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Even Vertex Gracefulness of Fan Graph

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Abstract: A graph is even vertex graceful if there exists an injective map $f : E(G) \to \{1,2,\ldots,2q\}$ so that the induced map $f^+: V(G) \to \{0,2,4,\ldots,2k-2\}$ defined by $f^+(x) = \sum f(xy) \pmod{2k}$ where $k = \max\{p, q\}$ makes all distinct.

In this paper, we prove that Fan graphs $F(nC_3)$, $F(nC_5)$ and $F(2nC_3)$ are all even vertex graceful, where n is any positive integer.

Introduction: A.Solairaju, and A.Sasikala [2008] got gracefulness of a spanning tree of the graph of product of P_m and C_n, A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A.Solairaju and C.Vimala [2008] also got gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n,

A.Solairaju and P.Muruganantham [2009] proved that ladder $P_2 \times P_n$ is even-edge graceful (even vertex graceful). They found [2010] the connected graphs $P_n \circ nC_3$ and $P_n \circ nC_7$ are both even vertex graceful, where n is any positive integer. They also obtained [2010] that the connected graph $P_n \Delta nC_4$ is even vertex graceful, where n is any even positive integer.

Section I : Preliminaries

Definition 1.1: Let G = (V,E) be a simple graph with p vertices and q edges.

A map $f : V(G) \to \{0,1,2,\ldots,q\}$ is called a graceful labeling if f is one – to – one

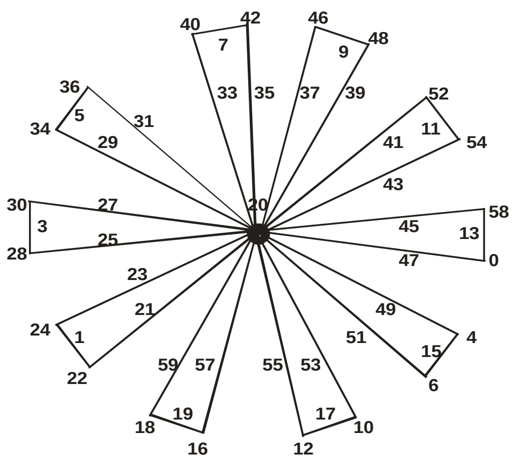
The edges receive all the labels (numbers) from 1 to q where the label of an edge is the absolute value of the difference between the vertex labels at its ends.

A graph having a graceful labeling is called a graceful graph.

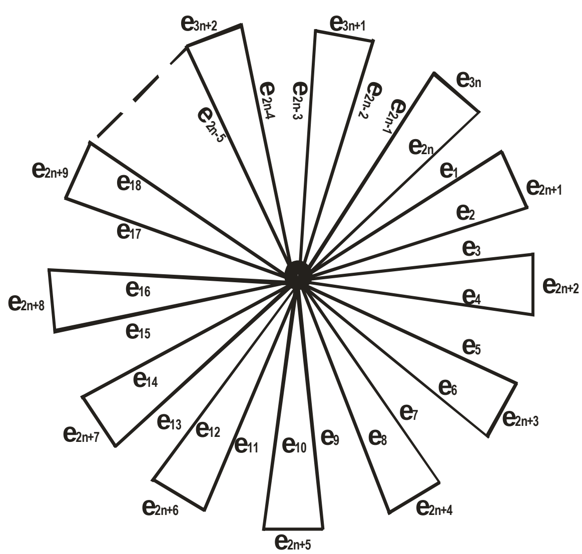
Definition 1.2: A graph is even vertex graceful if there exists an injective map $f : E(G) \to \{1,2,\ldots,2q\}$ so that the induced map $f^+: V(G) \to \{0,2,4,\ldots,2k-2\}$ defined by $f^+(x) = \sum f(xy) \pmod{2k}$ where $k = \max\{p, q\}$ makes all distinct.

Definition 1.3: A graph is odd-edge graceful if there exists an injective map $f : E(G) \to \{1,3,5, \ldots, 2q\}$ so that the induced map $f^+: V(G) \to \{0,1,2,3, \ldots, 2k-2\}$ defined by $f^+(x) = \sum f(xy) \pmod{2k}$ where $k = \max\{p, q\}$ makes all distinct.

Example 1.4: The following connected graph is even vertex graceful.



Definition 1.5: A fan graph or a friendship graph **$F(nC_3)$** is defined as the following connected graph containing n copies of circuits of each length 3 with some arbitrary labeling of edges.



Section 2: Even vertex graceful of fan graph

Few contributions on (friendship graph) fan graph are given below.

Notation: p and q denote the number of vertices and edges of a graph respectively.

Theorem 2.1: If n = 0 (mod 3), the fan graph $F(nC_3)$ is even vertex graceful.

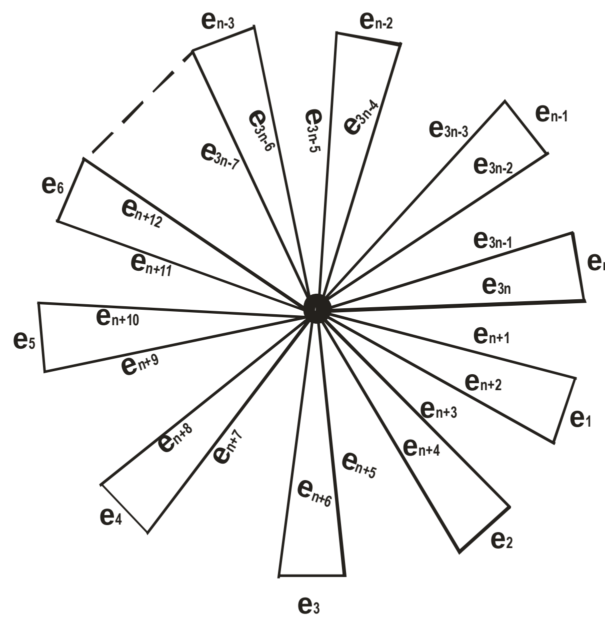
Proof: The graph $F(nC_3)$ is chosen with some arbitrary labeling of edges as in definition (1.5).

Define a map f: $E[F(nC_3)] \rightarrow \{0, 1,2,\ldots, 2q\}$ by $f(e_i) = (2i-1)$, i=1 ,…, 3n.

Then the induced map $f^+(u) = \sum f(uv)$ (mod 2q) where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling as well as edge–odd graceful labeling. Thus the connected graph $F(nC_3)$ is both even vertex graceful and odd-edge graceful.

Theorem 2.2: If n = 1 (mod 3), the fan graph $F(nC_3)$ is even vertex graceful.

Proof: The graph $F(nC_3)$ is chosen with some arbitrary labeling of edges as in the following diagram.

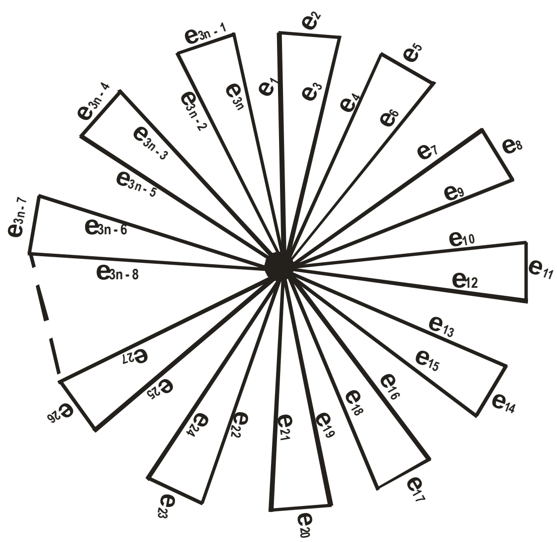


Define a map f: $E[F(nC_3)] \rightarrow \{0, 1,2,\ldots, 2q\}$ by $f(e_i) = 2i$, i = 1 ,…, 3n.

Then the induced map $f^+(u) = \sum f(uv)$ (mod 2q) where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling. Thus the connected graph $F(nC_3)$ is even vertex graceful .

Theorem 2.3: If n = 2 (mod 3), the fan graph $F(nC_3)$ is even vertex graceful.

Proof: The graph $F(nC_3)$ is chosen with some arbitrary labeling of edges as in the following diagram.



$f(e_i) = 2i$ for i = 1,2,3; $f(e_i) = (2i-1)$, i = 4,5,6;

$f(e_i) = (2i-6)$, i = 7, 8, 9, 13, 14, 15,…, (3n-5), (3n-4), (3n-3).

$f(e_i) = 2i-7$, i = 10, 11, 12 ,…, (3n-8), (3n-7), (3n-6).

Subcase (a): $n \equiv 2 \pmod 3 \equiv 2 \pmod 6$

$f(e_{3n-1}) = 2q$, $f(e_{3n}) = (2q-4)$, $f(e_{3n-2}) = (2q-8)$.

Subcase (b): $n \equiv 2 \pmod 3 \equiv 5 \pmod 6$

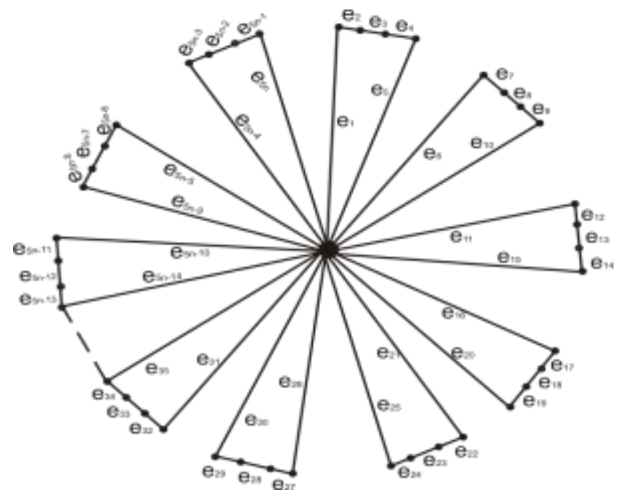
$f(e_{3n-1}) = 2q$, $f(e_{3n}) = (2q-2)$, $f(e_{3n-2}) = 2q-6$.

Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling. Thus the connected graph $F(nC_3)$ is even vertex graceful.

Main theorem 2.4: The fan graph $F(nC_3)$ is even vertex graceful.

Section 3 – Even vertex graceful of fan graph $F(nC_5)$.

Definition 3.1: The fan graph $F(nC_5)$ is a defined as the following connected graph containing n copies of circuits of each length 5 with some arbitrary labeling of edges.



Theorem 3.1: The fan graph $F(nC_5)$ is even vertex graceful.

Proof: An arbitrary labeling of edges of $F(nC_5)$ is followed from definition (3.1).

Define $f: E(F(nC_5)) \rightarrow \{1, 2, \ldots, 2q\}$ in the following manner:

Case (i) n is odd

$f(e_i) = (2i-1)$, i = 1, 2 ,3 ,..., q.

Case (ii) n is even

$f(e_i) = 2(i+2)$, i = 1, 2, 3, 4, 5;

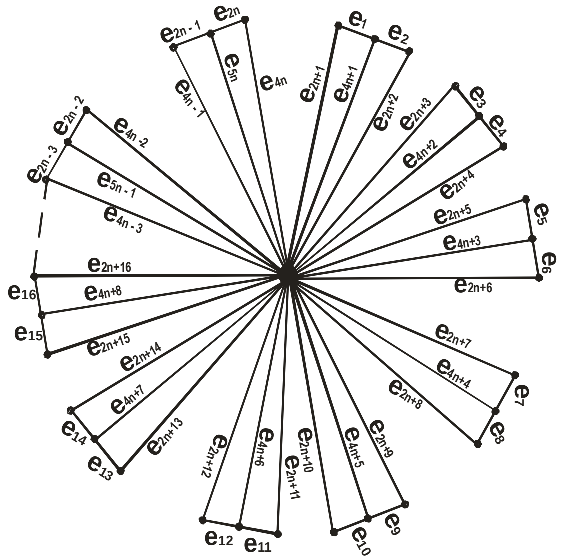
$f(e_{10k+i}) = f(e_i) + 10k$, k=1, 2 ,…., (n / 2), i = 1, 2, 3, 4, 5;

$f(e_i) = (2i-11)$, i = 6, 7, 8, 9, 10;

$f(e_{10k+i}) = f(e_i) + (10k)$, k = 1, 2 ,…., (n / 2), i = 6, 7, 8, 9, 10.

Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling. Thus the connected graph $F(nC_5)$ is even vertex graceful

Definition 3.2: The fan graph $F(2nC_3)$ is a defined as the following connected graph containing 2n copies of circuits of each length 3 with some arbitrary labeling of edges.



Theorem 3.2: The fan graph $F(2nC_3)$ is even vertex graceful.

Proof: An arbitrary labeling of edges of $F(2nC_3)$ is followed from the definition (3.2).

Define $f: E(F(2nC_3)) \rightarrow \{1,2, \ldots, 2q\}$ in the following manner:

$f(e_i) = 2i$, $i = 1, 2, 3, \ldots, 4n$;

Case (1) $n \geq 8$ and n is even : $f(e_{4n+i}) = (2q-4i)$, $i = 1, 2, 3, \ldots, n$.

Case (2) $n \equiv 1 \pmod 4$: $f(e_{4n+i}) = 2q-8(i-1)$, $i = 1, 2, 3, \ldots, n$.

Case (3) $n \equiv 3 \pmod 4$: $f(e_{4n+1}) = (2q-2)$; $f(e_{4n+i}) = f(e_{4n+1}) - 8(i-1)$, $i = 2, 3, \ldots, n$.

Then the induced map $f^+(u) = \sum f(uv) \pmod{2q}$ where the sum runs over all edges uv through v. Now, f and f^+ both satisfy even vertex graceful labeling. Thus the connected graph $F(2nC_3)$ is even vertex graceful.

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