Edge-odd Gracefulness of $P_M \Theta S_N$, for M = 5, 6, 7, 8

Dr. A. Solairaju
Associate Professor of Mathematics
Jamal Mohamed College, Tiruchirapalli – 620 020.
Tamil Nadu, India.

C. Vimala and A.Sasikala
Assistant Professors (SG), Department of Mathematics,
Periyar Maniammai University, Vallam
Thanjavur – Post. Tamil Nadu, India.

ABSTRACT

A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by $f_+(x) \equiv f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. In this article, the Edge-odd gracefulness of $P_m \Theta S_m$ m = 5, 6, 7, 8 is obtained.

Keywords: Graceful Graph, Edge-odd graceful labeling, Edge-odd graceful graph

INTRODUCTION

A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A.Solairaju, C.Vimala, A.Sasikala [2008] gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n, A. Solairaju et.al. [2010] that the cartesian product of path P_2 and circuit C_n for all integer n, $S_{m, n}$, $C_m \Theta S_n$ for n is even and the crown graph $C_3 \Theta P_n$ and $C_3 \Theta 2P_n$ are is edge-odd graceful. Here the edge-odd graceful labeling of $P_m \Theta S_n$, m = 5, 6, 7, 8 is obtained.

Section II: Basic concept

In this section, the following definitions are first listed.

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label │f(u) – f(v)│ the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: Edge – odd graceful graph [6]: A (p, q) connected graph is edge-odd graceful if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2, …, (2k-1)}defined by $f_+(x) \equiv \Sigma f(x, y) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd.

Section 3 - Edge-odd Gracefulness of $P_m \Theta S_n$, m = 5, 6, 7, 8

In this section edge-odd gracefulness of the fire - cracker graph $P_m \Theta S_n$, m = 5, 6, 7, and 8 is obtained.

Definition 2.1: The graph $P_m \Theta S_n$ is a tree obtained from the path P_m by adding a star graph S_n to each of the pendant vertices of P_m. It has (2n + m) vertices and ((2n + m - 1) edges.

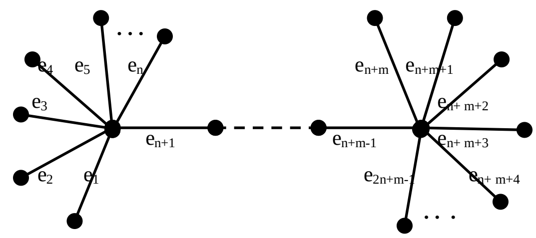

Figure 1: Graph of $P_m \Theta S_n$

Lemma 2.2: The connected graph $P_6 \Theta S_n$ is edge – odd graceful for n > 1.

Proof: The figure 2 is the graph of $P_6 \Theta S_n$ with (2n+6) vertices and (2n+5) edges, with some arbitrary graceful labeling in vertices and edges.

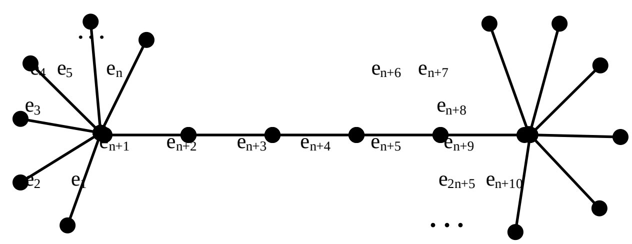

Figure 2: Edge-odd gracefulness of $P_6 \Theta S_n$

Define f: E(G) → {1, 3, …, 2q-1} by
$f(e_i) = (2i-1)$, for i = 1, 2, …, (n-1), (n+1), …, (n+5), (n+7), …, (2n+5)
$f(e_n) = 2n + 11$; $f(e_{n+6}) = 2n – 1$ [Rule 1]

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by
$f_+(v) \equiv \Sigma f(uv) \bmod (2k-1)$, where this sum run over all edges through v [Rule 2]

Thus the map f and the induced map f_+ provide labels as odd numbers for edges with all distinct and also the labelings for vertex set has distinct values in {1, 2,…. , 2k-1)}. Hence the graph $P_6 \Theta S_n$ is edge-odd graceful.

Lemma 2.3: The connected graph $P_7 \Theta S_n$ is edge – odd graceful for n > 1.

Proof: The figure 3 is the graph of $P_7 \Theta S_n$ with (2n+7) vertices and (2n+6) edges, with some arbitrary graceful labeling in vertices and edges.

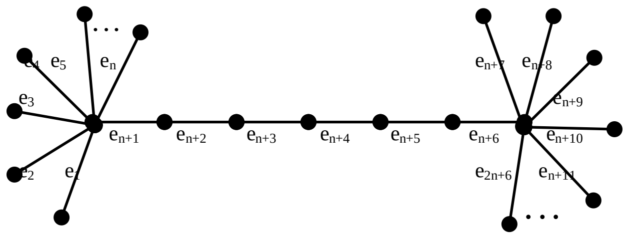

Figure 3: Edge-odd gracefulness of $P_7 \Theta S_n$

Define f: E(G) → {1, 3, …, 2q-1} by

For n ≡ 3, 5 (mod 8)

$f(e_i) = (2i-1)$, for i = 1, 2, …, (n-1), (n+1), …, (n+6), (n+8), …, (2n+6) (1)

$f(e_n) = 2n + 13$; $f(e_{n+7}) = 2n - 1$

For n ≡ 7, 9 (mod 8)

$f(e_i) = (2i-1)$, for i = 1, 2, …, (2n+ 6)

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by

$f_+(v) \equiv \Sigma f(uv) \bmod (2k-1)$, where this sum run over all edges through v (2)

Thus the map f and the induced map f_+ provide labels as odd numbers for edges with all distinct and also the labelings for vertex set has distinct values in {1, 2,…. ,(2k-1)}. Hence the graph $P_7 \Theta S_n$ is edge-odd graceful.

Lemma 2.3: The connected graphs $P_8 \Theta S_3$ and $P_8 \Theta S_5$ is edge – odd graceful.

Proof: The figure 4 and 5 is the connected graphs $P_8 \Theta S_3$ and $P_8 \Theta S_5$

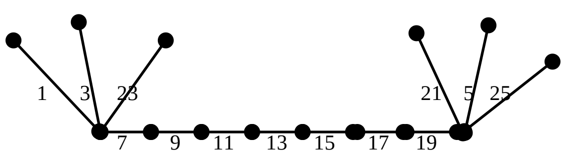

Figure 4: Edge-odd gracefulness of $P_8 \Theta S_3$

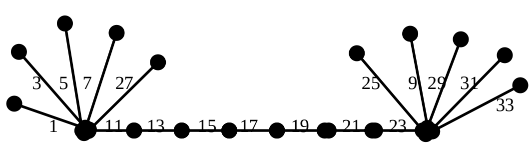

Figure 5: Edge-odd gracefulness of $P_8 \Theta S_5$

Lemma 2.4: The connected graph $P_8 \Theta S_n$ is edge – odd graceful for odd n > 5.

Proof: The figure 4 is the graph of $P_8 \Theta S_n$ with (2n+8) vertices and (2n+7) edges, with some arbitrary graceful labeling in vertices and edges.

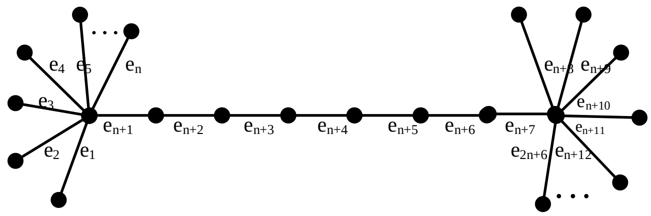

Figure 6: Edge-odd gracefulness of $P_8 \Theta S_n$

Define f: E(G) → {1, 3, …, 2q-1} by

$f(e_i) = (2i-1)$, for i = 7, 9, …, 2n+ 7 (1)

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by

$f_+(v) \equiv \Sigma f(uv) \bmod (2k-1)$, where this sum run over all edges through v (2)

Thus the map f and the induced map f_+ provide labels as odd numbers for edges with all distinct and also the labelings for vertex set has distinct values in {1, 2,…. ,(2k-1)}. Hence the graph $P_8 \Theta S_n$ is edge-odd graceful.

REFERENCES

1. A.Solairaju, A.Sasikala, C.Vimala, Gracefulness of a spanning tree of the graph of product of P_m and C_n, The Global Journal of Pure and Applied Mathematics of Mathematical Sciences, Vol. 1, No-2 (July-Dec 2008): pp 133-136

2. A.Solairaju and K.Chitra, Edge-odd graceful labeling of some graphs " Electronics Notes in Discrete Mathematics Volume 33,April 2009, Pages 15 - 20

3. A.Solairaju, C.Vimala, A.Sasikala, Gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n, The Global Journal of Pure and Applied Mathematics of Mathematical Sciences, Vol. 1, No-2 (July-Dec 2008): pp 117-120

4. A.Solairaju, A.Sasikala, C.Vimala, Edge-odd Gracefulness of a spanning tree of Cartesian product of P_2 and C_n ,Pacific-Asian Journal of Mathematics, Vol .3, No. 1-2. (Jan-Dec. 2009) pp:39-42

5. A.Solairaju, C. Vimala, A. Sasikala, Edge-Odd Gracefulness of $C_3 \Theta P_n$ and $C_3 \Theta 2P_n$ for n is even (communicated to Serial Publications)