Characterization of Fuzzy Lattices on a Group with Respect to T-Norms

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ABSTRACT

We introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. We also study the homomorphic image, pre-image of fuzzy lattices, arbitrary family of fuzzy lattices and lattice fuzzy normal groups using T-norms. we introduce the notion of sensible fuzzy lattice in groups and some related properties of lattices are discussed.

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1. Introduction

In the trajectory of stupendous growth of fuzzy set theory, fuzzy algebra has become an important are of research. A. Rosenfeld 1971 [10] used the concept of fuzzy set theory due to Zadeh 1965 [15]. Since then the study of fuzzy algebraic sub structures are important when viewed from a Lattice theoretic point of view. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. Nanda[4] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy sub groups and fuzzy normal sub groups are explored. G.S.V. Satya Saibaba [14] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l-groups. J.A. Goguen [6] replaced the valuation set [0, 1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [12] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. In this paper we modify the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. We study the homomorphic image, pre-image of fuzzy lattices, arbitrary family of fuzzy lattices and lattice fuzzy normal groups. We introduce the notion of sensible fuzzy lattices in groups using T-norms and some related properties of lattices are discussed.

SECTION – 2 PRELIMINARIES

Definition 2.1: A mapping $\mu : X \rightarrow [0, 1]$ where X is an arbitrary non empty set and is called fuzzy set in X.

Definition 2.2 : A Lattice ordered group (LG) is a system $G = (G, +, \leq)$.Where

(i) (G, +) is a group

(ii) (G, □) is a lattice

(iii) $x + a + y = b + y \rightarrow G(x) \leq \max \{G(a), G(b)\}$ for all $x, y, a, b \in G$

Definition 2.3: Let X be a fuzzy set and $\mu : X \to G$ be a Lattice ordered group of X, then μ is called fuzzy Lattice ordered group (FLG) if

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$
(ii) $\mu(-x) = \mu(x)$
(iii) $\mu(0) = 1$ for all $x, y \in G$.

Definition 2.4: Let μ be a fuzzy lattice ordered group of G and $\mu : X \to G$ is called Fuzzy Lattice if

(FL1) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$
(FL2) $\mu(-x) \geq \mu(x)$
(FL3) $\mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$
(FL4) $\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in G$

Definition 2.5: By a t-norm T, we mean a function $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying the following conditions

(T1) T (0, X) = 0
(T2) $T(x, y) \leq T(x, z)$ if $y \leq z$
(T3) T(x, y) = T (y, x)
(T4) T(x, T(y, z)) = T (T(x, y), z), for all x, y, z $\in [0, 1]$.

Definition 2.6: Let T be a t-norm. A fuzzy set A is said to be sensible under T if $I_m(A)$ C ΔT.
Where $\Delta T=\{T(\alpha, \alpha) =\alpha \mid \alpha \in [0, 1]\}$

Definition 2.7: For any fuzzy set μ in G and $t \in [0, 1]$, we define the set $U(\mu: t) = \{x \in G \mid \mu(x) \geq t\}$which is called an upper cut off μ and can be used to the characterization of μ.

Definition 2.8: Let f: $G \to G'$ be a lattice group homomorphism and A be a fuzzy lattice of G′ then $Af(x) = (A \circ f)(x) = f^{-1}(A)(x)$.

Definition 2.9: Let $\theta: X \to Y$ be a map. A and B are fuzzy lattices in X and Y respectively. Then the inverse image of B under θ is a fuzzy set defined by $\theta^{-1}(B) = \mu_{\theta^{-1}(B)}$ where $\mu_{\theta^{-1}(B)}(x) = \mu_B(\theta(x))$.

Definition 2.10: Any fuzzy lattice A of G is said to be Normal if there exist $x \in G$ such that A(x) = 1. Not that if A is normal fuzzy lattice of G then A(0) = 1 and hence A is normal iff A(0) = 1

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Definition 2.11: Let G be a group. Let A is fuzzy lattice of G then A is called Fuzzy Normal lattice (FLN) if ,A(xy) = A(yx). for all x ,y $\in$ G;

Definition 2.12: Let μ_A be a fuzzy set of G. Let θ: G $\to$ G′ be a map. Define the map $\mu_A(\theta): G \to [0, 1]$ by

$$\mu_{A^\theta}(x) = \mu_A(\theta(x))$$

SECTION – 3 PROPERITIES OF FUZZY LATTICE

Proposition 3.1: Let G and G′ be two Fuzzy Lattice ordered groups and $\theta : G \to G'$ be a homomorphism. If B is a Fuzzy Lattice of G′ then the pre-image $\theta^{-1}(B)$ is Fuzzy Lattice of G.

Proof : Assume that B is a Fuzzy Lattice of G. Let x, y $\in$ G.

(FL1) $\mu_{\theta^{-1}(B)}(x + y) = \mu_B\, \theta(x + y)$
$= \mu_B(\theta(x) + \theta(y))$
$\geq \min\{\mu_B(\theta(x), \mu_B\, \theta(y))\}$
$\geq \min\{\mu_{\theta^{-1}(B)}(x), \mu_{\theta^{-1}(B)}(y)\}$

(FL2) $\mu_{\theta^{-1}(B)}(-x) = \mu_B(\theta(-x))$
$\geq \mu_B(\theta(x))$
$\geq \mu_{\theta^{-1}(B)}(x)$

(FL3) $\mu_{\theta^{-1}(B)}(x \vee y) = \mu_B\, \theta(x \vee y)$
$= \mu_B(\theta x \vee \theta y)$
$\geq \min\{\mu_B(\theta(x)), \mu_B(\theta(y))\}$

$\geq \min\{\mu_{\theta^{-1}(B)}(x), \mu_{\theta^{-1}(B)}(y)\}$

(FL4) $\mu_{\theta^{-1}(B)}(x \wedge y) = \mu_B\, \theta(x \wedge y)$

$= \mu_B(\theta x \wedge \theta y)$

$\geq \min\{\mu_B(\theta(x)), \mu_B(\theta(y))\}$

$\geq \min\{\mu_{\theta^{-1}(B)}(x), \mu_{\theta^{-1}(B)}(y)\}$

$\therefore\ \theta^{-1}(B)$ is a Fuzzy Lattice of G.

Proposition 3.2 : Let θ: G $\to$ G' be an epimorphism and B is a Fuzzy set in G'. If $\theta^{-1}(B)$ is Fuzzy Lattice of G then B is Fuzzy Lattice of G′.

Proof : Let x, y $\in$ G, there exist an element a, b $\in$ G′ such that $\theta(a) = x$, $\theta(b) = y$

(FL1) $\mu_B(x + y) = \mu_B(\theta(a) + \theta(b))$

$= \mu_B\, \theta(a + b)$

$= \mu_{\theta^{-1}(B)}(a + b)$

$\geq \min\{\mu_{\theta^{-1}(B)}(a), \mu_{\theta^{-1}(B)}(b)\}$

$\geq \min\{\mu_B\, \theta(a), \mu_B\, \theta(b)\}$

$\geq \min\{\mu_B(x), \mu_B(y)\}$

(FL2) $\mu_B(-x) = \mu_B(-\theta(a))$

$= \mu_B(\theta(-a))$

$= \mu_{\theta^{-1}(B)}(-a)$

$\geq \mu_{\theta^{-1}(B)}(a)$

$\geq \mu_{(B)}\, \theta(a)$

$\geq \mu_{(B)}(x)$

(FL3) $\mu_B(x \vee y) = \mu_B(\theta(a) \vee \theta(b))$

$= \mu_B\, \theta(a \vee b)$

$= \mu_{\theta^{-1}(B)}(a \vee b)$

$\geq \min\{\mu_{\theta^{-1}(B)}(a), \mu_{\theta^{-1}(B)}(b)\}$

$\geq \min\{\mu_B\, \theta(a), \mu_B\, \theta(b)\}$

$\geq \min\{\mu_B(x), \mu_B(y)\}$

(FL4) $\mu_B(x \wedge y) = \mu_B(\theta(a) \wedge \theta(b))$

$= \mu_B(\theta(a \wedge b))$

$= \mu_{\theta^{-1}(B)}(a \wedge b)$

$\geq \min\{\mu_{\theta^{-1}(B)}(a), \mu_{\theta^{-1}(B)}(b)\}$

$\geq \min\{\mu_B\, \theta(a), \mu_B\, \theta(b)\}$

$\geq \min\{\mu_B(x), \mu_B(y)\}$

$\therefore$ B is a Fuzzy Lattice of G′.

Proposition 3.3 :If $\{A_i\}$ is a family of Fuzzy Lattice of G then $\cap A_i$ is a Fuzzy Lattice of G.

Where $\cap A_I = \{x, \wedge \mu_{A_i}(x) / x \in G\}$, $i \in A$.

Proof : Let x, y $\in$ G then for i $\in$ A it follows that

(FL1) $(\cap \mu_{A_i})(x + y) = \wedge \mu_{A_i}(x + y)$

$\geq \wedge \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\geq \wedge \min\{(\cap \mu_{A_i})(x), (\cap \mu_{A_i})(y)\}$

(FL2) $(\cap \mu_{A_i})(-x) = \wedge \mu_{A_i}(-x)$

$\geq \wedge \mu_{A_i}(x)$

$\geq (\cap \mu_{A_i})(x)$

(FL3) $(\cap \mu_{A_i})(x \vee y) = \wedge \mu_{A_i}(x \vee y)$

$\geq \wedge \min\{(\cap \mu_{A_i})(x), (\cap \mu_{A_i})(y)\}$

$\geq \min\{(\cap \mu_{A_i})(x), (\cap \mu_{A_i})(y)\}$

(FL4) $(\cap \mu_{A_i})(x \wedge y) = \wedge \mu_{A_i}(x \wedge y)$

$\geq \wedge \min\{\mu_{A_i}(x), \mu_{A_i}(y)\}$

$\geq \min\{(\cap \mu_{A_i})(x), (\cap \mu_{A_i})(y)\}$

$\therefore\ \cap A_i$ is a Fuzzy Lattice of G.

Proposition 3.4 : If A is Fuzzy set in G such that all non empty level subject U (A ; t) is a Fuzzy Lattice of G then A is Fuzzy Lattice of G.

Proof : Let x, y ∈ U (A; t), we have A(x) ≥ t and A(y) ≥ t. So that x + y ∈ U (A; t) we have A (x + y) ≥ t.

(FL1) A (x + y) ≥ t

≥ min {t, t}

≥ min{A(x), A(y)}

(FL2) A(–x) ≥ t = A (x)

(FL3) A (x V y) ≥ t

≥ min {t, t}

≥ min {A(x), A(y)}

(FL4) A (x Λ y) ≥ t

≥ min{t, t}

≥ min {A(x), B(y) } . A is a Fuzzy Lattice of G.

Proposition 3.5 : Let A be a Fuzzy Lattice of G. Let A* be a Fuzzy set in G defined by A*(x) = A(x) + 1 – A(0), for all x ∈ G. then A* is Normal Fuzzy Lattice of G which contains A.

Proof : For any x ∈ G, A* = A(x) + 1 – A(0)

(FL1) A* (x + y) = A (x + y) + 1 – A (0)

≥ min {A(x), A(y)} + 1 – A(0)

≥ min {A(x) +1 – A(0), A(y) + 1 – A(0)}

≥ min {A*(x), A*(y)}

(FL2) A* (–x) = A(–x) + 1 – A(0)

≥ A (x) + 1 – A(0)

≥ A* (x)

(FL3) A* (x V y)= A (x V y) + 1 – A(0)

≥ min {A(x), A(y)} + 1 – A(0)

≥ min {A(x) + 1 – A(0), A(y) + 1 – A(0)}

≥ min {A*(x), A*(y)}

(FL4) A* (x Λ y) = A (x Λ y) + 1 – A(0)

≥ min {A(x), A(y)} + 1 – A(0)

≥ min {A(x) +1 – A(0), A(y) + 1 – A(0)}

≥ min {A*(x), A*(y)}.A* is a Fuzzy Lattice of G and A* C A.

Proposition 3.6 : If A is a Fuzzy Lattice of G and θ is a homomorphism of G then the Fuzzy set A^{θ} of G given by A^{θ} = {< x; $\mu_{A^{\theta}}$ (x) >, x ∈ G} is a Fuzzy Lattice of G**.**

Proof : For any x, y ∈ G, we have

(FL1) $\mu_{A^{\theta}}$ (x + y) = μ_A θ (x + y)

= μ_A (θ (x) + θ (y))

≥ min {μ_A θ(x), μ_A θ(y)}

≥ min { $\mu_{A^{\theta}}$ (x), $\mu_{A^{\theta}}$ (y)}

(FL2) $\mu_{A^{\theta}}$ (–x) = $\mu_{A^{\theta}}$ (–x)

≥ $\mu_{A^{\theta}}$ (x)

(FL3) $\mu_{A^{\theta}}$ (x V y) = μ_A θ (x V y)

= μ_A (θ (x) V θ (y))

≥ min {μ_A θ(x), μ_A θ(y)}

≥ min { $\mu_{A^{\theta}}$ (x), $\mu_{A^{\theta}}$ (y)}

(FL4) $\mu_{A^{\theta}}$ (x Λ y) = $\mu_{A^{\theta}}$ (x Λ y)

= μ_A (θ(x) Λ θ(y))

≥ min {μ_A θ(x), μ_A θ(y)}

≥ min{ $\mu_{A^{\theta}}$ (x), $\mu_{A^{\theta}}$ (y)}. A^{θ} is a Fuzzy Lattice of G.

Proposition 3.7: Let T be a continuous t-norm and let f be a homomorphism on G. If μ is Fuzzy Lattice of G then μ^f is Fuzzy Lattice of f(G).

Proof :

Let $A_1 = f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$ and $A_{12} = f^{-1}(y_1 + y_2)$. Where $y_1 + y_2$ ∈ f(G). Consider the set

$A_1 + A_2$ = {x ∈ G / x = $a_1 + a_2$}

For some $a_1 \in A_1$ and $a_2 \in A_2$. If x ∈ $A_1 + A_2$ then x = $x_1 + x_2$. So that, we have f(x) = f ($x_1 + x_2$)

= f(x_1) + f(x_2)

= $y_1 + y_2$

So x ∈ f^{-1} ($y_1 + y_2$) = A_{12}, thus $A_1 + A_2$ C A_{12}

It follows that

(FL1) $\mu^f (y_1 + y_2) = \sup \{\mu (x) / \quad x \in f^{-1} (y_1 + y_2)\}$

$= \sup \{\mu (x) / x \in A_{12}\}$

$\geq \sup \{\mu (x) / x \in A_1 + A_2\}$

$\geq \sup \{\mu (x_1 + x_2) / x_1 \in A_1, \ x_2 \in A_2\}$

$\geq \sup \{T \{\mu (x_1), \mu (x_2) / x_1 \in A_1, x_2 \in A_2\}\}$

Since T is continuous for $\in > 0$,

We see that if

$\text{Sup} \{\mu (x_1) / x_1 \in A_1\} + x_1^* \leq \delta$ and

$\text{Sup} \{\mu (x_2) / x_2 \in A_2\} + x_2^* \leq \delta$

$T \{\sup \{\mu (x_1) x_1 \in A_1\}, \text{Sup} \{\mu (x_2) / x_2 \in A_2\}\} + T (x_1^*, x_2^*) \leq \in$ .... (1)

Choose $a_1 \in A_1$ and $a_2 \in A_2$, such that

$\text{Sup} \{\mu (x_1) / x_1 \in A_1\} + \mu(a_1) \leq \delta$ and

$\text{Sup} \{\mu (x_2) / x_2 \in A_2\} + \mu (a_2) \leq \delta$

Then

$T\{\text{Sup} \{\mu (x_1) / x_1 \in A_1\}, \text{Sup} \{\mu (x_2) / x_2 \in A_2\} + T (\mu (a_1), \ \mu (a_2)) \leq\in$ –––– (2)

Thus we have

$\mu^f(x + y) \ \geq \text{Sup} \{T(\mu (x_1), \mu (x_2)) / \quad x_1 \in A_1, x_2 \in A_2\}$

$= T \{\text{Sup} (\mu (x_1) / x_1 \in A_1), \text{Sup} (\mu (x_2) / x_2 \in A_2)\}$

$\geq T \{\mu^f(x), \mu^f(y)\}$

(FL2) $\mu^f (-x) \geq \mu^f (x)$

Similarly, we can show

(FL3) $\mu^f (x \vee y) \geq T \{ \mu^f (x), \mu^f (y)\}$

and

(FL4) $\mu^f (x \wedge y) \ \geq T \{ \mu^f (x), \mu^f (y)\}$

Proposition 3.8: Let T be a t-norm. Then every sensible Fuzzy Lattice A of G is Fuzzy Lattice of G.

Proof : Assume that A is sensible Fuzzy Lattice of G then we have

(FL1) $A (x + y) \geq \ T \{A(x), A(y)\}$

(FL2) $A (-x) \geq A(x)$

(FL3) $A (x \vee y) \geq T \{A(x), A(y)\}$

(FL4) $A (x \wedge y) \geq T \{A(x), A(y)\}$, for all $x, y \in G$.

Since A is sensible, we have

$\min \{A(x), A(y)\}= T \{\{\max (A(x), \ A (y)\}, \{\max (A(x), A (y)\}\}$

$\leq T \{A(x), A (y)\}$,

$T (A(x), A(y)) = \min \{A(x), A(y)\}$

It follows that

$A(x + y) \geq T (A(x), A(y))$

$= \min \{A(x), A(y)\}$, for all $x, y \in G$.

Similarly, we can show (FL2), (FL3) and (FL4) of Definition (2.4).

So A is a Fuzzy Lattice of G.

Proposition:3.9: Anonto homomorphic image of Fuzzy Lattice with sup property is Fuzzy Lattice.

Proof: Let $f : G \to G'$ be an onto homomorphism of G and let A be Fuzzy Lattice of G with sup property.

Given $x, y \in G$, we let $x_o \in f^{-1}(x')$ and $y_o \in f^{-1}(y')$ be such that

$A(x_o) = \sup_{h \in f^{-1}(x')} A(h), \qquad A(y_o) = \sup_{h \in f^{-1}(y')} A(h)$

Respectively. Then we can deduce that

(FL1) $A^f (x' + y') = \sup_{z \in f^{-1} (x_1 + y_1)} A(z)$

$\geq \text{Min} \{A(x_o), A(y_o)\}$

$\geq \text{Min} \{\sup_{h \in f^{-1}(x')} A(h) \quad \sup_{h \in f^{-1}(y')} A(h)\}$

$\geq \text{Min} \{A^f(x'), A^f(y')\}$

$A^f (x' + y') \geq \text{Min} \{A^f(x'), A^f(y')\}$

(FL2) $A^f(-x) = \sup_{z \in f^{-1} (-x')} \{A(z)\}$

$\geq A (y_o)$

$= \sup_{h \in f^{-1} (y')} A(h)$

$= A^f (y')$

Similarly, we can prove (FL3), (FL4) of Definition (2.4)

Preposition 3.10: Let f: G $\rightarrow$ G′ be a Lattice group homomorphism and A be a Fuzzy Lattice of G′ then $f^{-1}(A)$ is a Fuzzy Lattice of G.

Proof : Let x, y $\in$ G and A is a Fuzzy Lattice of G′.

(FL1) $f^{-1}(A)(x + y) = A\, f(x + y)$

$= A(f(x) + f(y))$

$\geq \min\{A(f(x) + f(y)\}$

$\geq \min\{f^{-1}(A)(x), f^{-1}(A)(y)\}$.

(FL2) $f^{-1}(A)(-x) = A\, f(-x)$

$\geq A\, f(x)$

$\geq f^{-1}(A)(x)$

(FL3) $f^{-1}(A)(x \vee y) = A\, f(x \vee y)$

$= A(f(x) \vee f(y))$

$\geq \min\{A\, f(x), A\, f(y))\}$

$\geq \min\{f^{-1}(A)(x), f^{-1}(A)(y)\}$

(FL4) $f^{-1}(A)(x \wedge y) = A\, f(x \wedge y)$

$= A(f(x) \wedge f(y))$

$\geq \min\{A\, f(x), A\, f(y))\}$

$\geq \min\{f^{-1}(A)(x), f^{1}(A)(y)\}$

$\therefore$ $f^{-1}(A)$ is a Fuzzy Lattice of G.

Proposition 3.11 : Let A be a fuzzy normal lattice of G. Then for all x, y $\in$ G, $A([x, y]) = A(0)$.

Proof : Since A is fuzzy normal lattice of G. We have

$A(x) = A(y\, x\, y^{-1})$ for all x, y $\in$ G.

Replacing x by x^{-1} and y by y^{-1}, it gives

$A(y^{-1}) = A(x^{-1}y^{-1}xy)$ or $A(x^{-1}y^{-1}xyy^{-1})$

$= A(y^{-1})$ or $A([x, y]y^{-1})$

$= A(y^{-1})$ or $A[x, y]$

$= A(0)$

SECTION – 4 Direct Product of Fuzzy Lattices

Definition 4.1: Let A_i be a Fuzzy Lattice of G_i, for i = 1, 2, 3 … n. Then the product of A_i (i = 1, 2, 3, … n) is the function $A_1 \times A_2 \times A_3 \times \ldots \times A_n : G_1 \times G_2 \times \ldots \times G_n \rightarrow L$ defined by $(A_1 \times A_2 \times A_3 \times \ldots \times A_n)(x_1, x_2, \ldots x_n) = \min\{A_1(x_1), A_2(x_2) \ldots A_n(x_n)\}$

Proposition 4.2: The Direct product of Fuzzy Lattices is a Fuzzy Lattices.

Proof :

Let $x = (x_1, x_2, \ldots x_n)$, $y = (y_1, y_2, \ldots y_n)$.

Let $(x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots y_n) \in A_1 \times A_2 \times A_3 \times \ldots \times A_n = \lambda$

(FL1) $\lambda(x + y) = \lambda((x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots y_n))$

$= A_1 \times A_2 \times A_3 \times \ldots \times A_n((x_1 + y_1), (x_2 + y_2) \ldots (x_n + y_n))$

$= \lambda((x_1 + y_1), (x_2 + y_2), \ldots, (x_n + y_n))$

$\geq \min\{A_1(x_1 + y_1), A_2(x_2 + y_2), \ldots, A_n(x_n + y_n)\}$

$\geq \min\{\min\{A_1(x_1), A_1(y_1)\}, \min\{A_2(x_2), A_2(y_2)\}, \text{---} \min\{A_n(x_n), A_n(y_n)\}\}$

$\geq \min\{\min\{A_1(x_1), A_2(x_2), \ldots, A_n(x_n)\}, \min\{A_1(y_1), A_2(y_2) \ldots, A_n(y_n)\}$

$\geq \min\{(A_1 \times A_2 \times \ldots \times A_n)(x_1, x_2, \ldots, x_n), (A_1 \times A_2 \times \ldots \times A_n)(y_1, y_2, \ldots, y_n)\}$

$\geq \min\{\lambda(x), \lambda(y)\}$

(FL2) $\lambda(-x) = \lambda(-x)$

$= (A_1 \times A_2 \times \ldots \times A_n)((-x_1), (-x_2), \ldots, (-x_n))$

$= \min\{A_1(-x_1), A_2(-x_2), \ldots A_n(-x_n)\}$

$\geq \min\{A_1(x_1), A_2(x_2) \ldots A_n(x_n)\}$

$\geq (A_1 \times A_2 \times \ldots \times A_n)(x_1, x_2, \ldots, x_n)$

$\geq \lambda(x)$

(FL3) $\lambda(x \vee y) = \lambda((x_1, x_2, \ldots x_n) \vee (y_1, y_2, \ldots y_n))$

$= \lambda(x_1 \vee y_1, x_2 \vee y_2, \ldots x_n \vee y_n)$

$= (A_1 \times A_2 \times \ldots \times A_n)(x_1 \vee y_1, x_2 \vee y_2, \ldots x_n \vee y_n)$

$= \min\{A_1(x_1 \vee y_1), A_2(x_x \vee y_2), \ldots, A_n(x_n \vee y_n)\}$

$\geq \min\{\min\{A_1(x_1), A_1(y_1)\}, \min\{A_2(x_2), A_2(y_2)\}, \ldots, \min\{A_n(x_n), A_n(y_n)\}\}$

$\geq \min\{\{\min A_1(x_1), A_2(x_2) \ldots A_n(x_n)\}, \min A_1(y_1), A_2(y_2) \ldots A_n(y_n)\}\}$

$\geq \min\{(A_1 \times A_2 \times \ldots \times A_n)(x_1, x_2, \ldots, x_n), (A_1 \times A_2 \times \ldots \times A_n)(y_1, y_2, \ldots, y_n)\}$

$\geq \min\{\lambda(x), \lambda(y)\}$

Similarly, we can show (FL4) of definition (2.4).

Proposition 4.3 : The intersection of two Fuzzy Lattices is a Fuzzy Lattice.

Proof :Since A and B are Fuzzy Lattices in G.

(FL1) $(A \cap B)(x + y) = \min\{A(x + y), B(x + y)\}$

$\geq \min\{\min\{A(x), A(y)\}\}, \min\{B(x), B(y)\}$

$\geq \min\{\min\{A(x), B(x)\}, \min\{A(y), B(y)\}$

$\geq \min\{(A \cap B)(x), (A \cap B)(y)\}$

(FL2) $(A\cap B)(-x)= \min\{A(-x),B(-x)\}$

$\geq \min\{A(x), B(x)\}$

$\geq (A \cap B)(x)$

(FL3) $(A \cap B)(x \vee y = \min\{A(x \vee y), B(x \vee y)\}$

$\geq \min\{\min\{A(x), A(y)\}, \min\{B(x), B(y)\}\}$

$\geq \min\{\min\{A(x), B(x)\}, \min\{A(y), B(y)\}\}$

$\geq \min\{(A \cap B)(x), (A \cap B)(y)\}$

Similarly, we can show (FL4 of Definition (2.4).

Proposition 4.4: If A is a Fuzzy Lattice then A^c is also a Fuzzy Lattice**.**

Proof : For any $x, y \in G$.

(FL1) $A^c(x + y) = 1 - A(x + y)$

$\geq \max\{1 - A(x), 1 - A(y)\}$

$\geq \max\{A^c(x), A^c(y)\}$

(FL2) $A^c(-x) = 1 - A(-x)$

$\leq 1 - A(x)$

$\leq A^c(x)$

(FL3) $A^c(x \vee y) = 1 - A(x \vee y)$

$\leq \max\{1 - A(x), 1 - A(y)\}$

$\leq \max\{A^c(x), A^c(y)\}$

Similarly we can show (FL4) of Definition (2.4)

CONCLUSION

Nandha [4] proposed the concept of Fuzzy Lattice using the notion of Fuzzy partial ordering. But after a critical situation, it has been observed that his definition contains some redundancy. In this paper as a consequence of this observation, we present a modified definition of Fuzzy Lattice and characterization of Fuzzy Lattices.

Applications: Lattice structure has been found to be extremely important in the areas of quantum logic, Erogodic theory, Reynold's operations, Soft Computing, Communication system, Information analysis system, artificial intelligences and physical sciences

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