International Journal of Computer Applications (0975 – 8887)
Volume 7– No.2, September 2010

Acyclic Coloring of Star Graph Families

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ABSTRACT
In this paper, we discuss the acyclic vertex colouring and acyclic chromatic number of middle graph, central graph and total graph of star graph.

General Terms
Middle graph, central graph and total graph of star graph are denoted by $M(K_{1,n})$, $C(K_{1,n})$ and $T(K_{1,n})$ respectively.

Keywords
Middle graph, central graph, total graph, acyclic colouring, acyclic chromatic number.

1. INTRODUCTION
Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The middle graph [6,9] of G, denoted by $M(G)$ is defined as follows. The vertex set of $M(G)$ is $V(G)\cup E(G)$. Two vertices x,y in the vertex set of $M(G)$ are adjacent in $M(G)$ in case one of following holds:

(i) x,y Are in $E(G)$ and x,y are adjacent in G. (ii) x Is in $V(G)$, y is in $E(G)$, and x,y are incident in G.

Let G be a finite undirected graph with no loops and multiple edges. The central graph [16] of a graph G, $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G. By the definition $p_{c\ G} = p+q$. For any p,q graph there exist exactly p vertices of degree $p-1$ and q vertices of degree 2 in $C(G)$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The total graph [6,9] of G, denoted by $T(G)$ is defined as follows. The vertex set of $T(G)$ is $V(G)\cup E(G)$. Two vertices x,y in the vertex set of $T(G)$ are adjacent in $T(G)$ in case one of the following holds:

(i) x,y are in $V(G)$ and x is adjacent to y in G. (ii) x,y are in $E(G)$ and x,y are adjacent in G. (ii) x is in $V(G)$, y is in $E(G)$, and x,y are incident in G.

A proper vertex colouring of a graph is acyclic if every cycle uses at least three colours [13]. The acyclic chromatic number of G, denoted by $a\ G$, is the minimum k such that G admits an acyclic k-colouring.

2. ACYCLIC COLOURING OF $M(K_{1,n})$

2.1 Theorem
For any star graph $K_{1,n}$ the acyclic chromatic number, $a[M(K_{1,n})] = n+1$.

Proof
Consider the star graph $K_{1,n}$ with $V\ K_{1,n} = v_1, v_2, v_3, ...v_n, v$ with v as the root vertex. In middle graph $M(K_{1,n})$, by the definition each edge vv_i for $1 \le i \le n$ of $K_{1,n}$ is subdivided by the vertex e_i in $M(K_{1,n})$. and the vertices $e_1, e_2, e_3, ...e_n, v$ induce a clique of order $n+1$ in $M(K_{1,n})$ i.e., $V(M(K_{1,n})) = \{v_i / 1 \le i \le n\} \cup \{e_i / 1 \le i \le n\} \cup \{v\}$. Now assign a proper colouring to these vertices as follows. Consider a colour class $C = c_1, c_2, c_3, ...c_{n+1}$. Assign the colour c_i to the vertex e_i for $i = 1,2,..n$ and the colour c_{n+1} to the vertex v and to v_i for $i = 1,2,..n$. The colouring is minimum, as $M(K_{1,n})$ contains a clique of order $n+1$, minimum $n+1$ colours are required for its proper colouring. Next we have to prove that the above said coloring is acyclic. It is obvious that to form a bichromatic cycle both colours should occur at least twice. But in the above said colouring, the only colour class which occur atleast twice is c_{n+1} and all other colour classes occur only once in the colouring procedure. Hence the above said colouring is acyclic.

Thus $a[M(K_{1,n})] = n+1$.

Example

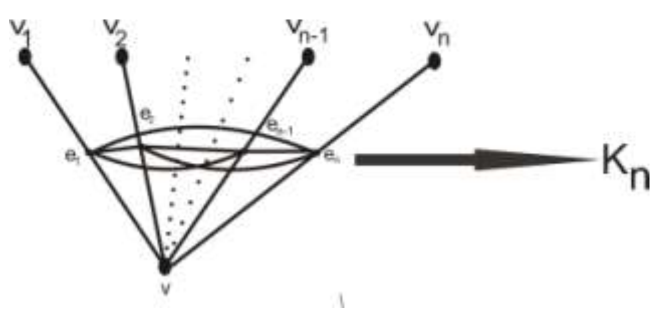

Figure 1

$a[M(K_{1,n})]=n+1$

3. ACYCLIC COLOURING OF $C(K_{1,n})$

3.1 Theorem

For the graph $K_{1,n}$ the acyclic chromatic number $a\left[C\;\;K_{1,n}\;\;\right]=n, n\geq 3$.

Proof

Consider the star graph $K_{1,n}$ with $V\;\;K_{1,n}\;\;=\;\;u_1,u_2,u_3,...u_n,u_0$ with u_0 as the root vertex. In $C(K_{1,n})$, let $u_{i,0}$ represents the newly introduced vertex in the edge connecting u_i and u_0 for $i=1,2...n$. Now assign a proper colouring to these vertices as follows. Consider a colour class $C=\;\;c_1,c_2,c_3,....c_n$. Assign the colour c_i to the vertex u_i for $i=1,2...n$ and the colour c_1 to the vertex u_0. Such a colouring excludes the newly introduced vertices. The colour c_2 is assigned to the newly introduced vertices $u_{i,0}$ for $i\neq 2$ and the colour c_3 is assigned to $u_{2,0}$. The colouring is minimum, as $C(K_{1,n})$ contains the subgraph K_n, minimum n colours are required for its proper colouring. Next we have to prove that the above said coloring is acyclic. It is obvious that to form a bichromatic cycle both colours should occur at least twice. So in the above said colouring, the color classes $c_k, 4\leq k\leq n$ never induce a 2-chromatic cycle (it occur only once in the colouring procedure). Now we need to examine the subgraphs induced by $\langle c_i,c_j\rangle$ for $i=1,2$ and $j=2,3$ with $i\neq j$ whether they induce a 2-chromatic cycle or not.

Case 1. If $i=1$ and $j=2$, **t**hen the sub graph induced by $\langle c_1,c_2\rangle$ is a tree with $n-3$ pendent vertices, which is clearly a forest.

Case 2. If $i=1$ and $j=3$, then the subgraph induced by $\langle c_1,c_3\rangle$ is the disjoint union of P_2 with itself. That is it forms a linear forest.

Case 3. If $j=2$ and $j=3$, then the subgraph induced by $\langle c_2,c_3\rangle$ is the union of a tree with $n-3$ isolated vertices, which also form a forest.

Thus any pair of the colour class will never induce a 2-chromatic cycle in the graph.

Therefore, $a\left[C\;\;K_{1,n}\;\;\right]=n, n\geq 3$.

Note: $a\left[C\;\;K_{1,n}\;\;\right]=3$, for $n=2$.

Example

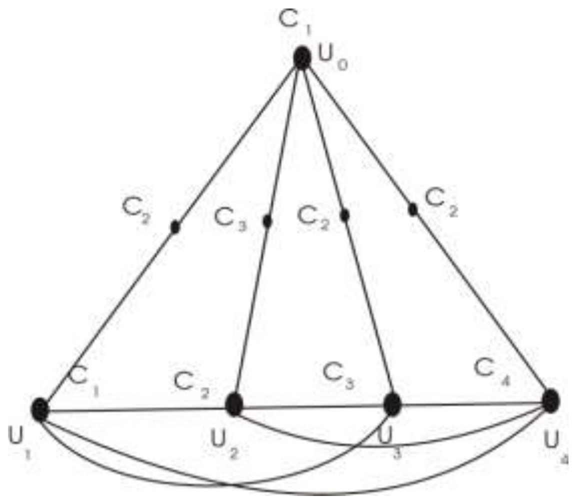

Figure 2

$a\left[C\;\;K_{1,4}\;\;\right]=4$

4. ACYCLIC COLOURING OF $T(K_{1,n})$

4.1 Theorem

For any star graph $K_{1,n}$ the acyclic chromatic number, $a[T(K_{1,n})]=n+1$.

Proof

Conside the star graph $K_{1,n}$ with $V\;\;K_{1,n}\;\;=\;\;v_1,v_2,v_3,...v_n,v$ with v as the root vertex. . In Total graph $T(K_{1,n})$, by the definition each edge vv_i for $1\leq i\leq n$ of $K_{1,n}$ is subdivided by the vertex e_i in $T(K_{1,n})$ and the vertices $e_1,e_2,e_3,...e_n,v$ induce a clique of order $n+1$ in $T(K_{1,n})$. i.e., $V(M(K_{1,n}))=\{v_i/1\leq i\leq n\}\bigcup\{e_i/1\leq i\leq n\}\bigcup\{v\}$. Now assign a proper colouring to these vertices as follows. Consider a colour class $C=\;\;c_1,c_2,c_3,....c_{n+1}$. Assign the colour c_i to the vertex e_i for $i=1,2,..n$ and the colour c_{n+1} to the vertex v. Now assign the colour c_1 to v_i for $2\leq i\leq n$ and c_2 to the remaining vertex v_1 . The colouring is minimum, as $T(K_{1,n})$ contains a clique of order $n+1$, minimum $n+1$ colours are required for its proper colouring. In the above said colouring, the color classes $c_k, 3\leq k\leq n+1$ never induce a 2-chromatic cycle (it occur only once in the colouring procedure). Also the subgraphs induced by $\langle c_1,c_2\rangle$ is the union of a path P_3 with $n-2$ isolated vertices, which is a forest. Thus any pair of the colour class will never induce a 2-chromatic cycle in the graph. That is the colouring is acyclic.

Thus $a[T(K_{1,n})] = n + 1$.

Example

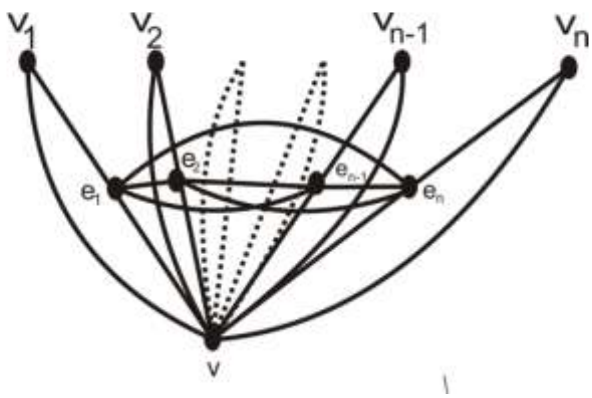

Figure 3

$$a[T(K_{1,n})] = n + 1.$$

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