Fixed Point Theorems in Fuzzy Metric Spaces Using Implicit Relations

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ABSTRACT
The aim of this paper is to present common fixed point theorem in fuzzy metric spaces, for four self maps, satisfying implicit relations. The results of B.Singh and M.S.Chauhan[16] are generalized in this paper. Also, the application of fixed points is studied for the Product spaces.

Keywords
Fuzzy metric space, ε-chainable fuzzy metric space, compatible mappings, weakly compatible mappings, implicit relation and common fixed point

1. INTRODUCTION
L.Zadeh's[18] investigation of the notion of fuzzy sets has led to the growth of fuzzy mathematics. The theory of fixed point equations is one of the preeminent basic tools to handle various physical formulations. Fixed point theorems in fuzzy mathematics has got a direction of vigorous hope and vital trust with the study of Kramosil and Michalek[10], who introduced the concept of fuzzy metric space. Later on, this concept of fuzzy metric space was modified by George and Veeramani[4].

Sessa[15] initiated the tradition of improving commutative condition in fixed point theorems by introducing the notion of weak commuting property . Further, Jungck[8] gave a more generalized condition defined as compatibility in metric spaces. Recently in 2006, Jungck and Rhoades [9] introduced the concept of weakly compatible maps which were found to be more generalized than compatible maps.

Grabiec[5] followed Kramosil and Michalek[10] and he obtained the fuzzy version of Banach contraction principle. Recently in 2000, B.Singh and M.S.Chauhan[16] brought forward the concept of compatibility in fuzzy metric space. Popa[11] proved some fixed point theorems for weakly compatible noncontinous mappings using implicit relations. His work was extended by Imdad[6] who used implicit relations for coincidence commuting property. Singh and Jain[14] extended the result of Popa[11] in fuzzy metric spaces.

This paper offers the fixed point theorems on fuzzy metric spaces, which generalize, extend and fuzzify several known fixed point theorems for compatible maps on metric space, by making use of implicit relations. The condition of ε-chainable fuzzy metric are characterized to get common fixed points. One of its corollaries is applied to obtain fixed point theorem on product of FM spaces.

2. PRELIMINARIES

Definition 1.1: A binary operation $*:[0,1]*[0,1]\to[0,1]$ is a continuous t-norm if it satisfies the following conditions:
(i) $*$ is a associative and commutative.
(ii) $*$ is continuous.
(iii) $a*a=a$, for all $a\in[0,1]$.
(iv) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for each $a,b,c,d\in[0,1]$.

Example of t-norm: $a*b=ab$ and $a*b=\min\{a,b\}$.

Definition 1.2: [4] The 3-tuple $(X,M,*)$ is called a fuzzy metric space . If X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2*(0,\infty)$, satisfying the following conditions for each $x,y,z\in X$ and $t,s>0$:
(i) $M(x,y,t)>0$.
(ii) $M(x,y,t)=1$, if and only if $x=y$.
(iii) $M(x,y,t)=M(y,x,t)$.
(iv) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$.
(v) $M(x,y,\cdot):(0,\infty)\to[0,1]$ is continuous.

Let $(X,M,*)$ be a fuzzy metric space. For $t>0$ there exists an open ball $B(x,r,t)$ with centre $x\in X$ and radius $0<r<1$, defined by: $B(x,r,t)=\{y\in X;M(x,y,t)>1-r\}$. Let (X,d) be a metric space and let $a*b=ab$ or $a*b=\min\{a,b\}$.Let $M(x,y,t)=t\div(t+d(x,y))$ $\forall x,y\in X$ and $t>0$. Then $(X,M,*)$ is a fuzzy metric M, induced by d is called the standard fuzzy metric.

Definition 1.3:[5] A sequence $\{x_n\}$ in a fuzzy metric space $(X,M,*)$ is said to be convergent to a point $x\in X$, such that $\lim_{n\to\infty} x_n = x$, if for each $\varepsilon > 0$, and for each $t > 0$, there exists $n_0 \in N$, such that $M(x_n, x, t) > 1-\varepsilon$ for all $n \geq n_0$.

It was proved by George and Veeramani[4] that a sequence $\{x_n\}$ in a fuzzy metric space $(X,M,*)$ converges to a point $x\in X$, if and only if $M(x_n, x, t) = 1$, for all $t > 0$.

Definition 1.4:[3] A sequence $\{x_n\}$ in a fuzzy metric space $(X,M,*)$ is defined as Cauchy sequence if for each $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in N$, such that $M(x_n, x_m, t) > 1-\varepsilon$ for all $n, m \geq n_0$.

A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 1.5:[16] Two self mappings A and B of a fuzzy metric space $(X,M,*)$ are said to be compatible if $\lim_{n\to\infty}(M(ABx_n, BAx_n, t) = 1$, for all $t > 0$, whenever $\{x_n\}$ is a sequence in X, such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$, for some $z\in X$.

Definition 1.6: Two self mappings A and B of a fuzzy metric space $(X,M,*)$ are said to be weakly compatible if $ABx = BAx$, when, $Ax = Bx$ for some $x\in X$.

If the self mappings A and B of a fuzzy metric space $(X,M,*)$ are compatible, then they are weakly compatible but its converse is not true.

Example 1.1: Let X=[0,4] and $a*b = \min\{a,b\}$.Let M be the standard fuzzy metric induced by d, where $d(x,y) = |x-y|$ for $x,y\in X$.

Define two self maps A and B of a fuzzy metric $(X,M,*)$ as follows:

$$Ax = \begin{cases} 4-x & 0\leq x\leq 2 \\ 4 & 2\leq x\leq 4 \end{cases} \qquad , Bx = \begin{cases} x & 0\leq x\leq 2 \\ 4 & 2\leq x\leq 4 \end{cases}$$

Let us consider $x_n = 1-(1/n)$, then [A, B] is proved to be not compatible but is weakly compatible.

Let $(X,M,*)$ is called a fuzzy metric space and $\varepsilon > 0$. A finite sequence $x = x_0, x_1, x_2, \ldots x_n = y$ is called to be ε-chain from x to y if $M(x_i, x_{i-1}, t) > 1-\varepsilon$ for all t > 0, and i = 1,2,3,4....n.

A fuzzy metric space $(X,M,*)$ is called as ε-chainable if for any $x, y\in X$, there exists a ε-chain from x to y.

Lemma 1.1:[5] Let $(X,M,*)$ be a fuzzy metric space, then for all $x, y\in X$, $M(x,y,\cdot)$ is non-decreasing.

Lemma 1.2: Let $(X,M,*)$ be a fuzzy metric space. If there exists $k\in(0,1)$, such that $M(x,y,kt)\geq M(x,y,t)$, for all $x,y\in X$ and t > 0, then x = y.

Proof: Let there exists, $k\in(0,1)$ such that $M(x,y,kt)\geq M(x,y,t)$, for all $x,y\in X$ and t > 0. Then $M(x,y,kt)\geq M(x,y,t/k)$ and so, $M(x,y,kt)\geq M(x,y,t/k^n)$ for positive integer n. Taking limit $n\to\infty$, $M(x,y,k)\geq 1$and hence $x = y$.

A class of Implicit relations: Let ψ be the set of all real and continuous functions: $F:(R^{+5})\to R$, nondecreasing in first argument satisfying the following conditions:

(a) For $u,v\geq 0, F(u,v,u,v,1)\geq 0$ implies that $u\geq v$.

(b) $F(u,1,1,u,1)\geq 0$ or $F(u,u,1,1,u)\geq 0$ or $F(u,1,u,1,u)\geq 0$ implies that $u\geq 1$.

Example 1.2: Let's consider $f(t_1,t_2,t_3,t_4,t_5)\geq 40t_1-18t_2+12t_3-14t_4-t_5+1$, then $F\in\psi$.

3. MAIN RESULTS

Theorem 2.1: Let $(X,M,*)$ be a complete ε-chainable fuzzy metric space and let A, B, S and T be the self mappings of X, satisfying the following conditions:

(i) $AX\subset TX$ and $BX\subset SX$.

(ii) The pairs (A, T) and (B, S) are weakly compatible.

(iii) T(X) or S(X) is complete.

(iv) there exists $k\in(0,1)$, such that

$$F\{M(Ax,By,kt), M(Sx,Ty,t), M(Ax,Sx,t), M(By,Ty,t), M(Ax,Ty,t)\}\geq 0$$

for every $x,y\in X$ and $t > 0$ .Then A, B, S and T have a unique common fixed point in X.

Proof: Let x_0 be any arbitrary point. As, $AX\subset TX$, $BX\subset SX$ so, there exists $x_1, x_2\in X$, such that $Ax_0 = Tx_1$ and $Bx_1 = Sx_2$. Inductively, we construct the sequences $\{y_n\}$ and $\{x_n\}$ in X, such that $y_{2n} = Tx_{2n+1} = Ax_{2n}$, and $y_{2n+1} = Sx_{2n+2} = Bx_{2n+1}$ for n

= 0, 1, 2… Now, using condition (iv) with $x = x_{2n},\ y = x_{2n+1}$, we get

$$F\{M(Ax_{2n}, Bx_{2n+1}, kt), M(Sx_{2n}, Tx_{2n+1}, t), M(Ax_{2n}, Sx_{2n}, t), M(Bx_{2n+1}, Tx_{2n+1}, t), M(Ax_{2n}, Tx_{2n+1}, t)\} \geq 0$$

That is,

$$F\{M(y_{2n}, y_{2n+1}, kt), M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n-1}, t), M(y_{2n+1}, y_{2n}, t), M(y_{2n}, y_{2n}, t)\} \geq 0$$

That is,

$$F\{M(y_{2n}, y_{2n+1}, kt), M(y_{2n-1}, y_{2n}, t), M(y_{2n}, y_{2n-1}, t), M(y_{2n+1}, y_{2n}, t),\ 1\} \geq 0.$$

Using condition (a), we have

$$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n}, y_{2n-1}, t) * M(y_{2n+1}, y_{2n}, t)$$

That is, $M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n}, y_{2n-1}, t)$.

Similarly, we have

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n+1}, y_{2n}, t)$.

Therefore, for all *n* even or odd, we have

$M(y_n, y_{n+1}, kt) \geq M(y_n, y_{n-1}, t)$.

Thus, for any n and t, we have

$M(y_n, y_{n+1}, kt) \geq M(y_n, y_{n-1}, t)$.

To prove that $\{y_n\}$ is a Cauchy sequence, we have

$$M(y_{n+1}, y_n, t) \geq M(y_n, y_{n-1}, t/k) \geq M(y_{n-1}, y_{n-2}, t/k^2) \geq --- \geq M(y_1, y_0, t/k^n) \to 1$$

as $n \to \infty$.

Thus, the result holds for *m* = 1.

By induction hypothesis suppose that result holds for *m* = *p*.

Now,

$$M(y_n, y_{n-p+1}, t) \geq M(y_n, y_{n-p}, t/2) * M(y_{n+1}, y_{n-p+1}, t/2) \to 1*1=1$$

Thus, the result holds for m = p+1. Hence, $\{y_n\}$ is a Cauchy sequence in X, which is complete. Therefore $\{y_n\}$ converges to z, such that $y_n \to z$, for some $z \in X$. So, it follows that $\{Ax_{2n}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n+1}\}$ and $\{Tx_{2n+1}\}$ also converges to z. To prove $\{x_n\}$ is a Cauchy sequence.

Since X is ε-chainable, so there exists ε-chain from x_n to x_{n+1} that is, there exists a finite sequence $x_n = y_1, y_2, \ldots\ldots, y_n = x_{n+1}$ such that $M(y_m, y_{m-1}, t) > 1-\in$ for all *t* > 0 and *i* = 1, 2, … , m. Thus we have

$$M(x_n, x_{n+1}, t) \geq M(y_1, y_2, t/l) * M(y_2, y_3, t/l) \ldots M(y_{m-1}, y_m, t/m)$$

$$\geq (1-\in) * (1-\in) * (1-\in) \ldots\ldots\ldots\ldots * (1-\in)$$

$$\geq (1-\in)$$

For m > n,

$$M(x_n, x_m, t) \geq M(x_n, x_{n+1}, t/m-n) * M(x_{n+1}, x_{n+2}, t/m-n) \ldots * M(x_{m-1}, x_m, t/m-n)$$

$$\geq (1-\in) * (1-\in) * (1-\in) \ldots\ldots * (1-\in)$$

$$\geq (1-\in)$$

Hence, $\{x_n\}$ is a Cauchy sequence in X, which is complete. Therefore $\{x_n\}$ converges to $z \in X$. Hence its subsequences $\{Ax_{2n}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n+1}\}$ and $\{Tx_{2n+1}\}$ also converges to z.

Case I: T(X) is complete.

If we take $z \in T(X)$, so there exists $u \in X$, such that $z = Tu$.

Step I: Put $x = x_{2n}$ and $y = u$ in (iv), so

$$F\{M(Ax_{2n}, Bu, kt), M(Sx_{2n}, Tu, t), M(Ax_{2n}, Sx_{2n}, t), M(Bu, Tu, t), M(Ax_{2n}, Tu, t)\} \geq 0$$

Taking Lim $n \to \infty$, we get

$$F\{M(z, Bu, kt), M(z, Tu, t), M(z, z, t), M(Bz, Tz, t), M(z, Tz, t)\} \geq 0$$

Since, F is nondecreasing in the first argument, so

$$F\{M(z, Bu, t), 1, 1, M(Bu, z, t), 1\} \geq 0$$

So, $F\{M(z, Bu, t) \geq 1$ ….{ by (b)}.

Hence, $z = Bu$ $\{\because B \subset S,\ z = Bu \in S,\ \therefore z = Bu = Su(say)\}$

Therefore, $z = Bu = Su = Tu$.

Now, (B, S) is weakly compatible, so $BSu = SBu$, thereby, $Bz = Sz$.

Step II: Put $x = x_{2n}$ and $y = z$ in (iv), we get

Taking Lim $n \to \infty$, we get

$$F\{M(z, Bz, kt), M(z, Tz, t), M(z, z, t), M(Bz, Tz, t), M(z, Tz, t)\} \geq 0$$

Since, F is nondecreasing in the first argument as well as $z = Tz$, since $z \in T(X)$, so we have

$$F\{M(z, Bz, t), M(z, z, t), 1, M(Bz, z, t), M(z, z, t)\} \geq 0$$

$$F\{M(z, Bz, t),\ 1,\ 1, M(Bz, z, t),\ 1\} \geq 0$$

That is $F\{M(z, Bz, t) \geq 1$….{by (b)}.

Hence, $z = Bz$. Therefore, $z = Bz = Tz$.

Step III: As, $B(X) \subset S(X)$ let there exists $v \in X$, such that $z = Bz = Sv$

Put x = v, y = z in (iv),

$$F\{M(Av, Bz, kt), M(Sv, Tz, t), M(Av, Sv, t), M(Bz, Tz, t), M(Av, Tz, t)\} \geq 0$$

That is,

$$F\{M(Av, z, kt), 1, M(Av, z, t), 1, M(Av, z, t)\} \geq 0$$

Since, F is nondecreasing in the first argument, we have

$$F\{M(Av, z, t), 1, M(Av, z, t), 1, M(Av, z, t)\} \geq 0$$

That is, $F\{M(Av, z, t) \geq 1$ …{by (b)}.

So, $z = Av$. Now since, $A \subset T$ $\therefore z = Av \in T$ or $z = Av = Tv$.

Therefore, $z = Av = Tv$.

Now as, (A, T) is weakly compatible, so $ATv = TAv$, such that $Az = Tz$.

So, combining all the results, we have $Az = Tz = Bz = Sz = z$.

Step IV: Put $x = Sz$ and $y = z$ in (iv), we get

$$F\{M(ASz, Bz, kt), M(SSz, Tz, t), M(ASz, SSz, t), M(Bz, Tz, t), M(ASz, Tz, t)\} \geq 0$$

$$F\{M(Az, z, kt), M(Sz, z, t), M(Az, Sz, t), M(z, z, t), M(Az, z, t)\} \geq 0$$

$$F\{M(Az, z, kt), M(z, z, t), M(Az, z, t), M(z, z, t), M(Az, z, t)\} \geq 0$$

As F is non decreasing in the first argument, we have

$$F\{M(Az, z, t), 1, M(Az, z, t), 1, M(Az, z, t)\} \geq 0$$

$$M(Az, z, t) \geq 1 \ldots \text{ \{by (b)\}}.$$

Therefore, $Az = z$.

Similarly, we can show that $Bz = z$, $Tz = z$ and $Sz = z$

Hence $z = Az = Tz = Bz = Sz$.

Case II: S(X) is complete.

If we take $z \in S(X)$, so there exists $w \in X$, such that, $z = Tw$.

The proof is likewise as in Case I. So, similarly, we can show that $Az = z$, $Bz = z$, $Tz = z$ and $Sz = z$

Hence $z = Az = Tz = Bz = Sz$.

Thus, z is the common fixed point of A, B, S and T.

Uniqueness: Let *w* and *z* be two common fixed points of maps A, B, S and T. Put *x* = *z* and *y* = *w* in (iv), we get

$$F\{M(Az, Bw, kt), M(Sz, Tw, t), M(Az, Sz, t), M(Bw, Tw, t), M(Az, Tw, t)\} \geq 0$$

$$F\{M(z, w, kt), M(z, w, t), M(z, z, t), M(w, w, t), M(z, w, t)\} \geq 0$$

Since, F is a nondecreasing, in the first argument, therefore, we have

$$F\{M(z, w, t), M(z, w, t), 1, 1, M(z, w, t)\} \geq 0$$

$$M(z, w, t) \geq 1 \ldots \text{ \{by (b)\}}$$

Thus, $z = w$. So, *z* is the unique common fixed point of A, B, S and T.

Remark 2.1: If S = T and A = B, then conditions (i)- (iii) says that the pair (A, T) is weakly compatible and $AX \subset TX$. In such a situation, the sequence occurs as $\{Ax_n\} = \{Tx_{n+1}\}$.

Remark 2.2: If S = T and A = B, then conditions (i)- (iii) says that the pairs (A, T) and (B,S) are weakly compatible and so, $A(X) \cup B(X) \subset T(X)$ and hence the sequences exists as follows: $Ax_{2n} = Tx_{2n+1}$ and $Bx_{2n+1} = Sx_{2n+2}$.

Theorem 2.1 with S = T = Identity map is:

Corollary 2.1: Let $(X, M, *)$ be a complete ε-chainable fuzzy metric space and let A, B, S and T be the self mappings of X, satisfying (i), (ii) and (iii) of Theorem 2.1and there exists $k \in (0,1)$, such that

$$F\{M(Ax, By, kt), M(x, y, t), M(x, Ax, t), M(y, By, t), M(y, Ax, t), M(x, By, t)\} \geq 0,$$

for every $x, y \in X$ and $t > 0$ .Then A and B have a unique common fixed point in X.

Corollary 2.2: Let $(X, M, *)$ be a complete ε-chainable fuzzy metric space and let A, B, S and T be the self mappings of X, satisfying (i), (ii) and (iii) of Theorem 2.1and there exists $k \in (0,1)$, such that

$$F\{M(Ax, By, kt), M(Sx, Ty, t), M(Ax, Sx, t), M(Sx, By, 2t), M(By, Ty, t), M(Ty, Ax, t)\} \geq 0,$$

for every $x, y \in X$ and $t > 0$ .Then A, B, S and T have a unique common fixed point in X.

Proof: From definition, we have

$$F\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(By, Sx, 2t), M(Ax, Ty, t)\}$$

$$\geq F\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Sx, Ty, t), M(Ty, By, t), M(Ax, Ty, t)\}$$

$$\geq F\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(Ax, Ty, t)\}$$

and hence from Theorem 2.1, A, B, S and T, we have a unique common fixed point in X.

Corollary 2.3: Let $(X, M, *)$ be a complete ε-chainable fuzzy metric space and let A, B, S and T be the self mappings of X, satisfying (i), (ii) and (iii) of Theorem 2.1and there exists $k \in (0,1)$, such that

$M(Ax, By, kt) \geq M(Sx, Ty, t)$, for every $x, y \in X$ and $t > 0$ .Then A, B, S and T have a unique common fixed point in X.

Proof: As we have $M(Sx, Ty, t) = M(Sx, Ty, t) * 1$

$$= M(Sx, Ty, t) * M(Ax, Ax, 5t)$$
$$\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) * M(By, Ty, t) * M(Ty, Ax, t)$$

and hence from Corollary 2.2, we have A, B, S and T, we have a unique common fixed point in X.

Corollary 2.4: Let $(X, M, *)$ be a complete ε-chainable fuzzy metric space and let A and B be the self mappings of X, which satisfies the condition $k \in (0,1)$, such that $M(Ax, By, kt) \geq M(x, y, t)$, for every $x, y \in X$ and $t > 0$ .Then A and B have a unique common fixed point in X.

Proof: In Corollary 2.3, if we take *A*= *B*, so the equation reduces to Grabiec's fuzzy Banach Contraction theorem (see [5]).

Corollary 2.5: Let $(X, M, *)$ be a complete ε-chainable fuzzy metric space and let A be the self mapping of X, which satisfies the condition $k \in (0,1)$, such that $M(Ax, Ay, kt) \geq M(x, y, t)$, for every $x, y \in X$ and $t > 0$. Then A has a unique common fixed point in X.

Proof: Let $(X, M, *)$ be a complete ε-chainable fuzzy metric space. For any $x, y \in X$ and $t > 0$, assume that

$$M_{\varepsilon}(x, y, t) = \sup\{M(x_0, x_1, t) * M(x_1, x_2, t) * \ldots * M(x_{n-1}, x_n, t)\}$$

Then, it is to be proved that M_{ε} is a fuzzy metric satisfying $M(x, y, t) \geq M_{\varepsilon}(x, y, t)$ and $M(x, y, t) = M_{\varepsilon}(x, y, t)$ whenever $M(x, y, t) > 1 - \varepsilon$.

To show that $(X, M_{\varepsilon}, *)$ is complete. Consider $\{x_n\}$ be a Cauchy sequence in $(X, M_{\varepsilon}, *)$. Then, for m > n, such that $M(y_{m-1}, y_m, t/m) > (1-\varepsilon) * (1-\varepsilon) * \ldots * (1-\varepsilon) \geq (1-\varepsilon)$ and hence, we have $M(x_n, x_m, t) = M_{\varepsilon}(x_n, x_m, t)$.

Since $(X, M, *)$ is complete, there exists $z \in X$, such that $M(x_n, z, t) > 1 - \in$ and hence $M_{\varepsilon}(x_n, z, t) = M(x_n, z, t)$ and $M_{\varepsilon}(x_n, z, t) > 1 - \in$. Hence $\{x_n\}$ converges to z and $(X, M_{\varepsilon}, *)$ is complete.

4. AN APPLICATION

Now, we shall apply the corollaries 2.1 and 2.4 to establish the following results:

Theorem 3.1: Let $(X, M, *)$ be a complete ε-chainable fuzzy metric space and let A and B be the two self maps on product X × X, with values in X. If there exists a constant $k \in (0,1)$, such that (i)

$$F\{M(A(x, y), B(u, v), kt), M(A(x, y), x, t), M(B(u, v), u, t), M(x, u, t), M(y, v, t), M(A(x, y), u, t)\} \geq 0$$

for all x, y, u, v in X, t> 0, then there exists exactly one point w in X ,such that $A(w, w) = w = B(w, w)$.

Proof: From above relation (i) we have,

$$F\{M(A(x, y), B(u, y), kt), M(A(x, y), x, t), M(B(u, y), u, t), M(x, u, t), M(A(x, y), u, t)\} \geq 0$$

for all x, y, u in X. Therefore by Corollary 2.1, we have, for each y in X, there exists one and only one z(y) in X such that $A(z(y), y) = z(y) = B(z(y), y)$ ……………… (ii)

For any $y, y' \in X$, by (i), and using relation $M(z(y), z(y'), kt) = M(A(z(y), y), B(z(y'), y), t)$ we get

$$F\{M(z(y), z(y'), kt), 1, 1, 1, M(y, y', t), M(z(y), z(y'), t)\} \geq 0$$

Since, F is nondecreasing in the first argument, therefore we have $F\{M(z(y), z(y'), t), 1, 1, 1, M(y, y', t), M(z(y), z(y'), t)\} \geq 0$, which implies that

$$M(z(y), z(y'), t) \geq M(y, y', t) * M(z(y), z(y'), t/k^n)$$

So, $\{\because M(z(y), z(y'), t/k^n) \to 1$, as $n \to \infty\}$.

Therefore, Corollary 2.4 yields that the map z (.) of X into itself has exactly one fixed point w in X *i.e.* $w = z(w)$. Hence, by (ii), $w = z(w) = A(w, w) = B(w, w)$. It is easily observed that A and B can have only one such common fixed point w in X.

5. REFERENCES

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