Some Characterization of Anti Q-L-Fuzzy ℓ-Group

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ABSTRACT

In this paper we introduce the notion of anti Q-L-fuzzy ℓ-group, anti Q-L-fuzzy ℓ-ideals with values in a complete lattice L which is infinite meet distributive, and investigate some of its properties.

Keywords

Fuzzy set, Q-fuzzy set, anti L-fuzzy sub ℓ-group, anti Q-L-fuzzy sub ℓ-group , anti Q-L-fuzzy ℓ-ideal.

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1. Introduction

L. A. Zadeh introduced the notion of a fuzzy subset A of a set X as a function from X into I = [0, 1]. Rosenfeld [18] and Kuroki[12] applied this concept in group theory and semi group theory, and developed the theory of fuzzy subgroups and fuzzy subsemigroupoids respectively. In 1982, Liu[13] defined and studied fuzzy subrings as well as fuzzy ideals in rings. Subsequently, Mukherjee and Sen[16], K. L. N. Swamy and U. M. Swamy[19], and Zhang Yue[22] fuzzified certain standard concepts on rings and ideals. Malik and Morderson[14] defined an extension of a fuzzy ideal of a ring and Further more Abou-Zaid [1] characterized prime fuzzy ideals in a near rings, and Jun [8] extended the Prime fuzzy ideals in rings to Γ-rings. J.A. Goguen [7] replaced the valuations set [0, 1],by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A.Solairaju and R.Nagarajan[11] introduce and define a new algebraic structure of Q-fuzzy groups. In fact it seems in order to obtain a complete analogy of crisp mathematics in terms of fuzzy mathematics, it is necessary to replace the valuation set by a system having more rich algebraic structure. These concepts ℓ-groups play a major role in mathematics and fuzzy mathematics. Satya Saibaba [23] introduce the concept of L-fuzzy ℓ-group and L-fuzzy ℓ-ideal of ℓ-group. In this paper, we initiate the study of Q-fuzzy lattice ℓ-groups . We also introduce the notion of an anti Q-L-fuzzy sub ℓ-group, anti Q-L-fuzzy ℓ-ideal and establishes the relation with Q-L-fuzzy sub ℓ- group and Q-L-fuzzy ℓ-ideal of an ℓ- group G and discussed some of its properties. The characterisations of an anti Q-L-fuzzy sub ℓ-group and anti Q-L-fuzzy ℓ-ideal under homomorphism and anti homomorphism are discussed.

2. Preliminaries

In this Section, we review some definitions and some results of L-fuzzy subgroups which will be used in the later sections. Throughout this section we mean that $(G,*)$ is a group, e is the identity of G and xy as $x*y$.

2.1 Definition

A lattice ordered group is a system $G=(G,*,\leq)$ where

i. $(G,*)$ is a group,

ii. $(G,\leq)$ is a lattice and

iii. the inclusion is invariant under all translations $x \mapsto a*x*b$.

That is, $x \leq y \Rightarrow a*x*b \leq a*y*b$ for all $a,b \in G$.

2.2 Definition

If a is an element of ℓ-group G, then $a \vee (-a)$ is called the absolute value of a and is denoted by $|a|$. Any element a of an ℓ-group G can be written as

$a = (a \vee 0) * (a \wedge 0)$. i.e, $a = a^{+} * a^{-}$, where a^{+} is called positive part of a and a^{-} is called negative part of a.

2.3 Definition

The function $f : G \to G^{1}$ is said to be a homomorphism if $f(xy) = f(x) f(y)$ for all $x, y \in G$.

2.4 Definition

The function $f : G \to G^{1}$ (G and G^{1} are not necessarily commutative) is said to be an anti homomorphism. if $f(xy) = f(y) f(x)$ for all $x, y \in G$.

2.1 Proposition

In any ℓ-group G, for all $a \in G$, We have

(i) $|a| \geq 0$, Moreover $|a| > 0$, unless $a = 0$

(ii) $a^{+} \wedge (-a^{+}) = 0$.

(iii) $|a| = a^{+} - a^{-}$.

2.2 Proposition

In any ℓ-group G, $a * (a \wedge b)^{-1} * b = a \vee b$ for all $a, b \in G$.

2.5 Definition

A L-Fuzzy subset λ of X is a mapping from X into L, where L is a complete lattice satisfying the infinite meet distributive law. If L is the unit interval [0,1] of real numbers, there are the usual fuzzy subset of X.

A L-fuzzy subset $\lambda : X \to L$ is said to be a nonempty, if it is not the constant map which assumes the values 0 of L.

2.6 Definition

Let $\lambda : X \to L$ be a L-fuzzy sub set of X. Then for t$\in L$, the set $\lambda_t = \{x \in X / \lambda(x) \leq t\}$ is called a lower t-cut or t-level set of λ.

2.7 Definition

Let $\lambda, \mu : X \to L$ be a L-fuzzy sub sets of X. If $\lambda(x) \leq \mu(x)$ for all $x \in X$, then we say that λ is contained in μ and we write $\lambda \subseteq \mu$.

2.8 Definition

Let $\lambda, \mu : X \to L$ be a L-fuzzy sub sets of X. Define $\lambda \cup \mu$, $\lambda \cap \mu$ are L-fuzzy subsets of X by all $x \in X$, $(\lambda \cup \mu)(x) = \lambda(x) \vee \mu(x)$ and $(\lambda \cap \mu)(x) = \lambda(x) \wedge \mu(x)$. Then $\lambda \cup \mu$, $\lambda \cap \mu$ are called union and intersection of λ and μ respectively.

2.10 Definition

A L-fuzzy subset λ of X is said to have sup property if, for any subset A of X, there exists $a_0 \in A$ such that $\lambda(a_0) = \vee_{a \in A} \lambda(a)$.

2.11 Definition

Let f be any function from a set X to a set Y, and let λ be any L-fuzzy subset of X. Then λ is called f-invariant if $f(x) = f(y)$ implies $\lambda(x) = \lambda(y)$, where $x, y \in X$.

2.12 Definition

A L-fuzzy subset λ of G is said to be a L-fuzzy subgroup of G, if, for all $x, y \in G$,

i. $\lambda(xy) \geq \lambda(x) \wedge \lambda(y)$,

ii. $\lambda(x^{-1}) = \lambda(x)$.

2.13 Definition

A L-fuzzy subset λ of G is said to be an anti L-fuzzy subgroup of G, if, for all $x, y \in G$,

i. $\lambda(xy) \leq \lambda(x) \vee \lambda(y)$,

ii. $\lambda(x^{-1}) = \lambda(x)$.

Remark: $\lambda(e) \leq \lambda(x)$ for all $x, y \in G$.

2.3 Proposition

A L-fuzzy subset λ of a group G is a anti L-fuzzy subgroup of G if and only if λ_t is a subgroup of G for all $\lambda(G) \cup \{t \in L / \lambda(e) \leq t\}$.

2.1 Theorem

Let λ be an anti L-fuzzy subgroup of G. Then $G_{\lambda} = \{x \in G / \lambda(x) = \lambda(e)\}$ is a subgroup of G.

Proof

Let $G_\lambda = \{x \in G / \lambda(x) = \lambda(e)\}$. Let $a, b \in G_\lambda$, then $\lambda(a) = \lambda(b) = \lambda(e)$.

$$\lambda(ab^{-1}) \leq \lambda(a) \vee \lambda(b^{-1})$$

$$\lambda(ab^{-1}) \leq \lambda(a) \vee \lambda(b)$$

$$\leq \lambda(e) \vee \lambda(e)$$

$$\leq \lambda(e)$$

That is, $\lambda(ab^{-1}) \leq \lambda(e)$, and $\lambda(e) \leq \lambda(ab^{-1})$. Hence $\lambda(ab^{-1}) = \lambda(e)$.Therefore, $ab^{-1} \in G_\lambda$. G_λ is a subgroup of G.

2.2 Theorem

λ is a L-fuzzy subgroup of G, iff λ^c is an anti L-fuzzy subgroup of G.

Proof

Suppose λ is a L-fuzzy subgroup of G. Then for all $x, y \in G$,

$$\lambda(xy) \geq \lambda(x) \wedge \lambda(y)$$

$$\Leftrightarrow \quad 1 - \lambda^c(xy) \geq (1-\lambda^c(x)) \wedge (1-\lambda^c(y))$$

$$\Leftrightarrow \quad \lambda^c(xy) \leq 1 - (1-\lambda^c(x)) \wedge (1-\lambda^c(y))$$

$$\Leftrightarrow \quad \lambda^c(xy) \leq \lambda^c(x) \vee \lambda^c(y).$$

We have, $\lambda(x) = \lambda(x^{-1})$ for all $x \in G$,

$$\Leftrightarrow \quad 1-\lambda^c(x) = 1-\lambda^c(x^{-1}) .$$

Therefore, $\lambda^c(x) = \lambda^c(x^{-1})$.Hence λ^c is an anti L-fuzzy subgroup of G.

2.14 Definition

Let Q and X be any two sets and λ be a L-fuzzy subset of X. A Q-L-Fuzzy subset λ of X is a mapping from $X \times Q$ into L, where L is a complete lattice satisfying the infinite meet distributive law. If L is the unit interval [0,1] of real numbers, there are the usual Q-fuzzy subset of X. A Q-L-fuzzy subset $\lambda : X \times Q \to L$ is said to be a nonempty, if it is not the constant map which assumes the values 0 of L.

2.15 Definition

Let $\lambda, \mu : X \times Q \to L$ be a Q-L-fuzzy subsets of X. If $\lambda(x,q) \leq \mu(x,q)$ for all $x \in X$ and $q \in Q$ then we say that λ is contained in μ and we write $\lambda \subseteq \mu$.

2.16 Definition

Let $\lambda, \mu : X \times Q \to L$ be a Q-L-fuzzy subsets of X. Define $\lambda \cup \mu$, $\lambda \cap \mu$ are Q-L-fuzzy subsets of X by all $x \in X$, $(\lambda \cup \mu)(x,q) = \lambda(x,q) \vee \mu(x,q)$ and $(\lambda \cap \mu)(x,q) = \lambda(x,q) \wedge \mu(x,q)$ for all $x \in X$ and $q \in Q$.Then $\lambda \cup \mu$, $\lambda \cap \mu$ are called union and intersection of λ and μ respectively.

2.17 Definition

Let $G = (G, *, \vee, \wedge)$ is a ℓ-group, e is the identity of G. Any sub ℓ- group $H \subseteq G$ is said to be an ℓ-ideal of G if H satisfies the following conditions

i. $x, y \in H \Rightarrow xy \in H$ and $yx \in H$,

ii. $x, y \in H$ and $y \in H \Rightarrow x \in H$.

2.18 Definition

A non trivial Complete Lattice L is said to have infinite meet distributive law if it satisfies,

$a \wedge \left(\bigvee_{x \in H} x\right) = \bigvee_{x \in H} (a \wedge x)$ for any $H \subseteq L$ and $a \in L$

3. Anti Q-L-fuzzy subgroups

In this section, we introduce the notion of anti Q-L-fuzzy subgroup of a group , and discussed some of its properties. Throughout this section, we mean that $(G, *)$ is a group, e is the identity of G and xy as $x * y$.

3.1 Definition

A Q-L-fuzzy subset λ of G is said to be a Q-L-fuzzy subgroup of G, if , for all $x, y \in G$ and $q \in Q$,

i. $\lambda(xy, q) \geq \lambda(x,q) \wedge \lambda(y,q)$,

ii. $\lambda(x^{-1}, q) = \lambda(x,q)$.

3.2 Definition

A Q-L-fuzzy subset λ of G is said to be an anti

Q-L-fuzzy subgroup of G, if , for all $x, y \in G$ and $q \in Q$,

i. $\lambda(xy, q) \leq \lambda(x, q) \vee \lambda(y, q)$,

ii. $\lambda(x^{-1}, q) = \lambda(x, q)$.

3.1 Proposition

A Q-L-fuzzy subset λ of a group G is an anti Q-L-fuzzy subgroup of G if and only if λ_t is a subgroup of G for all $\lambda(G) \cup \; t \in L / \lambda(e,q) \leq t, q \in Q$.

3.1 Theorem

Let λ be an anti Q-L-fuzzy subgroup of G. Then $G_\lambda = \; x \in G / \lambda(x,q) = \lambda(e,q), q \in Q$ is a subgroup of G.

Proof

Let $G_\lambda = \; x \in G / \lambda(x,q) = \lambda(e,q), q \in Q$.

Let $a, b \in G_\lambda$, then $\lambda(a,q) = \lambda(b,q) = \lambda(e,q)$.

$$\lambda(ab^{-1}, q) \leq \lambda(a,q) \vee \lambda(b^{-1}, q),$$

$$\lambda(ab^{-1}, q) \leq \lambda(a,q) \vee \lambda(b,q),$$

$$\leq \lambda(e,q) \vee \lambda(e,q),$$

$$\leq \lambda(e,q)$$

That is , $\lambda(ab^{-1}, q) \leq \lambda(e,q)$ and $\lambda(e,q) \leq \lambda(ab^{-1}, q)$ .Hence $\lambda(ab^{-1}, q) = \lambda(e,q)$.

Therefore, $ab^{-1} \in G_\lambda$. Hence G_λ is a subgroup of G.

3.2 Theorem

λ is a Q-L-fuzzy subgroup of G, iff λ^c is an anti Q-L-fuzzy subgroup of G.

Proof

Suppose λ is a Q-L-fuzzy subgroup of G. Then for all x, y $\in$ G.

$$\lambda(xy,q) \geq \lambda(x,q) \wedge \lambda(y,q)$$

$$\Leftrightarrow \; 1 - \lambda^c(xy,q) \geq (1 - \lambda^c(x,q)) \wedge (1 - \lambda^c(y,q))$$

$$\Leftrightarrow \lambda^c(xy,q) \leq 1 - (1 - \lambda^c(x,q)) \wedge (1 - \lambda^c(y,q))$$

$$\Leftrightarrow \; \lambda^c(xy,q) \leq \lambda^c(x,q) \vee \lambda^c(y,q).$$

We have, $\lambda(x,q) = \lambda(x^{-1}, q)$ for all x in G ,

$$\Leftrightarrow \quad 1 - \lambda^c(x,q) = 1 - \lambda^c(x^{-1}, q) \; .$$

Therefore, $\lambda^c(x,q) = \lambda^c(x^{-1}, q)$ .Hence λ^c is an anti Q-L fuzzy subgroup of G.

3.3 Definition

Let f be a mapping from X into Y , and let λ and μ be an anti Q-L-fuzzy subgroups of X and Y respectively. Then $f(\lambda)$ of Y and $f^{-1}(\mu)$ of X are defined by,

$$f(\lambda)(y) = \begin{cases} \wedge \; \lambda(x) / x \in X, \, f(x) = y \;\; if \;\; f^{-1}(y) \neq \phi; \\ 1 \; ; \; otherwise \end{cases}$$

Where $y \in Y$,and $f^{-1}(\mu)(x) = \mu(f(x))$, for all $x \in X$ are called image of λ under f and the pre-image of μ under f respectively.

3.3 Theorem

Let G and G^1 be any two groups. Let $f : G \rightarrow G^1$ be a homomorphism and onto. Let $\lambda : G \times Q \rightarrow L$ be an anti Q-L-fuzzy subgroup of G. Then $f(\lambda)$ is an anti Q-L-fuzzy subgroup of G^1, if λ has sup property and λ is f - invariant.

Proof

Let λ be an anti Q-L-fuzzy subgroup of G.

i. $f(\lambda)(xy,q) =$

$$\wedge \; \lambda(x_0 y_0, q) / x_0 y_0 \in G, \, f(x_0 y_0) = xy \; , q \in Q$$

$$= \lambda(x_0 y_0, q)$$

$$\leq \lambda(x_0, q) \vee \lambda(y_0, q)$$

$$\leq \; (\wedge \; \lambda(x_0, q) / x_0 \in G, \, f(x_0) = x, q \in Q \;) \; \vee$$

$$(\wedge \; \lambda(y_0, q) / y_0 \in G, \; f(y_0) = y \, , q \in Q \;\;)$$

$$\leq (\; f(\lambda)(x,q) \,) \vee (\; f(\lambda)(y,q) \,)$$

$$f(\lambda)(xy,q) \; \leq (\; f(\lambda)(x,q) \,) \vee (\; f(\lambda)(y,q) \,).$$

ii. $f(\lambda)(x^{-1}, q) = \wedge \; \lambda(x_0^{-1}, q) / x_0^{-1} \in G, \, f(x_0^{-1}) = x^{-1}, q \in Q$

$$= \lambda(x_0^{-1}, q)$$

$= \lambda(x_0, q)$

$= \wedge \; \lambda(x_0, q) / x_0 \in G, f(x_0) = x, q \in Q$

$= f(\lambda)(x, q)$

$f(\lambda)(x^{-1}, q) = f(\lambda)(x, q)$.

Hence $f(\lambda)$ is an anti Q-L- fuzzy subgroup of G^1.

3.4 Theorem

Let G and G^1 be any two groups. Let $f : G \to G^1$ be a homomorphism and onto . Let $\mu : G^1 \times Q \to L$ be an anti Q-L-fuzzy subgroup of G^1. Then $f^{-1}(\mu)$ is an anti Q-L-fuzzy subgroup of G.

Proof

Let μ be an anti Q-L-fuzzy subgroup of G^1.

i. $f^{-1}(\mu)(xy, q) = \mu(f(xy), q)$

$= \mu(f(x) f(y), q)$

$\leq \mu(f(x), q) \vee \mu(f(y), q)$

$\leq f^{-1}(\mu)(x, q) \vee f^{-1}(\mu)(y, q)$

$f^{-1}(\mu)(xy, q) \leq f^{-1}(\mu)(x, q) \vee f^{-1}(\mu)(y, q)$

ii . $f^{-1}(\mu)(x^{-1}, q) = \mu(f(x^{-1}), q)$

$= \mu((f(x))^{-1}, q)$

$= \mu((f(x)), q)$

$= f^{-1}(\mu)(x, q)$

$f^{-1}(\mu)(x^{-1}, q) = f^{-1}(\mu)(x, q)$.

Hence $f^{-1}(\mu)$ is an anti Q-L-fuzzy subgroup of G.

3.5 Theorem

Let G and G^1 be any two groups. Let $f : G \to G^1$ be an anti homomorphism and onto. Let $\lambda : G \times Q \to L$ be an anti Q-L-fuzzy subgroup of G. Then $f(\lambda)$ is an anti Q-L-fuzzy subgroup of G^1, if λ has sup property and λ is f - invariant.

Proof

Let λ be an anti Q-L-fuzzy subgroup of G.

i. $f(\lambda)(xy, q) = \wedge \; \lambda(x_0 y_0, q) / x_0 y_0 \in G, f(x_0 y_0) = xy, q \in Q$

$= \lambda(x_0 y_0, q)$

$\leq \lambda(x_0, q) \vee \lambda(y_0, q)$

$\leq (\wedge \; \lambda(x_0, q) / x_0 \in G, f(x_0) = x, q \in Q) \vee$

$(\wedge \; \lambda(y_0, q) / y_0 \in G, f(y_0) = y, q \in Q)$

$\leq (f(\lambda)(x, q)) \vee (f(\lambda)(y, q))$

$f(\lambda)(xy, q) \leq (f(\lambda)(x, q)) \vee (f(\lambda)(y, q))$.

ii. $f(\lambda)(x^{-1}, q) = \wedge \; \lambda(x_0^{-1}, q) / x_0^{-1} \in G, f(x_0^{-1}) = x^{-1}, q \in Q$

$= \lambda(x_0^{-1}, q)$

$= \lambda(x_0, q)$

$= \wedge \; \lambda(x_0, q) / x_0 \in G, f(x_0) = x, q \in Q$

$= f(\lambda)(x, q)$

$f(\lambda)(x^{-1}, q) = f(\lambda)(x, q)$

Hence $f(\lambda)$ is an anti Q-L- fuzzy subgroup of G^1.

3.6 Theorem

Let G and G^1 be any two groups. Let $f : G \to G^1$ be an anti homomorphism and onto . Let $\mu : G^1 \times Q \to L$ be an anti Q-L-fuzzy subgroup of G^1. Then $f^{-1}(\mu)$ is an anti Q-L-fuzzy subgroup of G.

Proof

Let μ be an anti Q-L-fuzzy subgroup of G^1.

i. $f^{-1}(\mu)(xy, q) = \mu(f(xy), q)$

$= \mu(f(y) f(x), q)$

$\leq \mu(f(y), q) \vee \mu(f(x), q)$

$\leq f^{-1}(\mu)(y, q) \vee f^{-1}(\mu)(x, q)$

$f^{-1}(\mu)(xy, q) \leq f^{-1}(\mu)(x, q) \vee f^{-1}(\mu)(y, q)$

ii . $f^{-1}(\mu)(x^{-1}, q) = \mu(f(x^{-1}), q)$

$$= \mu((f(x))^{-1}, q)$$

$$= \mu((f(x)), q)$$

$$= f^{-1}(\mu)(x, q)$$

$$f^{-1}(\mu)(x^{-1}, q) = f^{-1}(\mu)(x, q) .$$

Hence $f^{-1}(\mu)$ is an anti Q-L-fuzzy subgroup of G.

4. Anti Q-L-fuzzy ℓ-groups

In this section, we introduce the notion of anti Q- L-fuzzy sub ℓ-group of a ℓ-group , and discussed some of its properties. Throughout this section, we mean that $G=(G,*,\vee,\wedge)$ is a ℓ-group, e is the identity of G and xy as $x*y$.

4.1 Definition

A Q-L-fuzzy subset λ of G is said to be a Q-L-fuzzy sub ℓ-group of G, if , for all $x, y \in G$ and $q \in Q$,,

i. $\lambda(xy,q) \geq \lambda(x,q) \wedge \lambda(y,q)$,

ii. $\lambda(x^{-1},q) = \lambda(x,q)$,

iii. $\lambda(x \vee y,q) \geq \lambda(x,q) \wedge \lambda(y,q)$,

iv. $\lambda(x \wedge y,q) \geq \lambda(x,q) \wedge \lambda(y,q)$,

4.2 Definition

A Q-L-fuzzy subset λ of G is said to be an anti Q-L-fuzzy sub ℓ-group of G, if , for all $x, y \in G$ and $q \in Q$,

i. $\lambda(xy,q) \leq \lambda(x,q) \vee \lambda(y,q)$,

ii. $\lambda(x^{-1},q) = \lambda(x,q)$,

iii. $\lambda(x \vee y,q) \leq \lambda(x,q) \vee \lambda(y,q)$,

iv. $\lambda(x \wedge y,q) \leq \lambda(x,q) \vee \lambda(y,q)$,

4.1 Proposition

A Q-L-fuzzy subset λ of a group G is an anti Q-L-fuzzy sub ℓ-group of G if and only if λ_t is a sub ℓ-group of G for all $\lambda(G) \cup \; t \in L / \lambda(e,q) \geq t, q \in Q$.

4.1 Theorem

Let λ be an anti Q-L-fuzzy sub ℓ-group of G. then $G_\lambda = \; x \in G / \lambda(x,q) = \lambda(e,q), q \in Q$ is a sub ℓ-group of G.

Proof

Let $G_\lambda = \; x \in G / \lambda(x,q) = \lambda(e,q), q \in Q$.

Let $a, b \in G_\lambda$, then $\lambda(a,q) = \lambda(b,q) = \lambda(e,q)$.

$$\lambda(ab^{-1},q) \leq \lambda(a,q) \vee \lambda(b^{-1},q),$$

$$\lambda(ab^{-1},q) \leq \lambda(a,q) \vee \lambda(b,q),$$

$$\leq \lambda(e,q) \vee \lambda(e,q),$$

$$\leq \lambda(e,q)$$

That is , $\lambda(ab^{-1},q) \leq \lambda(e,q)$ and $\lambda(e,q) \leq \lambda(ab^{-1},q)$.

Hence $\lambda(ab^{-1},q) = \lambda(e,q)$. Therefore, $ab^{-1} \in G_\lambda$. Hence G_λ is a sub ℓ-group of G.

4.2 Theorem

λ is a Q-L-fuzzy sub ℓ-group of G, iff λ^c is an anti Q-L-fuzzy sub ℓ-group of G.

Proof

Suppose λ is a Q-L-fuzzy sub ℓ-group of G. Then for all x, y ∈ G,

i. $\lambda(xy,q) \geq \lambda(x,q) \wedge \lambda(y,q)$

$$\Leftrightarrow 1 - \lambda^c(xy,q) \geq (1-\lambda^c(x,q)) \wedge (1-\lambda^c(y,q))$$

$$\Leftrightarrow \lambda^c(xy,q) \leq 1 - (1-\lambda^c(x,q)) \wedge (1-\lambda^c(y,q))$$

$$\Leftrightarrow \lambda^c(xy,q) \leq \lambda^c(x,q) \vee \lambda^c(y,q).$$

ii. We have, $\lambda(x,q) = \lambda(x^{-1},q)$ for all x in G ,

$$\Leftrightarrow 1-\lambda^c(x,q) = 1-\lambda^c(x^{-1},q).$$

Therefore, $\lambda^c(x,q) = \lambda^c(x^{-1},q)$.

iii. $\lambda(x \vee y,q) \geq \lambda(x,q) \wedge \lambda(y,q)$

$$\Leftrightarrow 1 - \lambda^c(x \vee y,q) \geq (1-\lambda^c(x,q)) \wedge (1-\lambda^c(y,q))$$

$$\Leftrightarrow \lambda^c(x \vee y,q) \leq 1 - (1-\lambda^c(x,q)) \wedge (1-\lambda^c(y,q))$$

$$\Leftrightarrow \qquad \leq \lambda^c(x,q) \vee \lambda^c(y,q)$$

$\Leftrightarrow \quad \lambda^c(x \vee y, q) \leq \lambda^c(x,q) \vee \lambda^c(y,q)$

iv. $\lambda(x \wedge y, q) \geq \lambda(x,q) \wedge \lambda(y,q)$

$\Leftrightarrow \quad 1 - \lambda^c(x \wedge y, q) \geq (1-\lambda^c(x,q)) \wedge (1-\lambda^c(y,q))$

$\Leftrightarrow \quad \lambda^c(x \wedge y, q) \leq 1 - (1-\lambda^c(x,q)) \wedge (1-\lambda^c(y,q))$

$\Leftrightarrow \quad \leq \lambda^c(x,q) \vee \lambda^c(y,q)$

$\Leftrightarrow \quad \lambda^c(x \wedge y, q) \leq \lambda^c(x,q) \vee \lambda^c(y,q)$.

Hence λ^c is an anti Q-L fuzzy sub ℓ-group of G.

4.3 Theorem

Let G and G^1 be any two ℓ-groups. Let $f: G \to G^1$ be a homomorphism and onto. Let $\lambda: G \times Q \to L$ be an anti Q-L-fuzzy sub ℓ-group of G. Then $f(\lambda)$ is an anti Q-L-fuzzy sub ℓ-group of G^1, if λ has sup property and λ is f-invariant.

Proof

Let λ be an anti Q-L-fuzzy sub ℓ-group of G.

i. $f(\lambda)(xy,q) = \wedge \lambda(x_0y_0, q) / x_0y_0 \in G, f(x_0y_0) = xy \;, q \in Q$

$= \lambda(x_0y_0, q)$

$\leq \lambda(x_0,q) \vee \lambda(y_0,q)$

$\leq (\wedge\ \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q\ ) \vee$

$(\wedge\ \lambda(y_0,q) / y_0 \in G, f(y_0) = y\;, q \in Q\ )$

$\leq (\ f(\lambda)(x,q) \vee f(\lambda)(y,q))$

$f(\lambda)(xy,q) \leq (\ f(\lambda)(x,q) \vee f(\lambda)(y,q))$.

ii. $f(\lambda)(x^{-1},q) = \wedge\ \lambda(x_0^{-1},q) / x_0^{-1} \in G, f(x_0^{-1}) = x^{-1} \;, q \in Q$

$= \lambda(x_0^{-1}, q)$

$= \lambda(x_0, q)$

$= \wedge\ \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q$

$= f(\lambda)(x,q)$.

$f(\lambda)(x^{-1},q) = f(\lambda)(x,q)$.

iii. $f(\lambda)(x \vee y, q) =$

$\wedge\ \lambda(x_0 \vee y_0, q) / x_0 \vee y_0 \in G, f(x_0 \vee y_0) = x \vee y \;, q \in Q$

$= \lambda(x_0 \vee y_0, q)$

$\leq \lambda(x_0,q) \vee \lambda(y_0,q)$

$\leq (\wedge\ \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q\ ) \vee$

$(\wedge\ \lambda(y_0,q) / y_0 \in G, f(y_0) = y\;, q \in Q\ )$

$\leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

$f(\lambda)(x \vee y, q) \leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$.

iv. $f(\lambda)(x \wedge y, q) =$

$\wedge\ \lambda(x_0 \wedge y_0, q) / x_0 \wedge y_0 \in G, f(x_0 \wedge y_0) = x \wedge y \;, q \in Q$

$= \lambda(x_0 \wedge y_0, q)$

$\leq \lambda(x_0,q) \vee \lambda(y_0,q)$

$\leq (\wedge\ \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q\ ) \vee$

$(\wedge\ \lambda(y_0,q) / y_0 \in G, f(y_0) = y\;, q \in Q\ )$

$\leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

$f(\lambda)(x \wedge y, q) \leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

Hence $f(\lambda)$ is an anti Q-L- fuzzy sub ℓ-group of G^1.

4.4 Theorem

Let G and G^1 be any two ℓ-groups. Let $f: G \to G^1$ be a homomorphism and onto. Let $\mu: G^1 \times Q \to L$ be an anti Q-L-fuzzy sub ℓ-group of G^1. Then $f^{-1}(\mu)$ is an anti Q-L-fuzzy sub ℓ-group of G.

Proof

Let μ be an anti Q-L-fuzzy sub ℓ-group of G^1.

i. $f^{-1}(\mu)(xy,q) = \mu(f(xy),q)$

$= \mu(f(x)f(y),q)$

$\leq \mu(f(x),q) \vee \mu(f(y),q)$

$\leq f^{-1}(\mu)(x,q) \vee f^{-1}(\mu)(y,q)$

$f^{-1}(\mu)(xy,q) \leq f^{-1}(\mu)(x,q) \vee f^{-1}(\mu)(y,q)$.

ii . $f^{-1}(\mu)(x^{-1},q) = \mu(f(x^{-1}),q)$

$= \mu((f(x))^{-1},q)$

$= \mu((f(x)),q)$

$= f^{-1}(\mu)(x,q)$

$f^{-1}(\mu)(x^{-1},q) = f^{-1}(\mu)(x,q)$.

iii. $f^{-1}(\mu)(x \vee y, q) = \mu(f(x \vee y), q)$

$= \mu(f(x) \vee f(y), q)$

$\leq \mu(f(x),q) \vee \mu(f(y),q)$

$\leq f^{-1}(\mu)(x,q) \vee f^{-1}(\mu)(y,q)$

$f^{-1}(\mu)(x \vee y,q) \leq f^{-1}(\mu)(x,q) \vee f^{-1}(\mu)(y,q)$

iv. $f^{-1}(\mu)(x \wedge y,q) = \mu(f(x \wedge y),q)$

$= \mu(f(x) \wedge f(y),q)$

$\leq \mu(f(x),q) \vee \mu(f(y),q)$

$\leq f^{-1}(\mu)(x,q) \vee f^{-1}(\mu)(y,q)$

$f^{-1}(\mu)(x \wedge y,q) \leq f^{-1}(\mu)(x,q) \vee f^{-1}(\mu)(y,q)$

Hence $f^{-1}(\mu)$ is an anti Q-L-fuzzy sub ℓ-group of G.

4.5 Theorem

Let G and G^1 be any two ℓ-groups. Let $f: G \to G^1$ be an anti homomorphism and onto. Let $\lambda: G \times Q \to L$ be an anti Q-L-fuzzy sub ℓ-group of G. Then $f(\lambda)$ is an anti Q-L-fuzzy sub ℓ-group of G^1, if λ has sup property and λ is f - invariant.

Proof

Let λ be a Q-L-fuzzy sub ℓ-group of G.

i. $f(\lambda)(xy,q) = \wedge \lambda(x_0 y_0,q) / x_0 y_0 \in G, f(x_0 y_0) = xy, q \in Q$

$= \lambda(x_0 y_0, q)$

$\leq \lambda(x_0,q) \vee \lambda(y_0,q)$

$\leq (\wedge \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q) \vee$

$(\wedge \lambda(y_0,q) / y_0 \in G, f(y_0) = y, q \in Q)$

$\leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

$f(\lambda)(xy,q) \leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

ii. $f(\lambda)(x^{-1},q) = \wedge \lambda(x^{-1},q) / x \in G, f(x_0) = x^{-1}, q \in Q$

$= \lambda(x_0^{-1},q)$

$= \lambda(x_0,q)$

$= \wedge \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q$

$= f(\lambda)(x,q)$

$f(\lambda)(x^{-1},q) = f(\lambda)(x,q)$

iii. $f(\lambda)(x \vee y,q) =$

$\wedge \lambda(x_0 \vee y_0,q) / x_0 \vee y_0 \in G, f(x_0 \vee y_0) = x \vee y, q \in Q$

$= \lambda(x_0 \vee y_0,q)$

$\leq \lambda(x_0,q) \vee \lambda(y_0,q)$

$\leq (\wedge \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q) \vee$

$(\wedge \lambda(y_0,q) / y_0 \in G, f(y_0) = y, q \in Q)$

$\leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

$f(\lambda)(x \vee y,q) \leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$.

iv. $f(\lambda)(x \wedge y,q) =$

$\wedge \lambda(x_0 \wedge y_0,q) / x_0 \wedge y_0 \in G, f(x_0 \wedge y_0) = x \wedge y, q \in Q$

$= \lambda(x_0 \wedge y_0,q)$

$\leq \lambda(x_0,q) \vee \lambda(y_0,q)$

$\leq (\wedge \lambda(x_0,q) / x_0 \in G, f(x_0) = x, q \in Q) \vee$

$(\wedge \lambda(y_0,q) / y_0 \in G, f(y_0) = y, q \in Q)$

$\leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

$f(\lambda)(x \wedge y,q) \leq f(\lambda)(x,q) \vee f(\lambda)(y,q)$

Hence $f(\lambda)$ is an anti Q-L- fuzzy sub ℓ-group of G^1.

4.6 Theorem

Let G and G^1 be any two ℓ-groups. Let $f: G \to G^1$ be an anti homomorphism and onto. Let $\mu: G^1 \times Q \to L$ be an anti Q-L-fuzzy sub ℓ-group of G^1. Then $f^{-1}(\mu)$ is an anti Q-L-fuzzy sub ℓ-group of G.

Proof

Let μ be an anti Q-L-fuzzy sub ℓ-group of G^1.

i. $f^{-1}(\mu)(xy,q) = \mu(f(xy),q)$

$= \mu(f(y)f(x),q)$

$\leq \mu(f(y),q) \vee \mu(f(x),q)$

$\leq f^{-1}(\mu)(y,q) \vee f^{-1}(\mu)(x,q)$

$f^{-1}(\mu)(xy,q) \leq f^{-1}(\mu)(x,q) \vee f^{-1}(\mu)(y,q)$

ii. $f^{-1}(\mu)(x^{-1},q) = \mu(f(x^{-1}),q)$

$= \mu((f(x))^{-1},q)$

$= \mu((f(x)),q)$

$= f^{-1}(\mu)(x,q)$

$f^{-1}(\mu)(x^{-1},q) = f^{-1}(\mu)(x,q)$.

iii. $f^{-1}(\mu)(x \vee y, q) = \mu(f(x \vee y), q)$

$= \mu(f(y) \vee f(x), q)$

$\leq \mu(f(y), q) \vee \mu(f(x), q)$

$\leq f^{-1}(\mu)(y, q) \vee f^{-1}(\mu)(x, q)$

$f^{-1}(\mu)(x \vee y, q) \leq f^{-1}(\mu)(x, q) \vee f^{-1}(\mu)(y, q)$

iv . $f^{-1}(\mu)(x \wedge y, q) = \mu(f(x \wedge y), q)$

$= \mu(f(y) \wedge f(x), q)$

$\leq \mu(f(y), q) \vee \mu(f(x), q)$

$\leq f^{-1}(\mu)(y, q) \vee f^{-1}(\mu)(x, q)$

$f^{-1}(\mu)(x \wedge y, q) \leq f^{-1}(\mu)(x, q) \vee f^{-1}(\mu)(y, q)$.

Hence $f^{-1}(\mu)$ is an anti Q-L-fuzzy sub ℓ -group of G.

5. Anti Q-L-fuzzy ℓ -ideal in ℓ -group

In this section, we introduce the concept of anti Q-L-fuzzy ℓ -ideal in ℓ -group and discuss some of its properties. Throughout this section, we mean that $G = (G, *, \vee, \wedge)$ is a ℓ -group, e is the identity of G and xy as $x * y$.

5.1 Definition

A Q-L-fuzzy sub ℓ -group λ of a ℓ -group G is said to be a Q-L-fuzzy ℓ - ideal of G if , for all $x, y \in G$, $q \in Q$

i. $\lambda(xy, q) = \lambda(yx, q)$ and

ii. $x, y \in G$, $|x| \leq |y| \Rightarrow \lambda(x, q) \geq \lambda(y, q)$.

5.2 Definition

A Q-L-fuzzy sub ℓ -group λ of a ℓ -group G is said to be an anti Q-L-fuzzy ℓ - ideal of G if, for all $x, y \in G$, $q \in Q$,

i. $\lambda(xy, q) = \lambda(yx, q)$ and

ii. $x, y \in G$, $|x| \leq |y| \Rightarrow \lambda(x, q) \leq \lambda(y, q)$.

5.1 Theorem

A Q-L-fuzzy sub ℓ -group λ of G is an anti Q-L-fuzzy ℓ -ideal if and only if each level sub ℓ -group λ_t , $t \in \lambda(G) \cup \ t \in L / \lambda(e, q) \leq t, q \in Q$ is an ℓ -ideal of G.

Proof

Let λ be an anti Q-L-fuzzy ℓ -ideal of G.

Let $t \in \lambda(G) \cup \ t \in L / \lambda(e, q) \leq t, q \in Q$.

Let $x \in \lambda_t$ and $y \in G$ .Then

$|y| \leq |x| \Rightarrow \lambda(y, q) \geq \lambda(x, q) \geq t$.

$x, y \in \lambda_t$ and λ_t is a level sub ℓ -group then $xy \in \lambda_t$ and $yx \in \lambda_t$ .Let $xy \in \lambda_t$ and $y \in \lambda_t$ then $y^{-1} \in \lambda_t$,since λ_t is a level sub ℓ -group. Therefore, $(xy)y^{-1} = x \in \lambda_t$.

Hence λ_t is an ℓ -ideal of G.

Conversely, let each level sub ℓ -group λ_t , $t \in \lambda(G) \cup \ t \in L / \lambda(e, q) \geq t, q \in Q$ is an ℓ -ideal of G. Let $x, y \in G$ then $\lambda(xy, q) = \lambda(yx, q)$ for all $x, y \in G$, $q \in Q$ .Let $x, y \in G$ and $|x| \leq |y|$. Let $\lambda(y, q) = t$ so $y \in \lambda_t$ and $x \in \lambda_t$,since λ_t is an ℓ -ideal of G.

$\lambda(x, q) \leq t = \lambda(y, q) \Rightarrow \lambda(x, q) \leq \lambda(y, q)$ for all $x \in G$.

Hence λ be an anti Q-L-fuzzy ℓ - ideal of G.

5.2 Theorem

λ is a Q-L-fuzzy - ℓ -ideal of G iff λ^c is an anti Q-L-fuzzy - ℓ -ideal.

Proof

λ is a Q-L-fuzzy - ℓ -ideal of G. Then for all $x, y \in G$ and $q \in Q$,

i. $\lambda(xy, q) = \lambda(yx, q)$.

$\Leftrightarrow \quad 1 - \lambda(xy, q) = 1 - \lambda(yx, q)$

$\Leftrightarrow \quad \lambda^c(xy, q) = \lambda^c(yx, q)$.

ii. Let $x, y \in G$ such that $|x| \leq |y|$. Then ,

$\lambda(x, q) \geq \lambda(y, q)$

$\Leftrightarrow \quad 1 - \lambda(x, q) \leq 1 - \lambda(y, q)$.

$\Leftrightarrow \quad \lambda^c(x, q) \leq \lambda^c(y, q)$.

Hence λ^c is an anti Q-L- fuzzy - ℓ -ideal of G.

5.3 Theorem

Every anti Q-L-fuzzy ℓ -ideal is an anti Q-L-fuzzy sub ℓ -group of G.

Proof it is clear.

Remark

But the converse of the above theorem is not true.

5.1 Example

Let R be the set of all real numbers and $(R,+,\vee,\wedge)$ be a ℓ-group. Define $\lambda : R \times Q \to L$ by

$\lambda(x,q)=\begin{cases}0 \text{ if } x\in Z\\ 1 \text{ if } x\notin Z.\end{cases}$ Where Z is the set of all integers.

Clearly λ is a Q-L-fuzzy sub ℓ-group of R. Level set $\lambda_0 = Z$ is a ℓ-subgroup of R, but not ℓ-ideal of R. Since $x = 2 \in Z$, $y = 0.5 \in R$ and $|0.5| \le |2|$. But $\lambda(0.5,q) = 1 > 0 = \lambda(2,q)$. So λ is not a Q-L-fuzzy ℓ-ideal of R.

5.4 Theorem

If λ and μ are two anti Q-L-fuzzy ℓ-ideals of G, then $\lambda \cap \mu$ is an anti Q-L-fuzzy ℓ-ideal of G.

Proof

Let λ and μ are two anti Q-L-fuzzy ℓ-ideals of G. Then

i. $(\lambda\cap\mu)(xy,q) = \lambda(xy,q) \vee \mu(xy,q)$,

$= \lambda(yx,q) \vee \mu(yx,q)$,

$= (\lambda\cap\mu)(yx,q)$,

$(\lambda\cap\mu)(xy,q) = (\lambda\cap\mu)(yx,q)$.

ii. Let $x, y \in G$ such that $|x| \le |y|$.

$(\lambda\cap\mu)(x,q) = \lambda(x,q) \vee \mu(x,q) \le \lambda(y,q) \vee \mu(y,q)$

$= (\lambda\cap\mu)(y,q)$

Hence $\lambda\cap\mu$ is an anti Q-L-fuzzy ℓ-ideal of G.

Clearly , the intersection of any family of anti Q-L-fuzzy ℓ-ideals of G is an anti Q-L-fuzzy ℓ-ideal of G.

5.5 Theorem

If H is any ℓ-ideal of G, $H \ne G$, then the Q-L-fuzzy subset λ of G defined by

$\lambda(x,q)=\begin{cases}s \text{ if } x\in H\\ t \text{ if } x\notin H\end{cases}$ where $s,t \in L$ and $s < t$ and $t \ne 0$ is an anti Q-L-fuzzy ℓ-ideal of G.

Proof

Let $x, y \in G$ and $q \in Q$. Then the following cases arise.

Case i. Let $x \in H$ and $y \in H$ implies $xy \in H$ and $yx \in H$,That is, $\lambda(xy,q) = \lambda(yx,q) = s$.

ii. Let $x \in H$ and $y \notin H$ implies $xy \notin H$ and $yx \notin H$. That is, $\lambda(xy,q) = \lambda(yx,q) = t$.

iii. Let $x \notin H$ and $y \in H$ implies $xy \notin H$ and $yx \notin H$. That is, $\lambda(xy,q) = \lambda(yx,q) = t$.

iv. Let $x \notin H$ and $y \notin H$ implies $xy \in H$ and $yx \in H$ or $xy \notin H$ and $yx \notin H$.

That is, $\lambda(xy,q) = \lambda(yx,q) = s$ or

$\lambda(xy,q) = \lambda(yx,q) = t$.

Let $x \in G$ and $y \in H$ such that $|x| \le |y|$. Therefore $x \in G$, since H is ℓ-ideal and $\lambda(x,q) = s = \lambda(y,q)$. Hence λ is an anti Q-L-fuzzy ℓ-ideal of G.

5.6 Theorem

A non-empty subset H of G is a ℓ-ideal of G if and only if the function

$\psi_H(x)=\begin{cases}0 \text{ if } x\in H\\ 1 \text{ if } x\notin H\end{cases}$ is an anti Q-L fuzzy ℓ-ideal of G .

Proof

By **Theorem 5.5** , ψ_H is an anti Q-L fuzzy ℓ-ideal of G. Conversely,Let ψ_H be an anti Q-L fuzzy ℓ-ideal of G. Then $\lambda(xy,q) = \lambda(yx,q)$. Let $x \in G$ and $y \in H$ such that $|x| \le |y|$. $\psi_H(x) \le \psi_H(y) = 0$, since ψ_H is an ℓ-ideal. So $\psi_H(x) = 0$. So $x \in H$. Therefore H is ℓ-ideal of G.

5.7 Theorem

Let λ be an anti Q-L fuzzy ℓ-ideal of G. Then $G_\lambda = \ x \in G / \lambda(x,q) = \lambda(e,q), q \in Q$ is a ℓ-ideal of G .

Proof

Let $x \in G$ and $y \in G_\lambda$, Then

$\lambda(y,q) = \lambda(e,q), q \in Q$.

$\lambda(xy,q) \le \lambda(x,q) \vee \lambda(y,q)$

$= \lambda(x,q) \vee \lambda(e,q)$

$= \lambda(e,q)$.

$\lambda(xy,q) \le \lambda(e,q)$ and $\lambda(e,q) \le \lambda(xy,q)$.

Hence $\lambda(xy,q) = \lambda(e,q)$. Hence $xy \in G_\lambda$, Similarly $yx \in G_\lambda$ .Let $x \in G$ and $y \in G_\lambda$ such that $|x| \le |y|$.

$\lambda(x,q) \le \lambda(y,q) = \lambda(e,q)$, since λ is an anti Q-L- fuzzy ℓ -ideal and $\lambda(e,q) \le \lambda(x,q)$. So $\lambda(x,q) = \lambda(e,q)$, for all $x \in G$ and $q \in Q$. So $x \in G_\lambda$. Therefore G_λ is ℓ -ideal of G .

5.8 Theorem

Let G and G^1 be any two ℓ -groups. Let $f: G \to G^1$ be a homomorphism and onto. Let $\lambda: G \times Q \to L$ be an anti Q-L-fuzzy ℓ -ideal of G . Then $f(\lambda)$ is an anti Q-L-fuzzy ℓ -ideal of G^1 , if λ has sup property and λ is f - invariant.

Proof

Let λ be an anti Q-L-fuzzy ℓ -ideal of G .

i. $f(\lambda)(xy,q) = \wedge\ \lambda(x_0y_0,q) / x_0y_0 \in G,\ f(x_0y_0) = xy\ , q \in Q$

$= \lambda(x_0y_0, q)$

$= \lambda(y_0x_0, q)$

$= \wedge\ \lambda(y_0x_0,q) / y_0x_0 \in G,\ f(y_0x_0) = yx\ , q \in Q$

$= f(\lambda)(yx,q)$.

Therefore, $f(\lambda)(xy,q) = f(\lambda)(yx,q)$.

ii. Let $f(x), f(a) \in G^1$ such that $|f(x)| \le |f(a)|$. Then,

$f(\lambda)(f(x),q) = \underset{t \in f^{-1}(f(x))}{\wedge} \lambda(t,q) = \lambda(x,q)$ and

$f(\lambda)(f(a),q) = \underset{t \in f^{-1}(f(a))}{\wedge} \lambda(t,q) = \lambda(a,q)$

Now,

$|f(x)| \le |f(a)| \Rightarrow f(|x|) \le f(|a|) \Rightarrow f(|a|) = f(|x|) \vee f(|a|) = f(|x| \vee |a|)$

So $f(|a|) = f(|x| \vee |a|) \Rightarrow \lambda(|a|, q) = \lambda(|x| \vee |a|, q)$, since λ is f-invariant.. Since $|x| \le |x| \vee |a|$, we have $\lambda(x,q) \le \lambda(|x| \vee |a|, q) = \lambda(|a|, q) \le \lambda(a,q)$.

Therefore, $f(\lambda)(f(x),q) \le f(\lambda)(f(a),q)$. Hence $f(\lambda)$ is an anti Q-L- fuzzy ℓ -ideal of G^1 .

5.9 Theorem

Let G and G^1 be any two ℓ -groups. Let $f: G \to G^1$ be a homomorphism and onto. Let $\mu: G^1 \times Q \to L$ be an anti Q-L-fuzzy ℓ -ideal of G^1 . Then $f^{-1}(\mu)$ is an anti Q-L-fuzzy ℓ -ideal of G .

Proof

Let μ be an anti Q-L-fuzzy ℓ -ideal of G^1 .

i. $f^{-1}(\mu)(xy,q) = \mu(f(xy),q)$

$= \mu(f(x)f(y),q)$

$= \mu(f(y)f(x),q)$

$= \mu(f(yx),q)$

$= f^{-1}(\mu)(yx,q)$

Hence $f^{-1}(\mu)(xy,q) = f^{-1}(\mu)(yx,q)$

ii . Let $x, y \in G$ such that $|x| \le |y|$.

$f^{-1}(\mu)(x,q) = \mu(f(x),q)$ and $f^{-1}(\mu)(y,q) = \mu(f(y),q)$.

Now , $|x| \le |y| \Rightarrow f(|x|) \le f(|y|) \Rightarrow |f(x)| \le |f(y)|$.

Since μ be an anti Q-L-fuzzy ℓ -ideal of G^1 then $\mu((f(x)),q) \le \mu((f(y)),q)$.

That is, $f^{-1}(\mu)(x,q) \le f^{-1}(\mu)(y,q)$ .Hence $f^{-1}(\mu)$ is an anti Q-L-fuzzy ℓ -ideal of G .

5.10 Theorem

Let G and G^1 be any two ℓ -groups. Let $f: G \to G^1$ be an anti homomorphism and onto. Let $\lambda: G \times Q \to L$ be an anti Q-L-fuzzy ℓ-ideal of G. Then $f(\lambda)$ is an anti Q-L-fuzzy ℓ-ideal of G^1, if λ has sup property and λ is f - invariant.

Proof

Let λ be an anti Q-L-fuzzy ℓ-ideal of G.

i. $f(\lambda)(xy,q) = \wedge\ \lambda(x_0y_0,q) / x_0y_0 \in G,\ f(x_0y_0) = xy\ , q \in Q$

$= \lambda(x_0y_0,q)$

$= \lambda(y_0x_0,q)$

$= \wedge\ \lambda(y_0x_0,q) / y_0x_0 \in G,\ f(y_0x_0) = xy\ , q \in Q$

$= f(\lambda)(yx,q)$.

Therefore, $f(\lambda)(xy,q) = f(\lambda)(yx,q)$.

ii. Let $f(x), f(a) \in G^1$ such that $|f(x)| \le |f(a)|$. Then,

$(f(\lambda))(f(x),q) = \underset{t \in f^{-1}(f(x))}{\wedge} \lambda(t,q) = \lambda(x,q)$ and

$(f(\lambda))(f(a),q) = \underset{t \in f^{-1}(f(a))}{\wedge} \lambda(t,q) = \lambda(a,q)$

Now,

$|f(x)| \le |f(a)| \Rightarrow f(|x|) \le f(|a|) \Rightarrow f(|a|) = f(|x|) \vee f(|a|) = f(|x| \vee |a|)$

So $f(|a|) = f(|x| \vee |a|) \Rightarrow \lambda(|a|,q) = \lambda(|x| \vee |a|,q)$, since λ is f-invariant.. Since $|x| \le |x| \vee |a|$, we have $\lambda(x,q) \le \lambda(|x| \vee |a|,q) = \lambda(|a|,q) \le \lambda(a,q)$.

Therefore, $(f(\lambda))(f(x),q) \le (f(\lambda))(f(a),q)$. Hence $f(\lambda)$ is an anti Q-L- fuzzy ℓ-ideal of G^1.

5.11 Theorem

Let G and G^1 be any two ℓ-groups. Let $f : G \to G^1$ be an anti homomorphism and onto. Let $\mu : G^1 \times Q \to L$ be an anti Q-L-fuzzy ℓ-ideal of G^1. Then $f^{-1}(\mu)$ is an anti Q-L-fuzzy ℓ-ideal of G.

Proof

Let μ be an anti Q-L-fuzzy ℓ-ideal of G^1.

i. $f^{-1}(\mu)(xy,q) = \mu(f(xy),q)$

$= \mu(f(y)f(x),q)$

$= \mu(f(x)f(y),q)$

$= \mu(f(yx),q)$

$= f^{-1}(\mu)(yx,q)$

Hence $f^{-1}(\mu)(xy,q) = f^{-1}(\mu)(yx,q)$

ii . Let $x, y \in G$ such that $|x| \le |y|$.

$f^{-1}(\mu)(x,q) = \mu(f(x),q)$ and $f^{-1}(\mu)(y,q) = \mu(f(y),q)$.

Now , $|x| \le |y| \Rightarrow f(|x|) \le f(|y|) \Rightarrow |f(x)| \le |f(y)|$.

Since μ be an anti Q-L-fuzzy ℓ-ideal of G^1 then $\mu((f(x)),q) \le \mu((f(y)),q)$.

That is, $f^{-1}(\mu)(x,q) \le f^{-1}(\mu)(y,q)$ .Hence $f^{-1}(\mu)$ is an anti Q-L-fuzzy ℓ-ideal of G.

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