Anti Q-Fuzzy HX Group and Its Lower Level Sub HX Groups

R.Muthuraj , K.H.Manikandan, M.S.Muthuraman, P.M.Sitharselvam
Department of Mathematics, PSNA College of Engineering and Technology,
Dindigul-624 622, Tamilnadu , India.

ABSTRACT
In this paper, we redefine the definition of a fuzzy HX group and define a new algebraic structure of Q-fuzzy HX group and anti Q-fuzzy HX subgroup and some related properties are ivestigated. We establish the relation between Q-fuzzy HX group and anti Q-fuzzy HX group of a HX group . The purpose of this study is to implement the fuzzy set theory and group theory in anti Q-fuzzy HX subgroups. Characterizations of lower level subsets of an anti Q-fuzzy HX subgroup of a HX group are given.

Keywords
Fuzzy set , Q-fuzzy set, fuzzy subgroup , Q-fuzzy subgroup, anti-Q fuzzy subgroups. anti-Q fuzzy HX subgroups.

AMS Subject Classification (2000): 20N25, 03E72, 03F055 , 06F35, 03G25.

1.Introduction

K.H.Kim introduce the concept of intuitionistic Q-fuzzy semi prime ideals in semi groups and Osman kazanci, sultan yamark and serife yilmaz introduce the concept of intuitionistic Q-fuzzy R-subgroups of near rings and F.H. Rho, K.H.Kim, J.G Lu introduce the concept of intuitionistic Q-fuzzy subalgebras of BCK / BCI – algebras. A.Solairaju and R.Nagarajan introduce and define a new algebraic structure of Q-fuzzy groups. Li Hongxing introduce the concept of HX group and the authors Luo Chengzhong , Mi Honghai , Li Hongxing introduce the concept of fuzzy HX group. In this paper we define a new algebraic structure of Q-fuzzy HX group and anti Q-fuzzy HX subgroup and study some their related properties.

2. Preliminaries

In this section we site the fundamental definitions that will be used in the sequel. Through out this paper, G = (G , *) is a group, e is the identity element of G, and xy, we mean x * y.

2.1 Definition

In 2^G-{ϕ}, a nonempty set $\vartheta \subset 2^G$-{ϕ} is called a HX group on G, if ϑ is a group with respect to the algebraic operation defined by AB = { ab /a ∈ A and b ∈ B}, which its unit element is denoted by E.

2.2 Definition

Let X be any non empty set. A fuzzy subset λ of S is a function $\lambda : X \rightarrow [0,1]$.

2.3 Definition

A fuzzy set λ is called fuzzy HX subgroup of a HX group ϑ if for A , B $\in \vartheta$,

(i) $\lambda(AB) \geq \min\{\lambda(A), \lambda(B)\}$
(ii) $\lambda(A^{-1}) = \lambda(A)$.

2.4 Definition

A fuzzy set λ is called an anti fuzzy HX subgroup of a HX group ϑ if A , B $\in \vartheta$,

(i) $\lambda(AB) \leq \max\{\lambda(A), \lambda(B)\}$,
(ii) $\lambda(A^{-1}) = \lambda(A)$.

2.5 Definition

Let Q and ϑ be any two sets. A mapping

$\lambda : \vartheta \times Q \rightarrow [0,1]$ is called a Q–fuzzy set in ϑ .

2.6 Definition

A Q-fuzzy set λ is called Q-fuzzy HX subgroup of a HX group ϑ if for A , B $\in \vartheta$ and q∈Q,

(iii) $\lambda(AB, q) \geq \min\{\lambda(A,q), \lambda(B,q)\}$
(iv) $\lambda(A^{-1}, q) = \lambda(A, q)$.

2.7 Definition

A Q-fuzzy set λ is called an anti Q-fuzzy HX subgroup of a HX group ϑ if A , B $\in \vartheta$ and $q \in Q$,

(iii) $\lambda(AB, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$,

(iv) $\lambda(A^{-1}, q) = \lambda(A, q)$.

3. Properties of anti Q-fuzzy HX subgroup

In this section, we discuss some of the properties of anti Q-fuzzy HX subgroup.

3.1 Theorem

Let λ be an anti Q-fuzzy HX subgroup of a HX group ϑ then

i. $\lambda(A, q) \geq \lambda(E, q)$ for all $A \in \vartheta$, $q \in Q$ and E is the identity element of ϑ.

ii. The subset $H = \{A \in \vartheta \,/\, \lambda(A, q) = \lambda(E, q)\}$ is a sub HX group of ϑ.

Proof

(i) Let $A \in \vartheta$ and $q \in Q$.

$$\begin{aligned} \lambda(A, q) &= \max\{\lambda(A, q), \lambda(A, q)\} \\ &= \max\{\lambda(A, q), \lambda(A^{-1}, q)\} \\ &\geq \lambda(AA^{-1}, q) \\ &= \lambda(E, q). \end{aligned}$$

$$\lambda(A, q) \geq \lambda(E, q) \text{ for all } A \in \vartheta.$$

(ii) Let $H = \{A \in \vartheta \,/\, \lambda(A, q) = \lambda(E, q)\}$.

Clearly H is non-empty as $E \in H$. Let $A, B \in H$.

Then, $\lambda(A, q) = \lambda(B, q) = \lambda(E, q)$.

$$\begin{aligned} \lambda(AB^{-1}, q) &\leq \max\{\lambda(A, q), \lambda(B^{-1}, q)\} \\ &= \max\{\lambda(A, q), \lambda(B, q)\} \\ &= \max\{\lambda(E, q), \lambda(E, q)\} \\ &= \lambda(E, q). \end{aligned}$$

That is, $\lambda(AB^{-1}, q) \leq \lambda(E, q)$ and obviously

$$\lambda(AB^{-1}, q) \geq \lambda(E, q).$$

Hence, $\lambda(AB^{-1}, q) = \lambda(E, q)$ and $AB^{-1} \in H$.

Clearly, H is a sub HX group of ϑ.

3.2 Theorem

λ is a Q- fuzzy HX subgroup of ϑ, iff λ^C is an anti Q-fuzzy HX subgroup of ϑ.

Proof

Suppose λ is a Q-fuzzy HX subgroup of ϑ. Then for all

$A, B \in \vartheta$ and $q \in Q$,

$$\lambda(AB, q) \geq \min\{\lambda(x, q), \lambda(B, q)\}$$

$$\Leftrightarrow 1 - \lambda^c(AB, q) \geq \min\{(1 - \lambda^c(A, q)), (1 - \lambda^c(B, q))\}$$

$$\Leftrightarrow \lambda^c(AB, q) \leq 1 - \min\{(1 - \lambda^c(A, q)), (1 - \lambda^c(B, q))\}$$

$$\Leftrightarrow \lambda^c(AB, q) \leq \max\{\lambda^c(A, q), \lambda^c(B, q)\}.$$

We have, $\lambda(A, q) = \lambda(A^{-1}, q)$ for all A in ϑ and $q \in Q$,

$$\Leftrightarrow 1 - \lambda^c(A, q) = 1 - \lambda^c(A^{-1}, q) .$$

Therefore, $\lambda^c(A, q) = \lambda^c(A^{-1}, q)$.

Hence λ^c is an anti Q-fuzzy HX subgroup of ϑ.

3.3 Theorem

Let λ be any anti Q-fuzzy HX subgroup of a HX group ϑ with identity E. Then

$$\lambda(AB^{-1}, q) = \lambda(E, q) \Rightarrow \lambda(A, q) = \lambda(B, q)$$

for all A,B in ϑ and $q \in Q$.

Proof

Given λ is an anti Q-fuzzy HX subgroup of ϑ and $\lambda(AB^{-1}, q) = \lambda(E, q)$.

Then for all A , B in ϑ and $q \in Q$,

$$\lambda(A, q) = \lambda(A(B^{-1}B), q)$$

$= \lambda((AB^{-1})B, q)$

$\leq \max\{\lambda(AB^{-1}, q), \lambda(B, q)\}$

$= \max\{\lambda(E, q), \lambda(B, q)\}$

$= \lambda(B, q)$.

That is, $\lambda(A, q) \leq \lambda(B, q)$.

Now, $\lambda(B, q) = \lambda(B^{-1}, q)$, since λ is an anti Q-fuzzy HX subgroup of ϑ.

$= \lambda(EB^{-1}, q)$

$= \lambda((A^{-1}A)B^{-1}, q)$

$= \lambda(A^{-1}(AB^{-1}), q)$

$\leq \max\{\lambda(A^{-1}, q), \lambda(AB^{-1}, q)\}$

$= \max\{\lambda(A, q), \lambda(E, q)\}$

$= \lambda(A, q)$.

(i.e.) $\lambda(B, q) \leq \lambda(A, q)$.

Hence, $\lambda(A, q) = \lambda(B, q)$.

3.4 Theorem

λ is an anti Q-fuzzy HX subgroup of a HX group ϑ if and only if $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$, for all A , B in ϑ and $q \in Q$.

Proof

Let λ be an anti Q-fuzzy HX subgroup of a HX group ϑ. Then for all A , B in ϑ and $q \in Q$,

$$\lambda(AB, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$$

and $\lambda(A, q) = \lambda(A^{-1}, q)$.

Now, $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B^{-1}, q)\}$.

$= \max\{\lambda(A, q), \lambda(B, q)\}$

$\Leftrightarrow \lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$.

4. Properties of Lower level subsets of an anti Q-fuzzy HX subgroup

In this section, we introduce the concept of lower level subset of an anti Q-fuzzy HX subgroup and discuss some of its properties.

4.1 Definition

Let λ be an anti Q-fuzzy HX subgroup of a HX group ϑ. For any $t \in [0,1]$, we define the set $L(\lambda; t) = \{A \in \vartheta / \lambda(A, q) \leq t\}$ is called the lower level subset of A.

4.1 Theorem

Let λ be an anti Q-fuzzy HX subgroup of a HX group ϑ . Then for $t \in [0,1]$ such that $t \geq \lambda(E, q)$, $L(\lambda; t)$ is a sub HX group of G.

Proof

For all $A, B \in L(\lambda; t)$, we have,

$$\lambda(A, q) \leq t ; \lambda(B, q) \leq t.$$

Now, $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$.

$\lambda(AB^{-1}, q) \leq \max\{t, t\}$.

$\lambda(AB^{-1}, q) \leq t$.

$AB^{-1} \in L(\lambda; t)$.

Hence $L(\lambda; t)$ is a sub HX group of ϑ.

4.2 Theorem

Let ϑ be a HX group and λ be a Q-fuzzy subset of ϑ such that $L(\lambda; t)$ is a sub HX group of ϑ. For $t \in [0,1]$ $t \geq \lambda(E, q)$, λ is an anti Q-fuzzy HX subgroup of ϑ.

Proof

Let A, B in ϑ and $\lambda(A,q) = t_1$ and $\lambda(B,q) = t_2$.

Suppose $t_1 < t_2$, then A, B $\in$ L (λ ; t_2).

As L (λ ; t_2) is a subgroup of G, $AB^{-1} \in$ L (λ ; t_2).

Hence, $\lambda(AB^{-1}, q) \leq t_2 = \max\{t_1, t_2\}$

$$\leq \max\{\lambda(A, q), \lambda(B, q)\}$$

That is, $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$.

Hence λ is an anti Q-fuzzy HX subgroup of ϑ.

4.2 Definition

Let λ be an anti Q-fuzzy HX subgroup of a HX group ϑ. The sub HX groups L (λ ; t) for $t \in [0,1]$ and $t \geq \lambda(E, q)$, are called lower level sub HX groups of λ.

4.3 Theorem

Let λ be an anti Q-fuzzy HX subgroup of a HX group ϑ. If two lower level sub HX groups L (λ ; t_1), L (λ ; t_2), for, $t_1, t_2 \in [0,1]$ and $t_1, t_2 \geq \lambda(E, q)$ with $t_1 < t_2$ of λ are equal then there is no A in ϑ such that $t_1 < \lambda(A, q) \leq t_2$.

Proof

Let L (λ ; t_1) = L (λ ; t_2).

Suppose there exists $A \in \vartheta$ such that $t_1 < \lambda(A, q) \leq t_2$ then

L(λ ; t_1) $\subseteq$ L (λ ; t_2).

Then A $\in$ L (λ ; t_2) , but A $\notin$ L (λ ; t_1), which contradicts the assumption that, L(λ ; t_1) = L (λ ; t_2).Hence there is no A in ϑ such that $t_1 < \lambda(A, q) \leq t_2$.

Conversely, suppose that there is no A in ϑ such that

$t_1 < \lambda(A, q) \leq t_2$.

Then, by definition, L (λ ; t_1) $\subseteq$ L (λ ; t_2).

Let A $\in$ L (λ ; t_2) and there is no A in ϑ such that

$t_1 < \lambda(A, q) \leq t_2$.

Hence A $\in$ L (λ ; t_1) and L (λ ; t_2) $\subseteq$ L (λ ; t_1).

Hence L (λ ; t_1) = L (λ ; t_2).

4.4 Theorem

A Q-fuzzy subset λ of ϑ is an anti Q-fuzzy HX subgroup of a HX group ϑ if and only if the lower level subsets L (λ ; t), $t \in$ Image λ, are HX subgroups of ϑ.

Proof It is clear.

4.5 Theorem

Any sub HX group H of a HX group ϑ can be realized as a lower level sub HX group of some anti Q-fuzzy HX subgroup of ϑ.

Proof

Let λ be a Q-fuzzy subset and $A \in \vartheta$ and $q \in Q$.

Define,

$$\lambda(A, q) = \begin{cases} 0 & \text{if } A \in H \\ t & \text{if } A \notin H \text{, where } t \in (0,1]. \end{cases}$$

We shall prove that λ is an anti Q-fuzzy HX subgroup of ϑ.

Let A , B $\in \vartheta$ and $q \in Q$.

i. Suppose A, B $\in$ H, then AB $\in$ H and $AB^{-1} \in$ H.

$\lambda(A, q) = 0$, $\lambda(B, q) = 0$, and $\lambda(AB^{-1}, q) = 0$.

Hence $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$.

ii. Suppose A $\in$ H and B $\notin$ H, then AB $\notin$ H and $AB^{-1} \notin$ H.

$\lambda(A, q) = 0$, $\lambda(B, q) = t$ and $\lambda(AB^{-1}, q) = t$.

Hence $\lambda(AB^{-1}, q) \leq \max\{\lambda(A, q), \lambda(B, q)\}$.

iii. Suppose A, B $\notin$ H, then $AB^{-1} \in$ H or $AB^{-1} \notin$ H.

$\lambda(A, q) = t$, $\lambda(B, q) = t$ and $\lambda(AB^{-1}, q) = 0$ or t.

Hence $\lambda(AB^{-1}, q) \leq \max \{ \lambda(A, q), \lambda(B, q) \}$.

Thus in all cases, λ is an anti Q-fuzzy HX subgroup of ϑ.

For this anti Q-fuzzy HX subgroup, $L(\lambda ; t) = H$.

Remark

As a consequence of the **Theorem 4.3 and 4.4**, the lower level HX subgroups of an anti Q-fuzzy HX subgroup A of a HX group ϑ form a chain. Since $\lambda(E, q) \leq \lambda(A, q)$ for all A in ϑ and $q \in Q$, therefore $L(\lambda ; t_0)$, where $\lambda(E, q) = t_0$ is the smallest and we have the chain :

$\{E\} \subset L(\lambda ; t_0) \subset L(\lambda ; t_1) \subset L(\lambda ; t_2) \subset \ldots \subset L(\lambda ; t_n) = \vartheta$, where $t_0 < t_1 < t_2 < \ldots\ldots < t_n$.

REFERENCES

[**1**] Kim.K.H., Yun.Y.B., on fuzzy R- subgroups of near rings, J.fuzzy math 8 (3) (2000) 549-558.

[**2**] Kim.K.H., Yun.Y.B., Normal fuzzy R- subgroups in near rings, Fuzzy sets systems 121 (2001) 341-345.

[**3**] Kim.K.H., on intuitionistic Q- fuzzy semi prime ideals in semi groups, Advances in fuzzy mathematics, 1 (1) (2006) 15-21.

[**4**] Li Hongxing, HX group, BESEFAL,33(1987), pp(31-37).

[**5**] Luo Chengzhong , Mi Honghai , Li Hongxing , Fuzzy HX group , BUSEFAL.

[**6**] Muthuraj.R., Sithar Selvam.P.M., Muthuraman.M.S., Anti Q-fuzzy group and its lower Level subgroups, International journal of Computer Applications (0975-8887),Volume 3- no.3, June 2010, 16-20.

[**7**] Muthuraj.R., Sridharan.M., , Muthuraman.M.S., and Sithar Selvam.P.M., Anti Q-fuzzy BG-idals in BG-Algebra, International journal of Computer Applications (0975-8887),Volume 4, no.11, August 2010, 27-31.

[8] Muthuraj.R., Sridharan.M., and Sithar Selvam.P.M., Fuzzy BG-ideals in BG-Algebra, International Journal of Computer Applications (0975-8887) Volume 2-No.1 may 2010.

[**9**] Osman kazanci, sultan yamark and serife yilmaz "On intuitionistic Q - fuzzy R-subgroups of near rings" International mathematical forum, 2, 2007 no. 59, 2899-2910.

[10] Palaniappan.N., Muthuraj.R., , Anti fuzzy group and Lower level subgroups, Antartica J.Math., 1 (1) (2004) , 71-76.

[**11**] Rosenfeld.A., fuzzy groups, J. math. Anal.Appl. 35 (1971), 512-517.

[**12**] Solairaju.A., and Nagarajan.R., " Q- fuzzy left R- subgroups of near rings w.r.t T- norms", Antarctica journal of mathematics.5, (1-2), 2008.

[**13**] Solairaju.A., and Nagarajan.R., A New Structure and Construction of Q-Fuzzy Groups, Advances in fuzzy mathematics, Volume 4 , Number 1 (2009) pp.23-29.