The Inverse Domination in Semi-total Block Graphs

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ABSTRACT

Let G = (V, E) be a simple, finite, undirected graph with |V| = n and |E| = m. Kulli introduced the new graph valued function namely the semi-total block graph of a graph G. Let $B_1 = \{u_1,u_2,...,u_r, r \geq 2\}$ be a block of G. Then we say that the point u_1 and block B_1 are incident with each other, as are u_2 and B_1, u_3 and B_1 and so on. If two distinct blocks B_1 and B_2 are incident with a common cut point then they are called adjacent blocks. Let $B = \{B_1, B_2,...,B_p\}$ be the set of blocks of G. The semi-total block graph T_b (G) of a graph G is the graph whose point set is $V(G) \cup B(G)$ in which any two points are either adjacent or the corresponding members of G are incident. The points and blocks of G are members of $T_b(G)$. A non-empty set $D \subseteq V \cup B$ is a dominating set of $T_b(G)$ if every point in $(V \cup B)$-D is adjacent to atleast one point in D (Muddebihal, M.H. et al 2004). The domination number of $T_b(G)$ is denoted by $\gamma[T_b(G)]$ and it is defined as the minimum cardinality taken over all the minimal dominating sets of $T_b(G)$. In this paper, we defined Inverse domination in semi-total block graphs. Let D be the minimum dominating set of $T_b(G)$. If $(V \cup B)$-D contains a dominating set D' then D' is called the Inverse dominating set of $T_b(G)$. The Inverse domination number in semi-total block graph is denoted by $\gamma'[T_b(G)]$ and it is defined as the minimum cardinality taken over all the minimal Inverse dominating sets of $T_b(G)$. In this paper, many bounds on $\gamma'[T_b(G)]$ are attained and its exact values for some standard graphs are found**.** Its relationships with other parameters are investigated. Nordhaus-Gaddum type results are also obtained for this parameter.

Keywords: Domination number, Inverse domination number, semi-total block graph and independence number.

1.1. INTRODUCTION

Let G = (V, E) be a simple, finite, undirected graph with |V| = n and |E| = m. Kulli introduced the new graph valued function namely the semi-total block graph of a graph G. Let $B_1 = \{u_1,u_2,...,u_r, r \geq 2\}$ be a block of G. Then we say that the point u_1 and block B_1 are incident with each other, as are u_2 and B_1, u_3 and B_1 and so on. If two distinct blocks B_1 and B_2 are incident with a common cut point then they are called adjacent blocks. Let $B = \{B_1, B_2,...,B_p\}$ be the set of blocks of G. The semi-total block graph T_b (G) of a graph G is the graph whose point set is $V(G) \cup B(G)$ in which any two points are either adjacent or the corresponding members of G are incident. The points and blocks of G are members of $T_b(G)$. A non-empty set $D \subseteq V \cup B$ is a dominating set of $T_b(G)$ if every point in $(V \cup B)$-D is adjacent to atleast one point in D (Muddebihal, M.H. et al 2004). The domination number of $T_b(G)$ is denoted by $\gamma[T_b(G)]$ and it is defined as the minimum cardinality taken over all the minimal dominating sets of $T_b(G)$. In this paper, we defined Inverse domination in semi-total block graphs. Let D be the minimum dominating set of $T_b(G)$. If $(V \cup B)$-D contains a dominating set D' then D' is called the Inverse dominating set of $T_b(G)$. The Inverse domination number in semi-total block graph is denoted by $\gamma'[T_b(G)]$ and it is defined as the minimum cardinality taken over all the minimal Inverse dominating sets of $T_b(G)$. In this paper, many bounds on $\gamma'[T_b(G)]$ are attained and its exact values for some standard graphs are found**.** Its relationships with other parameters are investigated. Nordhaus-Gaddum type results are also obtained for this parameter.

A non-empty set $D \subseteq V$ is a dominating set of G if every point in V-D is adjacent to atleast one point in D. The domination number $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G (Haynes, T.W. et al 1998). A dominating set D of a graph G is a split dominating set of G if the induced subgraph <V-D> is disconnected (Kulli, V.R. and Janakiram,B. 1997). If V-D contains a dominating set D' then D' is called the Inverse dominating set of G. Then D' is called an Inverse split dominating set of G if the induced subgraph <V-D'> is disconnected (Ameenal Bibi, K. and Selvakumar,R. 2008). The Inverse split domination number $\gamma_s'(G)$ is the minimum cardinality of the minimal Inverse split dominating set of G. The point independence number or the Independence number $\beta_o(G)$ is the maximum cardinality among the independent set of points of G. An independent set has pairwise non – adjacent vertices.

1.2. RESULTS

We observed the following results for some standard graphs.

Observation 1.2.1:

For any path P_n with n ≥ 2 points,

$$\gamma' [T_b (P_n)] = \left\lceil \frac{n}{2} \right\rceil$$

Observation 1.2.2:

For any non-separable graph G,

$$\gamma' [T_b (G)] = 1$$

Note:

We needed the following theorem for our later results

Theorem 1.2.3:

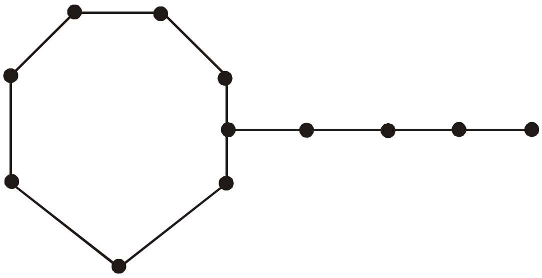
(K.Ameenal Bibi and R.Selvakumar (2008))

Let T be a tree such that any two adjacent cut points u and v with atleast one of u and v is adjacent to an end point then

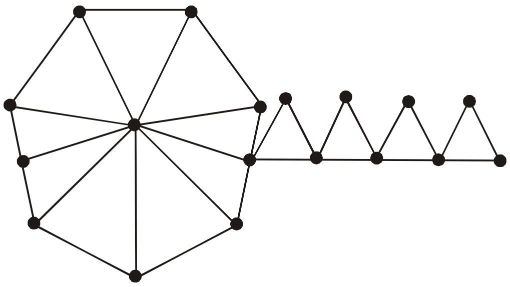
$$\gamma'(T) = \gamma_s'(T).$$

Example 1.2.4:

G:



$T_b(G)$:



Total number of blocks in G: p = 5,
$\gamma(G) = 4$, $\gamma'(G) = 4$, $\gamma[T_b(G)] = 3$ and $\gamma'[T_b(G)] = 3$.

Theorem 1.2.5:

For any tree T, $\gamma'[T_b(T)] \geq \gamma'(T)$.

Proof:

Let $D' = D_1' \cup D_2'$ be a minimum inverse dominating set of $T_b(T)$ where $D_1' \subset V(T)$ and $D_2' \subset [V(T_b(T)-V(T)]$. Let $D_2' = \{b_1,b_2,...,b_p\}$ where $b_1,b_2,...,b_p$ are points corresponding to the blocks $B_1, B_2,\ldots,B_p$ of T. In a tree, each block is k_2. Let $D_2''=\{v_1,v_2,...,v_p\}$ where $v_1,v_2,...,v_p$ are the points of T which are incident with $B_1, B_2,....,B_p$ respectively. Clearly $D' \cup D_2''$ is an inverse dominating set of T.

$$
\begin{aligned}
\text{Thus } \gamma'(T) &\leq |D_1' \cup D_2''| \\
&= |D_1' \cup D_2'| \\
&= |D'| \\
&= \gamma'[T_b(T)]
\end{aligned}
$$

Hence $\gamma'[T_b(T)] \geq \gamma'(T)$.

Theorem 1.2.6:

For any tree T, $\gamma'[T_b (T)] \geq \gamma_s'(T)$

Proof:

From theorem 1.2.3, $\gamma'(T) = \gamma_s'(T)$…............ (1)
From theorem 1.2.5, $\gamma'[T_b(T)] \geq \gamma'(T)$...........(2)
From (1) and (2) we get the required result.

1.3. BOUNDS ON γ' $[T_b(G)]$

Theorem 1.3.1:

For any graph G with p blocks
γ' $[T_b(G)] \leq p$.

Proof:

Let B_1, B_2,...,B_p be the p blocks of G.

Let $\{b_1,b_2,...,b_p\}$ be the corresponding block points in $T_b(G)$ and $\{c_1,c_2,...,c_t\}$ be the cut points of G. If G is non-separable then $\gamma'[T_b(G)]=1$. If G is separable with t=1 and $p \geq 2$ blocks, then consider the following two cases:

Case (i)

Suppose each block is k_p then $\gamma[T_b(G)] = t$
Thus $\gamma'[T_b(G)] < p$.

Case (ii)

Suppose each block of G is either C_p (p>3) or a block which is incomplete. Then $D'=\{b_1,b_2,...,b_p\}$ is such that each point of $T_b(G)$ not in D' is adjacent to atleast one point in D'. Thus D' is the minimum Inverse dominating set of $T_b(G)$. Hence $\gamma'[T_b(G)] = p$. Combining both the cases, we have $\gamma'[T_b(G)] \leq p$.

If G has $t \geq 2$ cut points then the minimum Inverse dominating set may be either $\{b_1, b_2,\ldots,b_p\}$ or $\{c_1,c_2,...,c_t\}$. In each case, we have $\gamma'[T_b(G)] \leq p$ since $t < p$. Hence the proof.

Theorem 1.3.2:

If $\gamma'(G) \leq \gamma'[T_b(G)]$ then $\gamma'(G) \leq p$ where p is the number of blocks of G.

Proof:

Assume $\gamma'(G) \leq \gamma'$ $[T_b(G)]$.
By theorem 1.3.1, $\gamma'[T_b(G)] \leq p$.
Hence $\gamma'(G) \leq \gamma'$ $[T_b(G)] \leq p$.
Thus $\gamma'(G) \leq p$.

Theorem 1.3.3:

For any connected graph G, $\gamma'[T_b(G)] \leq \dfrac{n}{2}$

Proof:

Let $\{B_1, B_2,...,B_p\}$ be the p number of blocks of G and two adjacent blocks of G have atleast one point v in common. Starting from v, consider alternate points of G and that forms an Inverse dominating set D' of G. If G contains only one block then start from any point v. Let $D' = \{v_1, v_2,...,v_{n/2}\}$. Since each point of G belongs to atleast one block of G, each B_i, $1 \leq i \leq p$ is adjacent to atleast one element of D'. Hence $\gamma'[T_b(G)] \leq |D'| \leq \dfrac{n}{2}$.

Theorem 1.3.4:

For any graph G, $\gamma'[T_b(G)] \leq n-\delta(G)$.

Proof:

Let $\{B_1,B_2,...,B_p\}$ be the number of blocks of G. Let v be the point of minimum degree $\delta(G)$. Let v_1, v_2,..., v_δ be the points adjacent to v. Without loss of generality, let $v \in B_1$. Let $b_1,b_2,...,b_p$ be the block points in $T_b(G)$ corresponding to the blocks $B_1,B_2,...,B_p$.

Let $D' = V(G) – \{v_1,v_2,...,v_\delta\}$

We claim tat D' contains atleast one point from each B_i, if not, let all the points of $B_j \not\in D'$.

Let $V(B_j) = \{v_1,v_2,...,v_r\} \subset \{v_1,v_2,...,v_\delta\}$. Then $\{(v_1,v_2,...,v_r) \cup v\}$ is a block which contradicts that B_j is a block. Therefore, D' contains atleast one point from each B_i, where $1 \leq i \leq p$. Obviously, D' is an Inverse dominating set of $T_b(G)$.

Hence γ' $[T_b(G)] \leq |D'| = n-\delta(G)$

Theorem 1.3.5:

For any graph G,

$\gamma'[T_b(G)] \leq \beta_o(G)$ where $\beta_o(G)$ is the independence number of G.

Proof:

Let $\{b_1,b_2,...,b_p\}$ be the block points in $T_b(G)$ corresponding to the blocks $B_1,B_2,...,B_p$ in G. Let $D' = \{v_1,v_2,...,v_p\}$ be the maximal independent set of G which is also an Inverse dominating set of G. Then we have the following two cases.

Case (i)

Suppose b_1 is a block point in $T_b(G)$ which is not adjacent to any point of D'. Then V(G)-D' forms a dominating set of $T_b(G)$ and $|V(G)-D'| \leq |D'|$

Therefore $\gamma'[T_b(G)] \leq \beta_o(G)$.

Case (ii)

Suppose each block is adjacent to atleast one point of D', then

$\gamma'[T_b(G)] \leq |D'| = \beta_o(G)$.

Hence $\gamma'[T_b(G)] \leq \beta_o(G)$.

Theorem 1.3.6:

For any graph G without end points,

$\gamma'[T_b(G)] \leq$ diam (G).

Proof:

Let $A=\{e_1,e_2,...,e_k\}$ be the set of lines which constitutes the largest path between any two points of G such that |A| = diam (G) and p be the number of blocks of G.

Case (i)

If all the elements of A belongs to a single block, then $\gamma'[T_b(G)]=1 \leq |A|$.

Case (ii)

If all the elements of A belongs to different blocks and G is without end points, then $p \leq |A|$. Then by theorem 3.3.1, $\gamma'[T_b(G)] \leq p \leq |A|$.

Hence $\gamma'[T_b(G)] \leq$ diam (G).

1.4. NORDHAUS – GADDUM TYPE RESULTS

Theorem 1.4.1:

For any connected graph G with $n \geq 4$ points,

(i) $\gamma'[T_b(G)] + \gamma'[T_b(\overline{G})] \leq 2n-1$

(ii) $\gamma'[T_b(G)] . \gamma'[T_b(\overline{G})] \leq n(n-1)$

Proof:

For any connected graph G, from theorem 3.3.3,

$\gamma'[T_b(G)] \leq \frac{n}{2}$

Since G is a graph with $n \geq 4$ points and G has no isolates, the number of points of $\overline{G}$ is less than or equal to n-1. Hence $\gamma'[T_b(\overline{G})] \leq n-1$.

Hence the theorem.

Theorem 1.4.2:

For any connected graph G with $p \geq 2$ blocks,

(i) $\gamma'[T_b(G)] + \gamma'[T_b(\overline{G})] \leq 2p$.

(ii) $\gamma'[T_b(G)] . \gamma'[T_b(\overline{G})] \leq p^2$.

Proof:

From theorem 1.3.1, $\gamma'[T_b(G)] \leq p$. Since G is a connected graph with $p \geq 2$ blocks, the number of blocks of $\overline{G}$ is less than or equal to p.

Hence $\gamma'[T_b(\overline{G})] \leq \gamma'[T_b(G)] \leq p$.

Hence the theorem.

1.5. CONCLUSION

Graphs can be used to study the structure of World Wide Web. We can determine whether two computers are connected by a communication link using graph models of computer network. We can also use graphs to schedule examinations and assign channels to television stations. Graphs are used as models to represent the competition of different species in an ecological niche, in computing the number of different combinations of flights between two cities in an airline network, in finding the number of colours needed to colour the regions of a map.

In this paper, many bounds on $\gamma'[T_b(G)]$ are attained and its exact values for some standard graphs are found**.** Its relationships with other parameters are investigated. Nordhaus-Gaddum type results are also obtained for this parameter.

1.6. REFERENCES

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