|y| - 1\right) X \frac{y}{|y|} \quad (9)$$

The weight update equation for this case becomes

$$w_{n+1} = w_n - 2\mu\left(y - \frac{y}{|y|}\right) X_n \quad (10)$$

where μ represents the rate of adaptation, controlled by the processing gain of the antenna array. If a large value of μ is taken then convergence becomes faster but makes the array system unstable/noisy. Conversely if a small value is taken then convergence becomes slow that is also not desirable. Therefore, value of μ is taken in between that satisfy the following conditions for good convergence and to avoid instability.

$$0 \le \mu \le \frac{2}{\lambda_{\max}} \quad (11)$$

Equation (10) looks like a LMS algorithm little bit with difference in cost function.

2.2 Simulation Results

Computer simulation is carried out, to illustrate that how various parameters such as number of elements (Ne), element spacing (d) and mu (µ) affect the beam formation. The simulations are designed to analyze the properties of CMA and LMS algorithms. The desired signal is phase modulated with $SNR = 35$ dB, used for simulation purpose. It is given by

$$S(t) = Ae^{j\sin(2*\pi*f*t) + \Phi_0} \quad (12)$$

where A is the constant magnitude i.e. modulus and Φ_0 is the phase angle, of all incoming signals includes desired plus interference and noise respectively.

2.2.1 Effect of Number of Elements on Array Factor

Uniform linear array is taken with different number of elements for simulation purpose. The spacing between array elements is taken as $\lambda / 2$. The gain versus angle is shown in Fig. 2 to 4 for $Ne = 4$ and 10 respectively. It is observed that pencil beam is obtained when number of element is increased from 4 to 10.

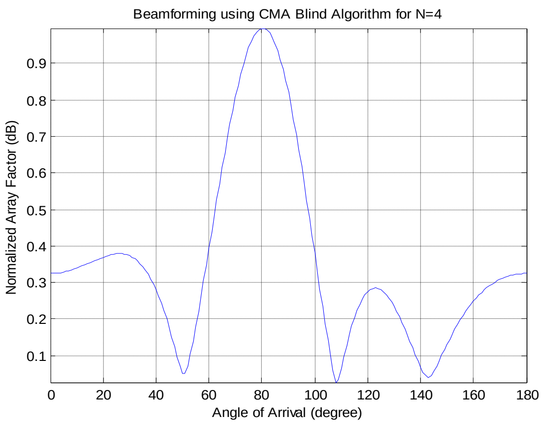

Fig.2. Normalized *a*rray factor plot for CMA algorithm with AOA for desired user is 80 degree and **three interferer with 30, 110 and 140 degrees** with constant space of $\lambda/2$ between elements for $Ne = 4$

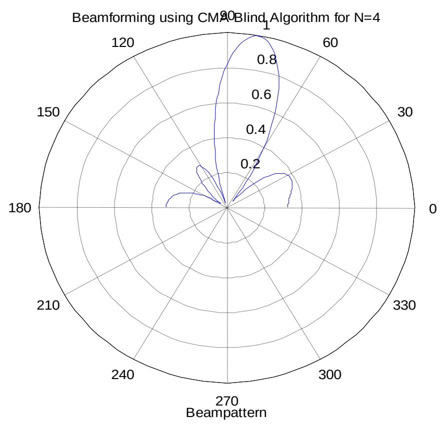

Fig.3. Polar plot for CMA algorithm with AOA for desired user is 80 degree and **three interferer with 30, 110 and 140 degrees** with constant space of $\lambda/2$ between elements for $Ne = 4$

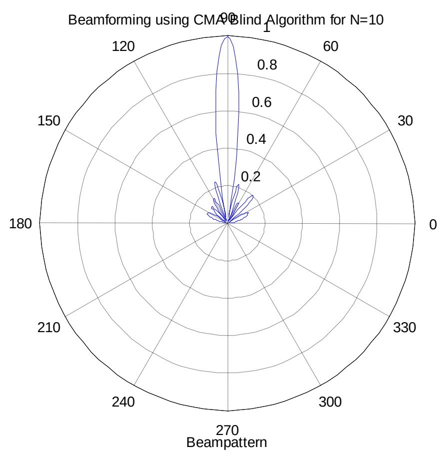

Fig.4. Polar plot for CMA algorithm with AOA for desired user is 90 degree and **three interferer with 50, 10 and 20 degrees** with constant space of $\lambda/2$ between elements for $Ne = 10$

The optimum weight vector for $Ne = 10$ with spacing $\lambda/2$ is given by [9.8154 - 9.1146i, -2.6558 -10.9057i, -10.5821 - 5.1980i, -10.7608 + 8.6798i, 4.3686 +12.2025i, 10.5975 + 2.3055i, 9.2850 - 7.7989i, -4.2285 -13.3422i, -12.6545 - 0.7419i, -6.8470 + 8.4256i] and is shown in Fig 5. Similarly optimum weight vector for, $Ne = 4$ with different elements spacing (d) can be computed.

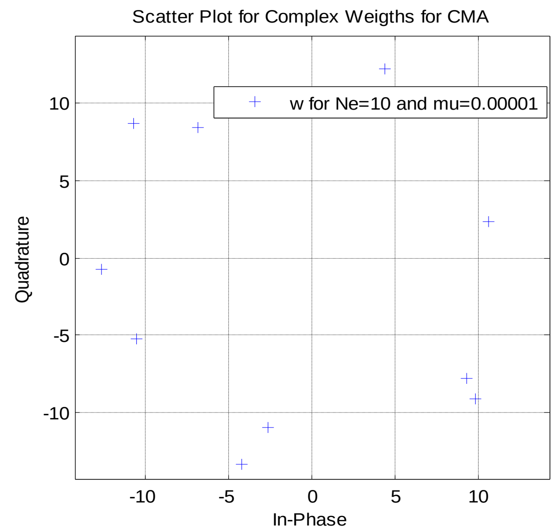

Fig.5 Scatter plot for **complex weights** for CMA for $Ne = 10$

2.2.2 Effect of Spacing Between Elements on Array Factor

The effect of array spacing for $\lambda/2$, $\lambda/4$ and $\lambda/8$ is shown in Fig. 6 for $Ne = 10$. Since the spacing between the elements is critical, due to sidelobes problems, which causes spurious echoes and diffraction secondaries, which are repetitions of the main beam within the range of real angles.

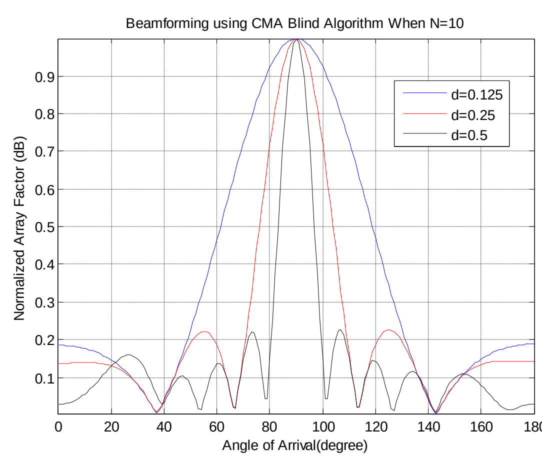

Fig.6. Normalized *a*rray factor plot for CMA algorithm with AOA for desired user is 90 degree and **three interferer with 30, 110 and 140 degrees** for $Ne = 10$

2.2.3 Effect of Step Size (μ) on Array Factor

If a large value of μ is taken then convergence becomes faster but makes the array system unstable/noisy. Conversely if a small value is taken then convergence becomes slow that is also undesirable. Therefore, value of μ is taken in between that satisfy the conditions imposed in (11) for good convergence as shown in Fig 7 for $Ne = 4$

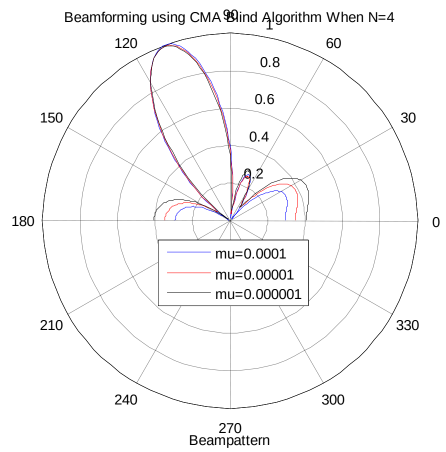

Fig.7. Polar plot for CMA algorithm with AOA for desired user is 110 degree and **three interferer with 30, 50 and 140 degrees** for $Ne = 4$

2.2.4 Effect of AOA on Array Factor

CMA is compared on the basis of AOA as shown in Fig. 8 and has shown best response for beamforming keeping $\lambda / 2$ spacing between elements for $Ne = 6$.

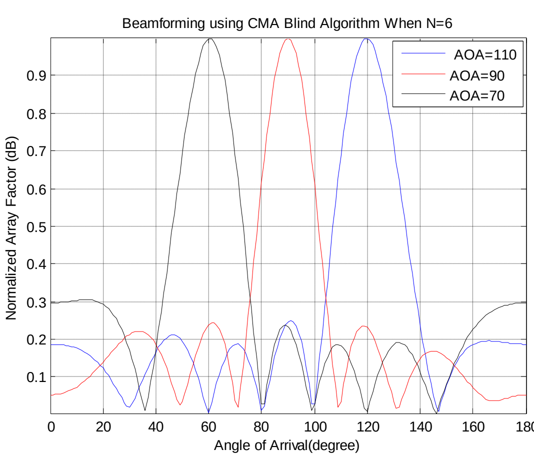

Fig.8. Normalized array factor plot for CMA algorithm for **three AOA with 60, 90 and 120 degrees** for $Ne = 6$

2.2.5 Effect of Number of Elements on MSE

The effect of number of elements on MSE for constant space $d = \lambda / 2$ between elements is shown in Fig. 9 and 10 for $Ne = 4$ and 10 respectively. From these figures, it is clear that minimum MSE is obtained for $Ne = 4$ when same $\mu = 0.00001$ is taken for comparison.

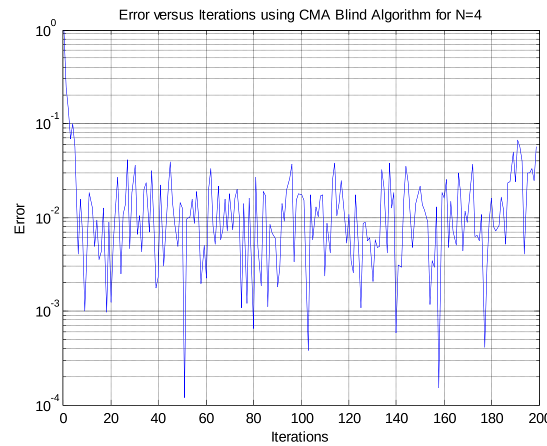

Fig.9. Mean square error for CMA algorithm for $Ne = 4$ and **space** $(d = \lambda / 2)$ **is kept constant**

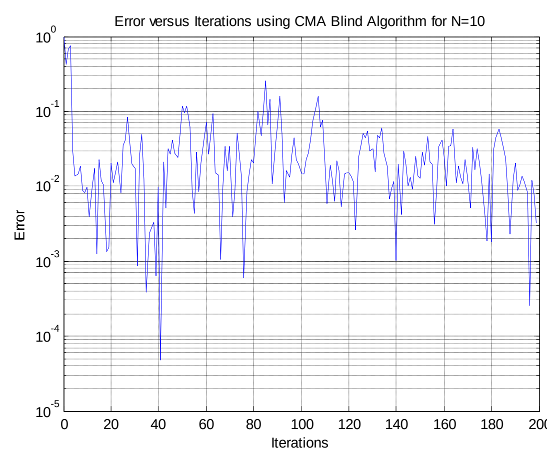

Fig.10. Mean square error for CMA algorithm for $Ne = 10$ and **space** $(d = \lambda / 2)$ **is kept constant**

3. LMS ALGORITHM

3.1 Theory

As said earlier that CMA is a blind algorithm whereas LMS is nonblind algorithm which requires a training sequence of known symbols $d(n)$, to train the adaptive weights. It uses the estimate of the gradient vector from the available data. This algorithm makes successive corrections to the weight vector in the direction of the negative of the gradient vector which finally concludes to minimum MSE. This successive correction to the weight vector is the point at which optimum value w_0 is obtained that relies on autocorrelation matrix R and cross correlation matrix p of the filter. LMS is an adaptive beamforming algorithm, defined by the following equations [3] [4] [5] [8] [9] with input signal $u(n)$:

$$y(n) = w^T(n-1)u(n) \qquad (12)$$

$$e(n) = d(n) - y(n) \qquad (13)$$

$$w(n) = w(n-1) + \mu e(n)u^*(n) \qquad (14)$$

$$\begin{aligned}\xi &= E[e^2(n)] \\ &= E[(d^2(n))] - 2w^T p + w^T R w \qquad (15)\end{aligned}$$

where $y(n)$ is the filter output, $e(n)$ is the error signal between filter output and desired signal $d(n)$ at step n. $d(n)$ is the training sequence of known symbols (also called a pilot signal), is required to train the adaptive weights. Enough training sequence of known symbols must be available to ensure convergence but it is important to realize that training signal represents wasted of resources in terms of energy and time both.

Equation (14) is the weight $w(n)$ update function for the LMS algorithm, where μ is the rate of adaptation, controlled by the processing gain of the antenna array as described by (3). The convergence conditions imposed on step size μ is given by

$$0 \le \mu \le \frac{1}{\lambda_{\max}} \qquad (16)$$

Where $\lambda_{\max}$ is the largest eigen value of autocorrelation matrix R. If μ must select within bounded conditions as defined in (16) to ensure better convergence. ξ is the performance cost function describing quadratic function of filter tap-weight vector w in terms of MSE. R is the autocorrelation matrix of filter input and is given by

$$R = E[u(n)u^T(n)] \qquad (17)$$

and p is the cross correlation matrix between input and desired signal and is defined by

$$p = E[u(n)d(n)] \qquad (18)$$

Solving (15) for optimum solution, we have:

$$w_0 = pR^{-1} \qquad (19)$$

This equation is known as Wiener Hopf.

If p and R are not available to solve Wiener Hopf directly, then we employ an iterative search method in which starting with an initial guess for w_0, say $w(0)$, a recursive search method that require many iterations to converge to w_0 is used. With an initial guess for w_0 at $n=0$, the tap-weight vector at the nth iterations is denoted as $W(n)$ that finally depends on μ for convergence to obtain optimum solution w_0 for smart antenna array consisting of number of elements (Ne) that finally leads to obtain minimum MSE.

3.2 Simulation Results

3.2.1 Effect of Number of Elements on Array Factor

Uniform linear array with same number of sample $(N=200)$ is taken for simulation purpose. The space $\lambda/2$ is maintained between elements. AOA for desired user is set at 100 degree and three interferers are set at 50, 30 & 130 degrees for $Ne=4$. Null is obtained as shown in Fig 11 and 12 respectively.

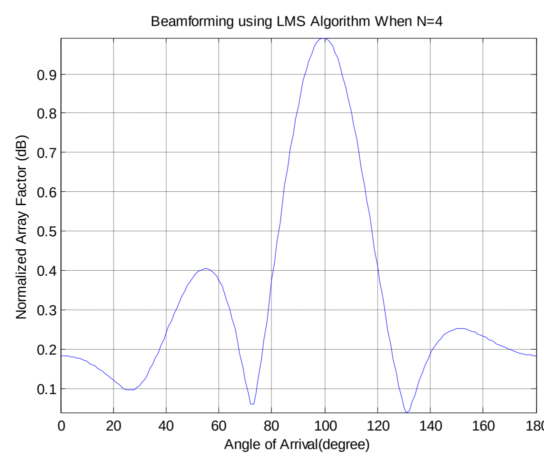

Fig.11. Normalized array factor plot for LMS algorithm with AOA for desired user is 100 degree and **three interferer with 50, 30 and 130 degrees** for $Ne=4$

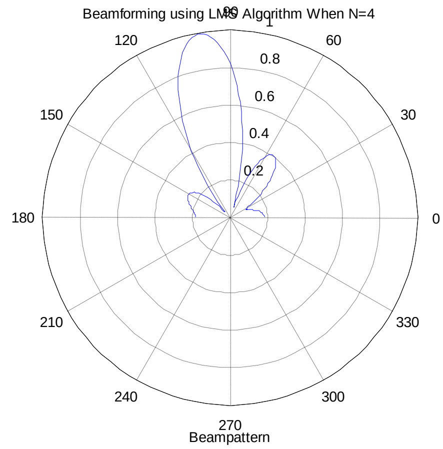

Fig.12. Polar plot for LMS algorithm with AOA for desired user is 100 degree and **three interferer with 50, 30 and 130 degrees** for $Ne = 4$

Similarly, AOA for desired user is set at 90 degree and three interferers are set at 50, 110 & 130 degrees for $Ne = 10$. Null is obtained as shown in Fig 13.

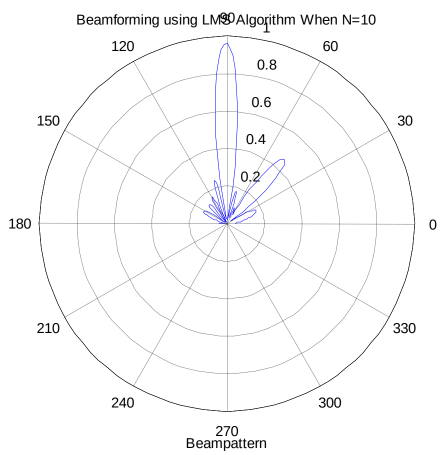

Fig.13. Polar plot for LMS algorithm with AOA for desired user is 90 degree and **three interferer with 50, 110 and 130 degrees** for $Ne = 10$

The optimum weight vector for $Ne = 10$ with spacing $\lambda / 2$ is given by [0.1190 - 0.0008i, 0.0240 - 0.0893i, -0.0295 - 0.0578i, -0.0947 - 0.0152i, -0.0347 + 0.1078i, 0.0716 + 0.0503i, 0.0735 - 0.0004i, 0.0569 - 0.0911i, -0.0819 - 0.0714i, -0.0644 + 0.0369i] and is shown in Fig 14. Similarly optimum weight vector for $Ne = 4$ and $Ne = 6$ with different elements spacing (d) can be computed.

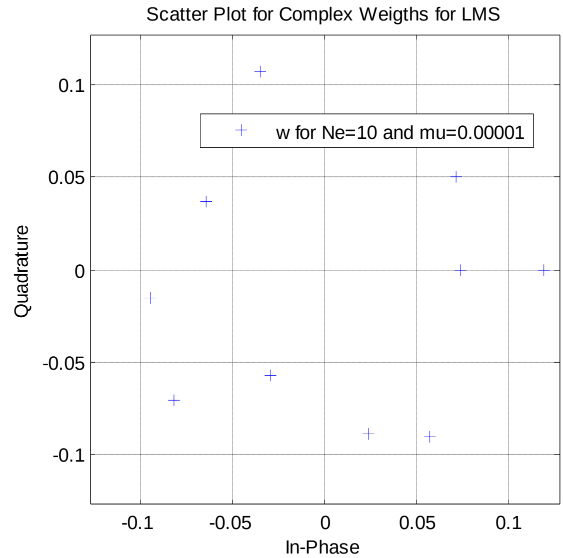

Fig.14 Scatter plot for **complex weights** for LMS for $Ne = 10$

3.2.2 Effect of Spacing Between Elements on Array Factor

When number of elements is kept constant for different array spacing $\lambda / 2$, $\lambda / 4$ and $\lambda / 8$. Then its effect is shown in Fig. 15 for $Ne = 4$

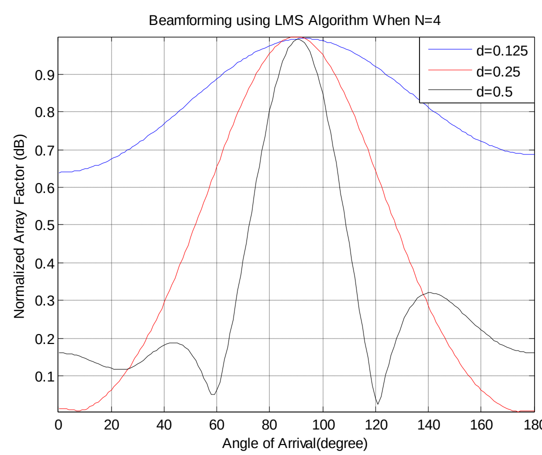

Fig.15. Normalized array factor plot for LMS algorithm with AOA for desired user is 90 degree and **three interferer with 30, 50 and 140 degrees** for $Ne = 4$

3.*2.3 Effect of Step Size (μ) on Array Factor*

If a large value of μ is taken then convergence becomes faster but makes the array system unstable/noisy. Conversely if a small value is taken then convergence becomes slow that is also not desirable. Therefore, value of μ is taken in between that satisfy the conditions imposed in (16) for good convergence as shown in Fig 16 for $Ne = 4$

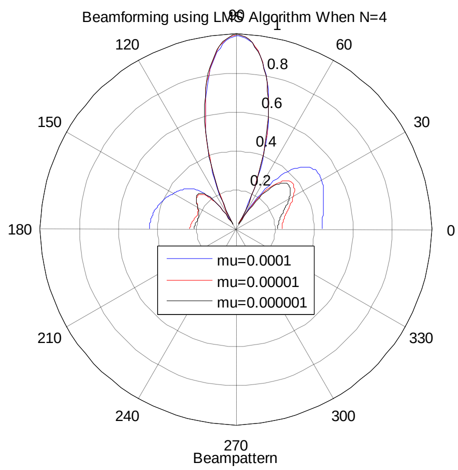

Fig.16. Polar plot for LMS algorithm with AOA for desired user is 90 degree and **three interferer with 20, 30 and 40 degrees** for $Ne = 4$

3.2.4 Effect of AOA on Array Factor

LMS algorithm is also compared on the basis of AOA as shown in Fig. 17 and has shown best response for beamforming keeping $\lambda / 2$ spacing between elements for $Ne = 6$.

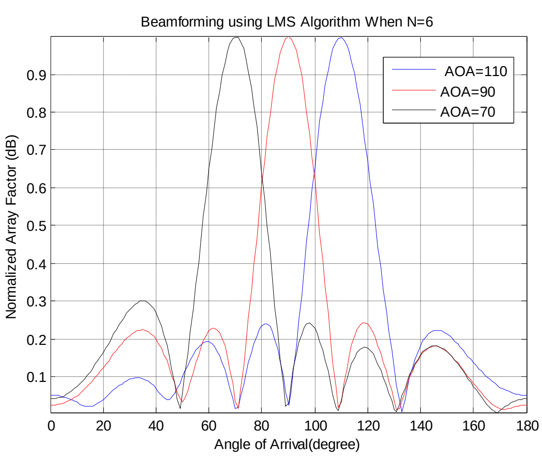

Fig.17. Normalized array factor plot for LMS algorithm for **three AOA with 70, 90 and 110 degrees** for $Ne = 6$

3.2.5 Effect of Number of Elements on MSE

The effect of number of elements on MSE for constant space $d = \lambda / 2$ between elements is shown in Fig. 18 and 19 for $Ne = 4$ and 10 respectively. From these figures, it is clear that minimum MSE is obtained for $Ne = 4$ when same $\mu = 0.00001$ is taken for comparison.

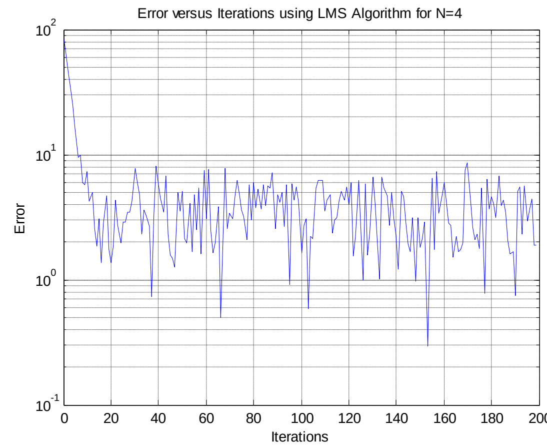

Fig.18. Mean square error for LMS algorithm for $Ne = 4$ and **space** $(d = \lambda / 2)$ **is kept constant**

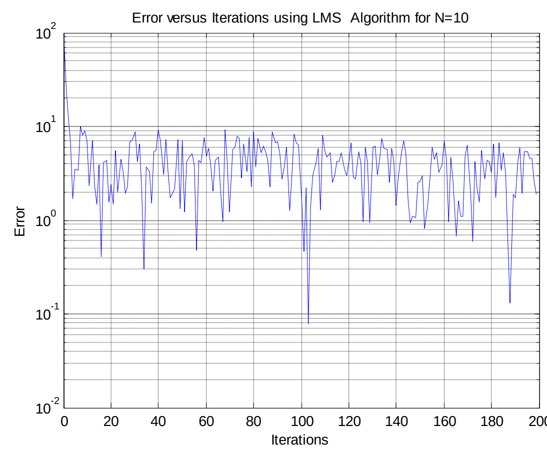

Fig.19. Mean square error for LMS algorithm for $Ne = 10$ and **space** $(d = \lambda / 2)$ **is kept constant**

4. CONCLUSIONS

In this paper, two adaptive beamforming algorithms are discussed. One is blind algorithm i.e. CMA and other is nonblind algorithm called LMS which needs pilot signal to train the beamformer weights. These algorithms are used in smart/adaptive antenna array system in coded form to generate beam in the look direction and null towards interferers, thus enhancing mobile communication performance both in quality and capacity. It is confirmed from the simulation results that narrow beam of smart antenna can be steered towards the desired direction by steering beam angle Φ_0, keeping different number of elements and spacing between elements for both algorithms using adaptive weights $w(n)$. Both these algorithms have good response towards desired direction and have better capability to place null towards interferer. However, it is ascertained from the simulation results that the performance of CMA is better to minimize MSE for different number of elements using performance cost function of the algorithm that minimized the average power in the error signal as compared to LMS algorithm which shows some deficiency to minimize MSE taking same number of iteration and elements. Therefore, CMA is found the most efficient algorithm as compared to LMS. CMA a blind algorithm is, therefore, a better option to implement at base station of mobile communication systems to reduce system overhead, avoid interference and optimize capacity as it doesn't require pilot signal which represents wasted resources in terms of time and energy.

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