Application of Edge Coloring of a Fuzzy Graph

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ABSTRACT
In this paper, we have developed an algorithm to generate a course Time table satisfying certain constraints using Edge coloring of a fuzzy graph. With examples, we show how these constraints are satisfied.

General Terms
Fuzzy Logic, Fuzzy Graph.

Keywords
Edge coloring of a Fuzzy Graph, Time table problem.

1. INTRODUCTION
Edge coloring of graph arises in a variety of scheduling applications. One such application [1] is Time table scheduling problem. In Time tabling problem, it is required to schedule all the courses in such a way that no two classes of the same course are scheduled at the same time. If each edge color [2] represents a time slot in the schedule, then an edge coloring of a graph represents a feasible time table for courses.

Time tabling problem involves factors such as teachers, classes and courses. Various resources are rooms, time slots etc. Time tabling problem [3] is concerned with maximum utilization of the available resources subject to a set of constraints and can be classified [4] into three main classes namely school time tabling, course time tabling and examination time tabling. In this paper, we have attempted to give solution to course time tabling problem using the concept of Edge coloring of a fuzzy graph.

Using the definition of edge coloring of a fuzzy graph and algorithm developed in our paper [5], we have attempted to generate course Time table satisfying the following constraints. Nevertheless, the solutions we propose can be applied to general time tabling problem.

1. We assume that the number of semesters is three.
2. Six subjects are to be taught by different teachers for each semester.
3. Every teacher will be handling two subjects (of course, for different semesters).
4. Each subject should be taught for four hours in a week.
5. Classes are held for seven hours from Monday to Friday and for four hours on Saturdays.

We note that the department requires totally nine teachers for teaching all the subjects and the department has to allocate time slots for 72 hours in a week.

2. COURSE TIME TABLING PROBLEM
In general time tabling problem, we assume there are m teachers T_1, $T_2 \ldots T_m$ who teach for n semesters S_1, S_2, …..S_n. Each teacher T_i teaches a subject for semester S_j for p_{ij} periods in a week. This problem can be represented using a fuzzy bi − partite graph and the required time table can be generated using edge coloring of a fuzzy graph. Edge coloring is applied at two different stages. In the first stage, days are allocated for each teacher for each semester the teacher will be teaching. In the second stage, for the days allotted in the first stage, time slots will be allocated.

2.1 First Stage
Coming to the constraints we have imposed, the fuzzy bi - partite graph used in the first stage consists of four nodes on the left side which correspond to four hours per week handled by a teacher for a particular semester. Two nodes on the right side correspond to the two semesters to which the concerned teacher is teaching.

In the fuzzy graph shown below, T_{i1}, T_{i2} , T_{i3} and T_{i4} represent the 1st, 2nd, 3rd and 4th time slots assigned to a teacher T_i in a week

(i varies from 1 to 9). S_j and S_k are the two semesters handled by T_i (j and k vary from 1 to 3, $j \neq k$). The corresponding adjacency matrix is as shown in the Table 1.

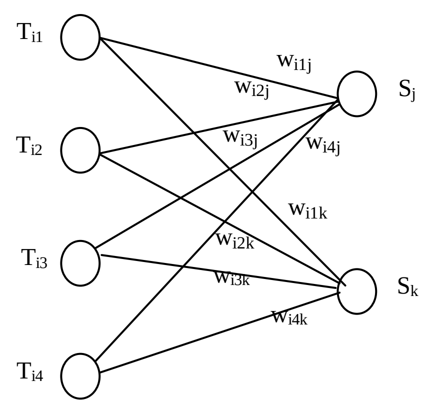

Figure 1. Fuzzy graph of first stage.

Table 1. Adjacency matrix for the first stage.

	T_{i1}	T_{i2}	T_{i3}	T_{i4}	S_j	S_k
T_{i1}	0	0	0	0	1	1
T_{i2}	0	0	0	0	1	1
T_{i3}	0	0	0	0	1	1
T_{i4}	0	0	0	0	1	1
S_j	1	1	1	1	0	0
S_k	1	1	1	1	0	0

Since there are six working days in a week (Monday to Saturday), we use six colors to color the above fuzzy graph. If r and s are two different colors, we use the color difference table d (r, s) = $|r - s|$. Membership table is based on the priorities given by a teacher. The membership values can be 'l', 'm' or 'h' which correspond to 'low', 'medium' and 'high' respectively. Block diagram for the first stage is shown below.

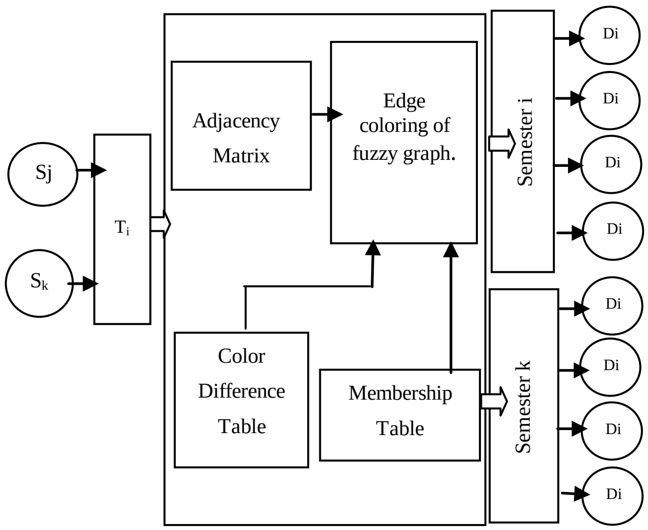

Figure 2. Block diagram of first Stage.

Edge coloring algorithm in the first stage takes as input adjacency matrix, color difference table and corresponding membership table and is executed nine times corresponding to nine teachers. In each iteration, the color allocated for an edge corresponds to a day of the week and indicates the day on which the particular teacher engages the particular semester. The output of the first stage becomes the input to the second stage.

The constraints that are satisfied in the first stage are the following.

1. For each semester, exactly four slots in a week are allocated for a teacher who teaches that semester.
2. Each teacher will engage a particular semester for maximum one slot in a day.

2.2 Second Stage

The fuzzy bi – partite graph used in the second stage consists of nine nodes on the left side which correspond to nine teachers. There are three nodes on the right side which correspond to three semesters.

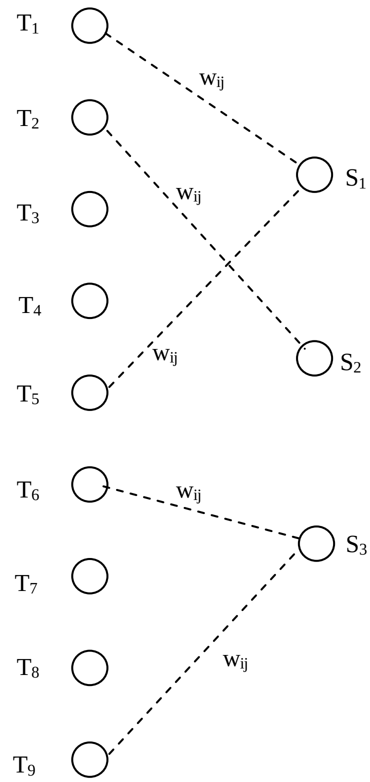

Figure 3. Fuzzy graph of second stage.

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	S_1	S_2	S_3
T_1	0	0	0	0	0	0	0	0	0	…	…	…
T_2	0	0	0	0	0	0	0	0	0	…	…	…
T_3	0	0	0	0	0	0	0	0	0	…	…	…
T_4	0	0	0	0	0	0	0	0	0	…	…	…
T_5	0	0	0	0	0	0	0	0	0	…	…	…
T_6	0	0	0	0	0	0	0	0	0	…	…	…
T_7	0	0	0	0	0	0	0	0	0	…	…	…
T_8	0	0	0	0	0	0	0	0	0	…	…	…
T_9	0	0	0	0	0	0	0	0	0	…	…	…
S_1	…	…	…	…	…	…	…	…	…	0	0	0
S_2	…	…	…	…	…	…	…	…	…	0	0	0
S_3	…	…	…	…	…	…	…	…	…	0	0	0

Table 2. Adjacency matrix for the second stage.

In the second stage, the edge coloring algorithm is executed six times corresponding to six days in a week. In each iteration, the adjacency matrix is created dynamically by extracting the data available at the output of the first stage. In other words, by referring to the teachers who are teaching on a given day for a given semester, the adjacency matrix is created dynamically in each iteration. Hence the edges in figure 3 are shown by dotted lines and the entries connecting the edges T_i with S_j ($1 \le i \le 9$ and $1 \le j \le 3$) in Table 2 are also shown by dotted lines.

Seven colors are considered for Monday to Friday (iteration 1 to iteration 5) which correspond to seven slots in a day. For Saturday (iteration 6), the number of colors considered is four corresponding to four slots on Saturday. The membership table for each iteration is also created dynamically. The block diagram for the second stage is shown in the figure given below.

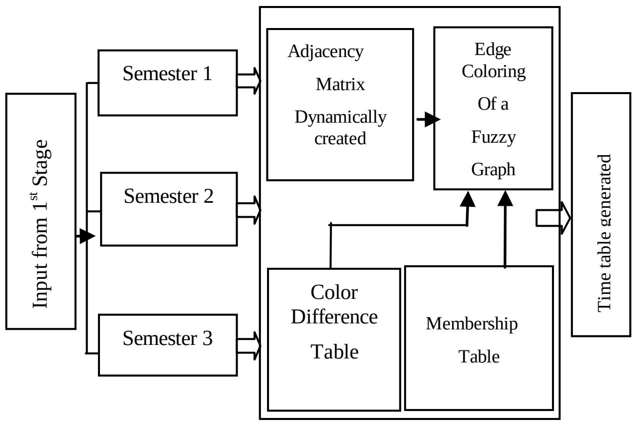

Figure 4. Block diagram of second Stage.

In each iteration which represents a particular day, timeslots are allocated for all the nine teachers. Finally, the entire Time table is generated.

The constraints that are satisfied in the second stage are the following.

1. In any given time slot, each teacher can teach at most one semester and each semester can be taught by at most one teacher.
2. If a teacher handles classes for two semesters in a day, the gap between the two slots is controlled by membership values assigned to the edges connecting teachers and semesters.

3. ALGORITHM TO GENERATE TIME TABLE USING EDGE COLORING OF A FUZZY GRAPH

STAGE 1:

Step 1: Allocate two semesters for each teacher.

Step 2: Generate the adjacency matrix corresponding to four hours in a week for each semester for each teacher.

Step 3: Generate the color difference table for six colors which corresponding to six days in a week.

Step 4: For each teacher, get the membership table and execute the edge coloring algorithm which takes adjacency matrix, membership table and color difference table as inputs.

Step 5: The output generated by the edge coloring algorithm assigns the colors to days of a week.

STAGE 2:

Step 6: For each day, generate the adjacency matrix and membership table dynamically for the second stage by mapping the output generated in the first stage.

Step 7: For each day, generate the color difference table and execute the edge coloring algorithm which again takes adjacency matrix, membership table and color difference table as inputs.

Step 8: Time table is generated by assigning the colors generated by the edge coloring algorithm to different time slots in a day.

Step 9: Display the final Timetable of each semester and also of Individual Teachers.

Example 3.1:

Consider an example where there are nine teachers $T_1, T_2, \ldots, T_9$ who take classes for three semesters S_1, S_2 and S_3. Assume that teachers T_1, T_2 and T_3 teach for semesters S_1 and S_2, teachers T_4, T_5 and T_6 teach for semesters S_1 and S_3, teachers T_7, T_8 and T_9 teach for semesters S_2 and S_3.

Using the above algorithm, we generate Time table which satisfies the constraints given in section 1. For the first stage, we consider the fuzzy graph of figure 1 and the adjacency matrix of Table 1. Nine membership tables for the first stage based on the priorities given by nine teachers $T_1, T_2, \ldots T_9$ are given below.

Table 3 Membership tables for nine teachers for the first stage of example 3.1

Teacher 1:

	T_{11}	T_{12}	T_{13}	T_{14}	S_1	S_2
T_{11}	0	0	0	0	l	l
T_{12}	0	0	0	0	m	h
T_{13}	0	0	0	0	m	m
T_{14}	0	0	0	0	h	h
S_1	l	m	m	h	0	0
S_2	l	h	m	h	0	0

Teacher 2:

	T_{21}	T_{22}	T_{23}	T_{24}	S_1	S_2
T_{21}	0	0	0	0	m	l
T_{22}	0	0	0	0	l	h
T_{23}	0	0	0	0	l	m
T_{24}	0	0	0	0	h	l
S_1	m	l	l	h	0	0
S_2	l	h	m	l	0	0

Teacher 3:

	T_{31}	T_{32}	T_{33}	T_{34}	S_1	S_2
T_{31}	0	0	0	0	h	h
T_{32}	0	0	0	0	m	m
T_{33}	0	0	0	0	h	l
T_{34}	0	0	0	0	l	m
S_1	h	m	h	l	0	0
S_2	h	m	l	m	0	0

Teacher 4:

	T_{41}	T_{42}	T_{43}	T_{44}	S_1	S_3
T_{41}	0	0	0	0	h	m
T_{42}	0	0	0	0	m	h
T_{43}	0	0	0	0	l	h
T_{44}	0	0	0	0	h	l
S_1	h	m	l	h	0	0
S_3	m	h	h	l	0	0

Teacher 5:

	T_{51}	T_{52}	T_{53}	T_{54}	S_1	S_3
T_{51}	0	0	0	0	l	h
T_{52}	0	0	0	0	m	h
T_{53}	0	0	0	0	l	m
T_{54}	0	0	0	0	h	m
S_1	l	m	l	h	0	0
S_3	h	h	m	m	0	0

Teacher 6:

	T_{61}	T_{62}	T_{63}	T_{64}	S_1	S_3
T_{61}	0	0	0	0	m	m
T_{62}	0	0	0	0	h	l
T_{63}	0	0	0	0	l	m
T_{64}	0	0	0	0	m	h
S_1	m	h	l	m	0	0
S_3	m	l	m	h	0	0

Teacher 7:

	T_{71}	T_{72}	T_{73}	T_{74}	S_2	S_3
T_{71}	0	0	0	0	m	m
T_{72}	0	0	0	0	l	h
T_{73}	0	0	0	0	m	h
T_{74}	0	0	0	0	h	l
S_2	m	l	m	h	0	0
S_3	m	h	h	l	0	0

Teacher 8:

	T_{81}	T_{82}	T_{83}	T_{84}	S_2	S_3
T_{81}	0	0	0	0	h	l
T_{82}	0	0	0	0	m	h
T_{83}	0	0	0	0	l	m
T_{84}	0	0	0	0	h	h
S_2	h	m	l	h	0	0
S_3	l	h	m	h	0	0

Teacher 9:

	T_{91}	T_{92}	T_{93}	T_{94}	S_2	S_3
T_{91}	0	0	0	0	l	m
T_{92}	0	0	0	0	h	l
T_{93}	0	0	0	0	m	h
T_{94}	0	0	0	0	m	m
S_2	l	h	m	m	0	0
S_3	m	l	h	m	0	0

Output generated by the first stage for the membership tables considered above is as shown in the Table given below.

Table 4. Output generated by first stage for example 3.1.

Semester 1:

	Mon	Tue	Wed	Thur	Fri	Sat
T_1	1	1	0	1	0	1
T_2	1	1	1	1	0	0
T_3	1	1	0	1	0	1
T_4	1	1	1	0	1	0
T_5	1	1	0	1	0	1
T_6	1	1	1	1	0	0

Semester 2:

	Mon	Tue	Wed	Thur	Fri	Sat
T_1	1	1	0	1	0	1
T_2	1	1	1	1	0	0
T_3	1	1	0	1	0	1
T_7	1	1	1	0	1	0
T_8	1	1	0	1	0	1
T_9	1	1	1	0	1	0

Semester 3:

	Mon	Tue	Wed	Thur	Fri	Sat
T_4	1	1	1	0	1	0
T_5	1	1	0	1	0	1
T_6	1	1	1	1	0	0
T_7	1	1	1	0	1	0
T_8	1	1	0	1	0	1
T_9	1	1	1	0	1	0

The output generated by the first stage gives information about the teachers who are all taking classes for a particular semester on a particular day. For example, on Wednesday, teachers T_2, T_4 and T_6 are engaging classes for semester 1, teachers T_2, T_7 and T_9 are engaging classes for semester 2 and teachers T_4, T_6, T_7 and T_9 are engaging classes for semester 3.

We now look at the adjacency matrix generated dynamically for the second stage by mapping the output generated in the first stage. The corresponding membership tables for the second stage are given below.

Table 5. The membership table for the second stage of example 3.1.

Monday:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	S_1	S_2	S_3
T_1	0	0	0	0	0	0	0	0	0	l	h	0
T_2	0	0	0	0	0	0	0	0	0	h	m	0
T_3	0	0	0	0	0	0	0	0	0	l	h	0
T_4	0	0	0	0	0	0	0	0	0	m	0	h
T_5	0	0	0	0	0	0	0	0	0	h	0	l
T_6	0	0	0	0	0	0	0	0	0	m	0	m
T_7	0	0	0	0	0	0	0	0	0	0	m	l
T_8	0	0	0	0	0	0	0	0	0	0	l	m
T_9	0	0	0	0	0	0	0	0	0	0	l	h
S_1	l	h	l	m	h	m	0	0	0	0	0	0
S_2	h	m	h	0	0	0	m	l	l	0	0	0
S_3	0	0	0	h	l	m	l	m	h	0	0	0

Tuesday:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	S_1	S_2	S_3
T_1	0	0	0	0	0	0	0	0	0	h	m	0
T_2	0	0	0	0	0	0	0	0	0	h	l	0
T_3	0	0	0	0	0	0	0	0	0	l	h	0
T_4	0	0	0	0	0	0	0	0	0	m	0	h
T_5	0	0	0	0	0	0	0	0	0	l	0	m
T_6	0	0	0	0	0	0	0	0	0	l	0	h
T_7	0	0	0	0	0	0	0	0	0	0	h	l
T_8	0	0	0	0	0	0	0	0	0	0	m	l
T_9	0	0	0	0	0	0	0	0	0	0	l	m
S_1	h	h	l	m	l	l	0	0	0	0	0	0
S_2	m	l	h	0	0	0	h	m	l	0	0	0
S_3	0	0	0	h	m	h	l	l	m	0	0	0

Wednesday:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	S_1	S_2	S_3
T_1	0	0	0	0	0	0	0	0	0	m	h	0
T_2	0	0	0	0	0	0	0	0	0	0	0	0
T_3	0	0	0	0	0	0	0	0	0	l	m	0
T_4	0	0	0	0	0	0	0	0	0	0	0	h
T_5	0	0	0	0	0	0	0	0	0	h	0	m
T_6	0	0	0	0	0	0	0	0	0	0	0	0
T_7	0	0	0	0	0	0	0	0	0	0	0	0
T_8	0	0	0	0	0	0	0	0	0	0	l	h
T_9	0	0	0	0	0	0	0	0	0	0	h	m
S_1	m	0	l	0	h	0	0	0	0	0	0	0
S_2	h	0	m	0	0	0	0	l	h	0	0	0
S_3	0	0	0	h	m	0	0	h	m	0	0	0

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	S_1	S_2	S_3
T_1	0	0	0	0	0	0	0	0	0	0	0	0
T_2	0	0	0	0	0	0	0	0	0	0	0	0
T_3	0	0	0	0	0	0	0	0	0	0	0	0
T_4	0	0	0	0	0	0	0	0	0	m	0	h
T_5	0	0	0	0	0	0	0	0	0	0	0	0
T_6	0	0	0	0	0	0	0	0	0	0	0	0
T_7	0	0	0	0	0	0	0	0	0	0	h	m
T_8	0	0	0	0	0	0	0	0	0	0	0	0
T_9	0	0	0	0	0	0	0	0	0	0	m	m
S_1	0	0	0	m	0	0	0	0	0	0	0	0
S_2	0	0	0	0	0	0	h	0	m	0	0	0
S_3	0	0	0	h	0	0	m	0	m	0	0	0

Thursday:

Friday:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	S_1	S_2	S_3
T_1	0	0	0	0	0	0	0	0	0	m	h	0
T_2	0	0	0	0	0	0	0	0	0	l	h	0
T_3	0	0	0	0	0	0	0	0	0	h	m	0
T_4	0	0	0	0	0	0	0	0	0	0	0	0
T_5	0	0	0	0	0	0	0	0	0	m	0	h
T_6	0	0	0	0	0	0	0	0	0	m	0	l
T_7	0	0	0	0	0	0	0	0	0	0	0	0
T_8	0	0	0	0	0	0	0	0	0	0	m	h
T_9	0	0	0	0	0	0	0	0	0	0	h	m
S_1	m	l	h	0	m	m	0	0	0	0	0	0
S_2	h	h	m	0	0	0	0	m	0	0	0	0
S_3	0	0	0	0	h	l	0	h	0	0	0	0

Saturday:

	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	S_1	S_2	S_3
T_1	0	0	0	0	0	0	0	0	0	0	0	0
T_2	0	0	0	0	0	0	0	0	0	m	h	0
T_3	0	0	0	0	0	0	0	0	0	0	0	0
T_4	0	0	0	0	0	0	0	0	0	h	0	m
T_5	0	0	0	0	0	0	0	0	0	0	0	0
T_6	0	0	0	0	0	0	0	0	0	h	0	h
T_7	0	0	0	0	0	0	0	0	0	0	m	m
T_8	0	0	0	0	0	0	0	0	0	0	0	0
T_9	0	0	0	0	0	0	0	0	0	0	h	m
S_1	0	m	0	h	0	h	0	0	0	0	0	0
S_2	0	h	0	0	0	0	m	0	h	0	0	0
S_3	0	0	0	m	0	h	m	0	m	0	0	0

Considering the priorities of nine teachers given in Table 3, the final time table is generated and is shown in the Table given below.

Table 6. Final Time Table generated.

Semester 1

	slot1	slot2	slot3	slot4	slot5	slot6	slot7
Mon	T_2	T_1	T_4	T_3	T_5	-	T_6
Tue	T_1	T_3	T_5	T_2	T_6	T_4	-
Wed	T_2	-	T_4	-	-	T_6	-
Thurs	T_1	T_2	T_3	-	T_5	-	T_6
Fri	-	-	T_4	-	-	-	-
Sat	T_1	T_3	T_5	-	-	-	-

Semester 2

	slot1	slot2	slot3	slot4	slot5	slot6	slot7
Mon	T_1	T_8	T_2	T_9	T_3	-	T_7
Tue	T_3	T_2	T_1	T_9	T_7	-	T_8
Wed	T_9	-	-	T_2	-	-	T_7
Thurs	T_3	-	T_1	-	T_8	-	T_2
Fri	T_7	-	T_9	-	-	-	-
Sat	T_3	T_8	T_1	-	-	-	-

Semester 3

	slot1	slot2	slot3	slot4	slot5	slot6	slot7
Mon	T_4	T_5	T_6	T_7	T_8	-	T_9
Tue	T_4	T_7	T_9	T_8	T_5	-	T_6
Wed	T_4	-	T_6	-	T_7	-	T_9
Thurs	T_5	T_6	-	-	-	-	T_8
Fri	T_4	-	T_7	-	T_9	-	-
Sat	T_5	-	T_8	-	-	-	-

4. CONCLUSION

Time table generated using Edge coloring of a Fuzzy graph always ensures non conflicting course time table. However, it is difficult to implement certain constraints in Time Table problem using edge coloring algorithm because it always starts coloring from first available color. If the algorithm can be modified in such a way that it starts coloring the edges of a fuzzy graph by choosing the available colors randomly instead of in order, we should be able to implement other constraints too.

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