

Financial Market Analysis of Bombay Stock Exchange using an Agent Based Model

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ABSTRACT

Returns on stocks have traditionally been modelled by fitting a suitable statistical process to empirical returns. Studies on agent based models of stock market have been carried out by researchers, primarily on US markets. This paper analyzes the empirical features generated using historical data from the Bombay Stock Exchange (BSE), employing the concept of agent based model proposed by LeBaron[2,3,8]. Agent-based approach to stock market considers stock prices as arising from the interaction of a number of individual investors. These investors are modeled as intelligent agents, using differing lengths of past information, each trading with its own rules adapting and evolving over time, and this in turn determines the market prices. It is seen that the model generates some features that are similar to those from actual data of the BSE.

General Terms

Agent Based Modeling, Financial Forecasting, Neural Networks, Genetic Algorithm.

Key Words

Agents, Assets, Bombay Stock Exchange, Returns, Risky Assets, Risk Free Assets, Feed Forward Neural Networks, Rational Expectations Price, Forward Testing.

1. AGENT BASED MODELLING (ABM) OF STOCK MARKETS

1.1 Introduction

Returns achieved on stock markets contain certain characteristic features[1]. These features include a distribution of returns that is more peaked than the Gaussian distribution, periods of persistent high volatility, and correlation between volatility and trading volume. It has been shown that agent based models are able to demonstrate this, unlike the traditional financial models. The traditional economic models generally use either a simple distribution of returns such as the Gaussian and treat extreme events as outliers, or construct a statistical process which reproduces some of these features. The agent-based approach considers a population of intelligent adaptive agents and let them interact in order to maximize their financial performance. It has been shown that such an approach can replicate features of real stock markets[2,3,4,5,6,7,12,13]. This paper is an extension of the work of LeBaron and aims to study the Bombay Stock Exchange by forward testing [8] an Agent Based Model where a

market is run using real data as the price input up to the current date, and then allowed to continue on into its future to enable the study of the empirical features. An attempt is made to study the behavior of agents with varying memory to see whether all horizon agents dominate the BSE market, preventing the stable long horizon agents to play a crucial role, as was established for the US market by LeBaron[2,3,8,9]. In this study, the financial data of BSE from the years 2003 to 2009 is considered for building the model, as against LeBaron's model, where the data is entirely generated.

1.2 Why use an agent-based model for the stock market?

Agent-based simulation is a bottom-up system approach to forecast and understand the behaviour of non-linear systems[5]. Interaction between agents is a key feature of the agent-based systems. As an alternative to regarding stock prices as stochastic processes, in ABM, prices arise from simulating the interactions of autonomous entities with different profit-making strategies. The collective behaviour of such groups of individuals is not determined by a single mechanism but by the interaction of individual behaviours distributed across the group and it is only by the individual behaviours that the group behaviour can emerge. This indeed is the mechanism prevailing in stock markets and hence the aptness of agent based models for analysis. A large number of agent-based stock market models have been proposed by researchers. [2,4,6,7,8,9,10,12,13,14,15].

1.3 Features of a Stock Market

The returns achieved from investing in shares in a stock market accrue partly from changes in the share price as capital gains and partly from dividends paid. Returns on shares are volatile and are consequently expected to be higher than for a safer investment such as risk free bonds. A key property of returns on shares is their apparent randomness which follows from the concept of an efficient market based on the Random Walk Hypothesis. This hypothesis states that it is impossible to predict future price changes based on historic data. Therefore a realistic model of a stock market should show autocorrelation of price return close to zero; a distribution of returns which is non-Gaussian having a kurtosis significantly above three with "fat tails" and persistence in asset return volatility. However in practice the Gaussian is often used to represent the stock market returns since it is amenable to easier analytical tractability, notwithstanding the fact that in actuality, the returns are non-Gaussian.

2. Review of Agent Based Model of LeBaron

The design proposed by LeBaron [2,3,8], reviewed in the following paragraphs is used for analysis of the BSE.

2.1 Agent Based Model

The agents used has some “intelligence” and the investment decisions specified on a proportion of wealth are based upon an information set. A key concept of the model is that of “bounded rationality” – that an agent cannot feasibly analyse all the information in the market, and can at best combine limited “bounded” rational decision-making with empirical evidence. A Walrasian auction is adopted, wherein each agent calculates its demand for shares at every possible price and submits this to an auctioneer. The price is then set so that total demand across all agents equals total shares that can be issued.

2.2 Assets

The model contains two assets for investment – cash and equity. Cash pays a constant guaranteed rate of return r_f (risk free). The equity pays a dividend at each time step. This is random and the log-dividend follows a random walk:

$$\log(d_{t+1}) = \log(d_t) + \varepsilon_t$$

where d_t is the dividend and ε_t is a Gaussian random variable $N(\mu, \sigma)$. The equity is available in a fixed supply of one share for the population. If s_i is the share holding of agent i , the

$$\text{constraint that } \sum_{i=1}^I s_i = 1 \text{ will be always maintained,}$$

where I is the number of agents. The equity price arises through the interactions of the agents.

2.3 Agents

The model contains a number of agents. Each agent has a certain wealth and at each time step it decides how much of its wealth to consume and how much to save, and how much of its savings wealth to allocate to equity and how much to cash. The agents are of Constant Relative Risk Aversion (CRRA) of logarithmic form and at time t makes these decisions in an attempt to maximise its lifetime utility

$$u_{i,t} = E_t \sum_{s=0}^{\infty} \beta^s \log c_{i,t+s}$$

subject to the constraint

$$w_{i,t} = p_t s_{i,t} + b_{i,t} + c_{i,t} = (p_t + d_t) s_{i,t-1} + (1 + r_f) b_{i,t-1}$$

where $u_{i,t}$ is the lifetime utility of agent i from time t onwards, β is a constant, $c_{i,t}$ is the consumption of agent i at time t , and $w_{i,t}$ is wealth at time t , $s_{i,t}$ and $b_{i,t}$ are the risky and risk-free asset holdings, p_t the share price, d_t the dividend paid. The two decisions, choosing $c_{i,t}$ and $s_{i,t}$, will affect the agent's pattern of consumption. Utility would model the benefit obtained from an amount of money and utility of wealth is optimized rather than actual wealth. The optimal amount of wealth to consume at a single time step can be shown to be a constant proportion of wealth

$$c_{i,t} = (1 - \beta) w_{i,t}$$

The time rate of discount β is set to $1/1+r$ where $r = 0.01$ is the discount rate. The agent does not consider what happens over the whole of the future, but restricts itself only over the next single time step. In order to maximise $u_{i,t}$ it is sufficient to maximise the expected log-return:

$$E_t \log [1 + \alpha_t r_{t+1} + (1 - \alpha_t) r_f]$$

where α_t is the proportion of wealth allocated to equity, r_{t+1} is the return achieved from equity in the period $(t, t+1)$ and r_f the constant cash return. It is not possible to perform the maximisation deductively since the equity returns distribution is not known in advance (these arise from the interaction of the agents). Therefore the agents maximise a sample expectation taken from historic returns. Because the distribution of returns may change over time, agents do not look at the whole past history, but rather look at the last T_i periods, to maximise

$$1/T_i \sum_{k=1}^{T_i} \log [1 + (\alpha_{t-k} r_{t+1-k}) + (1 - \alpha_{t-k}) r_f]$$

where T_i is a constant for agent i . The choice of T_i will affect an agent's performance. There are many ways in which an agent could determine its allocation to equities. Under LeBaron's model it does this by using one of a pool of rules.

2.4 Rules

A rule recommends the proportion of savings an agent should allocate to equities, taking information about the current state of the market and produces an output $\alpha \in (0,1)$. The rules are implemented as simple feed forward neural network (Figure 1) with a single hidden unit with restricted inputs giving an output. The equations given below define the network, where z_t is time t information and w_j are parameters. k takes values from 1 to 6 so that the weight array $\{w\}$ consists of 19 parameters. The output from the intermediate neuron k is denoted h_k .

$$h_k = g_1(w_{0,k} z_{t,k} + w_{1,k})$$

$$\alpha(z_t) = g_2(w_2 + \sum_{k=1}^6 w_{3,k} h_k)$$

$$g_1(x) = \tanh(x)$$

$$g_2(x) = 1/2(1 + \tanh(x/2))$$

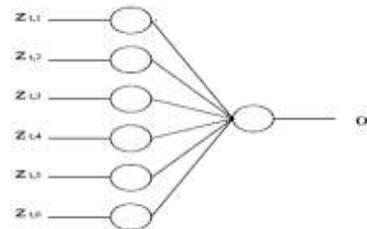


Figure 1: Structure of FFNN

2.5 Input Values to the FFNN - The Information Set

The information set consists of six items reflecting various fundamental and technical trading strategies, and its combinations. These six items extract potentially useful information from the large quantity of historic data and simplifies the decision making process. The first three inputs are the returns on equity in the previous three time-steps, useful for technical trading. The fourth is a measure of how the current price differs from the rational-expectations price. The last two inputs measure the ratio between the current price and exponentially weighted moving averages of the price. The Information set is:

$$\begin{aligned} z_{t,1} &= r_t = \log((p_t + d_t)/p_{t-1}) \\ z_{t,2} &= r_{t-1} \\ z_{t,3} &= r_{t-2} \\ z_{t,4} &= \log(r p_t / d_t) \\ z_{t,5} &= \log(p_t / m_{1,t}) \\ z_{t,6} &= \log(p_t / m_{2,t}) \end{aligned}$$

Where p_t is the share price, d_t is the dividend paid, r is a constant and $m_{i,t}$ is the moving average given by

$$\begin{aligned} m_{i,t} &= \rho_1 m_{i,t-1} + (1 - \rho_1) p_t \\ \text{with } \rho_1 &= 0.8 \text{ and } \rho_2 = 0.99. \end{aligned}$$

2.6 Trading and price-setting

For a given share price p , each agent can determine how much of its wealth it wishes to invest and how much of this is to be invested in shares. Consequently it arrives at a demand function for shares

$$d_{i,t}(p_t) = [\alpha_i(p_t, I_t) \beta W_{i,t}] / p_t$$

where i denotes the agent, t refers to time, and I_t represents the information set. A Walrasian auction is then used to find the price p_t . Walrasian auction is one in which the price is set by an auctioneer in order that the total demand for shares at that price is equal to the available supply:

$$\sum_{I=1}^{N_{agents}} d_{i,t}(p_t) = N_{shares}$$

where N_{agents} is the number of agents and N_{shares} the number of shares. Because the new price affects the information set, and this affects the rule output which in turn affects the agents' demand, the equations to solve are non-linear. This equation is solved using complex recursive function which searches for a value of p_t that satisfies these equations starting from the price at the previous time-step.

2.7 Adaptation and evolution

The model contains three forms of adaptation and evolution

- Agents can adapt by selecting a different rule. At each time step a proportion of the agents can adapt. An agent adapts by comparing the performance of its current rule with a randomly chosen rule, and switching to the new rule if the new rule scores more than the old.
- Agents evolve at each time step, wherein agents with the least wealth is removed from the population. It is replaced by a new agent which is given the median cash and equity holding. The new agent is given a memory length taken from a random distribution.
- The rules are also evolved. A rule is replaced if it has not been used for 10 time steps, and is replaced using one of three genetic operators: copying a parent and changing a single weight to a random number in $(-1, 1)$; copying a parent and adding a random number in $(-0.25, 0.25)$ to a single weight, and copying a parent and replacing the weights for one neuron with those for the corresponding neuron of another parent.

3. Implementation – Agent Based Model of BSE

3.1 Validating the FFNN structure for Rule

BSE Sensex, the most popular Indian stock index has been chosen for the study. The time step considered here is one month. Prior to adopting the FFNN as the structure for the Rules, we have validated it by using the BSE Index data. The MATLAB Neural Network Toolbox has been chosen for creating, training and testing the network. The FFNN was trained with inputs from historical prices of BSE index, calculated taking monthly closing prices of BSE stock index from the year 2003 to 2008. The output is a simple function $\alpha(z_i, w_j)$. The output from the network, α is a 0 or 1 which would suggest where to invest in the next time-step, a 0 indicating that risk-free asset (Public Provident Fund data considered) would give higher returns for the particular time-step and a 1 indicating that investing in an Index fund tracking the BSE would render higher returns. The network was tested with data pertaining to the year 2009 and the results have been found to validate the market scenario. It has been found that the FFNN with one hidden layer with six neurons has produced quite accurate results. Thus the FFNN has been found to establish the functional dependency between the input parameters and the market behavior and hence has been validated for the purpose of generating the rules for the agents of BSE.

3.2 Agents

The model contains of a number of agents. Each agent has properties that define its behavior. The properties of agents defined in this model are given in Table 1 below:

Table 1 Agent Class

NAME	PROPERTY
Rule now	Integer specifying the rule used by agent
Wealth	Wealth till (t-1) to be spent at time t
Memory	No of time steps agent looks back
Returns	Returns in the past
Proportions	Alpha values used in the past
Memwealth	Memory bound wealth
Volume	Volume it demands at each time step
Exist	No of time steps the agent has existed

3.3 Step by Step Process

The flow diagram of the implementation is illustrated as a Step by Step Process in Figure.2:-

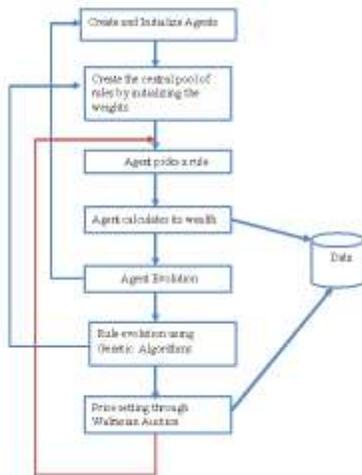


Figure 2 Step by Step Process

The simulation of the system is done for two cases

- All memory case- where agents with all memory lengths are present in the market.
- Long memory case- Only agents with long memory are present in the market.

The simulation runs for 1000 iterations for both the cases and the prices, returns and the interaction and evolution of agents are monitored.

3.4 Parameter settings

Table 2 gives the parameter settings used in the experiment. A total of 140 agents have been used. The model was initialised by setting agent memories and neural network parameters using a uniform random distribution over the allowable range. Each agent started with an equal shareholding set to 100.

Table 2. Initializing Various Parameters

PARAMETERS	VALUES
Number of Agents	140
Minimum Memory	1
Maximum Memory	70
Length of one Time-Step	1 month
Number of Rules	250
Number of Shares	1
Number of time-steps projected	1000
Minimum Neural Network weight	-1
Maximum Neural Network weight	1
Time steps before a new rule is discarded	10

3.5 Results

Forward Testing simulation was carried out by feeding data from the BSE index prices. For the first 70 months, data from BSE is used by the agents. Subsequently, the prices are generated by Walrasian auction. The process continues for 1000 iterations. The “all-memory case” followed by “long memory case” has been implemented. The following results emerge:

- In the “long-memory case” where agents’ memory lengths are in the range [51,70], the prices converge after an initial learning period to the rational-

expectations price, so that the returns are Gaussian. There is very little trading.

- In the “all-memory case” where agents’ memory lengths are in the range [1,70] prices do not settle to the rational-expectations price. Returns are more volatile, and the distribution has fat tails. Shares continue to be traded frequently.

3.6 Prices

- Figure 3 shows the variation in prices for the all memory case. The first 70 time steps show variation of BSE index prices. After the 70th time step prices are generated through Walrasian auction as a result of interaction between agents with different memory lengths and trading strategies.

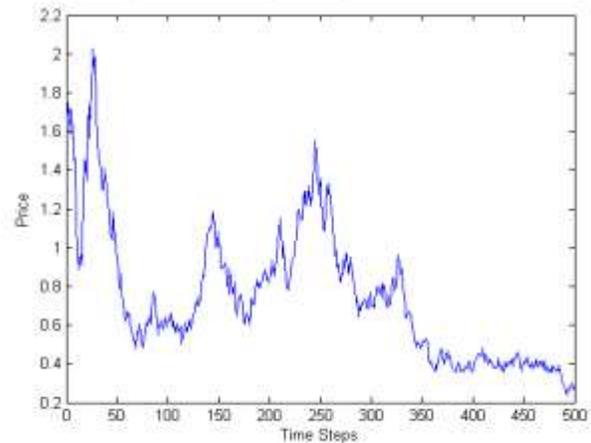


Figure 3. Price time series for all-memory agents

- It can be understood that the prices vary considerably in the presence of agents with all memory lengths. The volatility in prices is evident in the above case.
- Figure 4 shows the variation in prices for the long memory case. The first 70 time steps show variation of BSE index prices. After the 70th time step prices are generated through Walrasian auction as a result of interaction between agents with similar memory lengths (long) and slightly varying trading strategies.

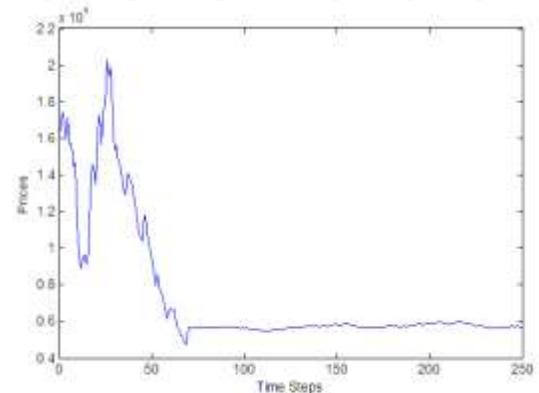


Figure 4. Price time series for long-memory agents

- Further, it is seen that prices tend to stabilize after some time. This is because agents converge to similar strategies over a period of time due to similar memory lengths. This in turn suppresses trading and prices stabilize to the rational expectations price. Hardly any shares are traded once the rational-expectations price has been reached. Comparing the two figures and the data from BSE, we could deduce that it is because of the presence of agents with varying memory and different trading strategies, trading takes place. Volatility in prices arises because of these differences in agents' properties.

3.7 Returns

- Logarithmic returns are considered here which is calculated from the prices and dividends. It is depending on these returns that an agent chooses his trading strategy (rules).
- The following graph, Figure 5 shows variation in returns for the all memory case. The returns for the first 70 times steps are calculated using prices and dividends from the BSE. After this the returns are calculated with prices generated by the Walrasian auction and random dividends.

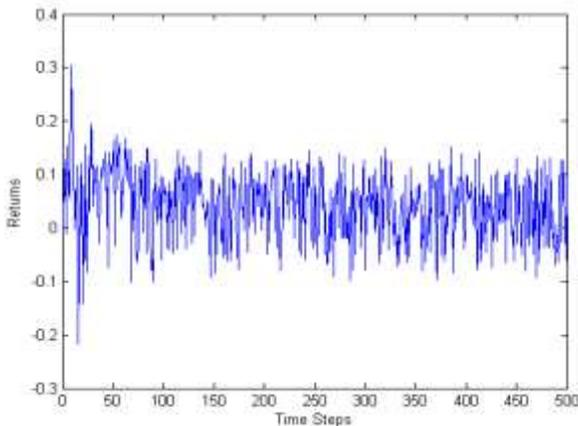


Figure 5. Return time series for all-memory agents

- The above graph shows how returns from the markets closely resemble the returns arising out of interaction between agents of varying memory lengths.
- The following graph, Figure 6, shows variation in returns for the long memory case. The returns for the first 70 times steps are calculated using prices and dividends from the BSE. After this the returns are calculated with prices generated by the Walrasian auction and random dividends.
- The variation in returns in this case is very less compared to the actual market and the all memory case. This is because of similarity in property of agents that interact in the market. These graphs further emphasize the fact that the market is comprised of agents with different strategies, different memory length and irrational behavior.

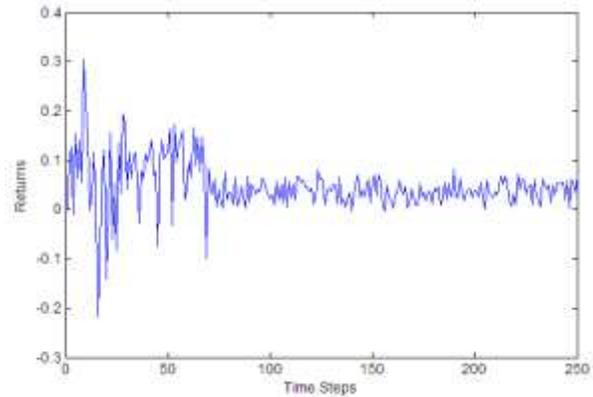


Figure 6. Return time series for long-memory agents

3.8 Statistical Observations

The statistical observations obtained from the runs of the model are given at Table 3. The table presents summary statistics for these returns in the two different cases along with comparison of the BSE Sensex. The first columns correspond to the series standard deviation, and the second to kurtosis. In the standard deviation, the all-memory case shows a value closer to the BSE data implying the existence of investors of all kinds of memory in a market rather than investors of only long memory. The column labeled kurtosis shows the flatness of the curve and also the peak value which in turn can give an idea about the variations from the mean.

Table 3 Return Summary Statistics

	Standard Deviation	Kurtosis
BSE	4.24	1.9
All Memory	3.98	1.78
Long Memory	1.87	1.2

3.9 Other Miscellaneous Observations

Other observations detected along with possible explanations for such behavior are given below:-

- Agents with memory length around 25 get deleted often. Though the reason for this could not be conclusively understood, one possible explanation could be because of the fact that in the data fed in, the BSE reaches the lowest values after a period of about 25 months. Hence this could have affected the agents' strategy, thereby affecting its performance.
- Agents with memory length from 50 to 60 stay for a longer period of time. Some agents even stay throughout the entire 1000 runs. A possible explanation is that this memory is the optimal period to look back in order to get the maximum returns from the market.
- Rules which gave a value of α near 0.5 were modified early. This could be attributed to two factors:-

- In cases of a fall in BSE Sensex, rules which have a much lower α value reduced the losses, thus making the earlier rules poor.
- In cases of rise in Sensex, rules which have much higher α value increased the profits, again making the $\alpha = 0.5$ rules poor.

3.10 Variation from LeBaron Model

- In order to reduce the computational complexity and to synchronize the model with the BSE, few variations have been introduced in LeBaron's model as follows:-
- The number of agents have been reduced from 250 to 140.
- Memory of the agents is between [1,70], limited to available BSE data.
- A tolerance value of 0.1 was introduced into the Walrasian auction.
- Since the computational complexity of 'fzero' function in MATLAB was very high, a customized version of the function which works in a similar manner, has been developed.

4. FUTURE ENHANCEMENTS

- The simulation was run for 1000 time steps due to high computational duration. It can be extended further to 10,000 runs (as suggested in the original paper) and patterns can be observed for a longer period of time. Further, owing to computational complexities, tolerance levels have been introduced, which could be removed in future simulations for better results.
- In the actual market, massive fluctuations occur due to natural disasters, calamities, war etc. These catastrophic events are not considered when generating prices in Walrasian auction, since in this simulation, the interaction between agents alone determines the price. Further research can be done, so as to embed theories underlying such events so as to give a more realistic approach to the price formation mechanism.
- Lastly, the Walrasian auction is used to generate prices in this model. Further studies could make this a prediction tool. For this to be possible, the agents may have to be redefined, their numbers and interactions increased and tested exhaustively to verify and validate.

5. CONCLUSION

LeBaron's model of artificial market [2,3,8] has been successfully replicated employing the concept of Agent Based Modeling with few modifications so as to synchronize it with the BSE. This was implemented by forward testing the BSE data (a strategy different from that adopted by LeBaron) and the prices and returns were observed over a period of 1000 time steps. Various empirical features of BSE market have been generated. The results obtained suggest that it is indeed the interaction between agents of different strategies that brings volatility and trading in the market. This also proves that Agent Based Models are capable of quantitatively replicating various features of actual financial markets. Observations on the behavior of agents were made to identify what type of agents and what strategies are

successful in the market. Further research would have to be carried out to refine this model as a prediction tool for the BSE.

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