Edge-odd Gracefulness of the Graph $S_2\square S_n$

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ABSTRACT
A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1,2, 3, …, (2k-1)}defined by $f_+(x) \equiv \Sigma f(x, y)$ (mod 2k), where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. In this article, the Edge-odd gracefulness of $S_2\square S_n$ is obtained.

Keywords: Graceful Graph, Edge-odd graceful labeling, Edge-odd graceful graph

INTRODUCTION
A.Solairaju and K.Chitra [2009] obtained edge-odd graceful labeling of some graphs related to paths. A. Solairaju et.al. [2010] that the cartesian product of P_2 and C_n for all integer n, and $S_{m,n}$.

Section-2: Edge-odd Gracefulness of book graph $S_2\square S_n$

Definition 2.1: Graceful Graph: A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0, 1, 2, …, m} such that when each edge uv is assigned the label │f(u) – f(v)│ and the resulting edge labels are distinct. Then the graph G is graceful.

Definition 2.2: The book graph $S_2\square S_n$ is a connected graph obtained by adding 'n' number of C_4 with one edge. It has 2n vertices and 3n – 2 edges. This graph is given in figure 1.

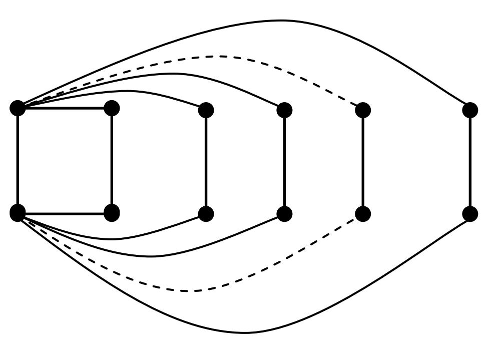

Figure 1: Graph of $S_2\square S_n$

Definition 2.2: Edge-odd graceful graph: A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2, …,(2k-1)} defined by $f_+(x) \equiv \Sigma f(x, y)$ (mod 2k), where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.

Theorem 2.1: The connected graph $S_2\square S_n$, for n ≥ 3, is edge – odd graceful.

Proof: The figure 2 is the Cartesian product graph $S_2\square S_n$ with 2n vertices and 3n-2 edges, with some arbitrary labeling to its vertices and edges. It is proved that the graph $S_2\square S_n$, for n ≥ 3, is edge – odd graceful by taking two cases such as n is odd and n is even.

Case (i) : n is odd

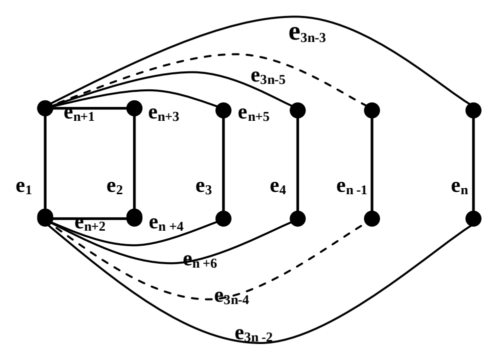

Figure 2: Edge – odd graceful Graph $S_2\square S_n$ for n is odd

Hence define f: E(G) → {1, 3, …, 2q-1} by
$f(e_i) = (2i-1)$, for i = 1, 2, …, (3n -2) (Rule 1)

Define f_+: V(G) → {0, 1, 2, …, (2k-1)} by

$f_+(v) \equiv \Sigma f(uv)$ mod (2k), where this sum run over all edges through v (Rule 2).

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in {0, 1, 2,…., (2k-1)}. Hence the graph $S_2\square S_n$, for n is odd, is edge-odd graceful.

Case (ii) : n is even

Here it is proved that $S_2\square S_n$ is graceful by taking 2 cases for n such as

(a). $n \equiv 2 \pmod 6$
(b). $n \equiv 0 \pmod 6$ and $n \equiv 4 \pmod 6$

Subcase (a): $n \equiv 2 \pmod 6$

Here edges are given labeling with odd numbers as in the figure 3

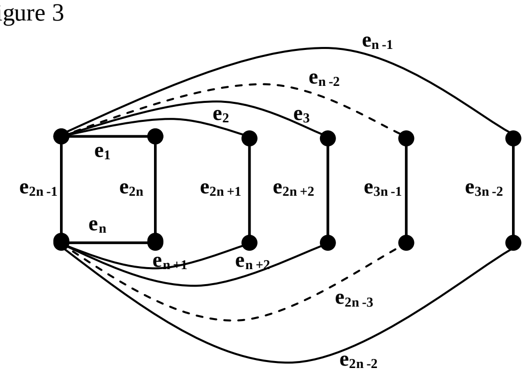

Figure 3: Edge – odd graceful Graph $S_2\square S_n$, for $n \equiv 2 \pmod 6$

Define f: $E(G) \rightarrow \{1, 3, \ldots, 2q-1\}$ by
$f(e_i) = (2i-1)$, for $i = 1, 2, \ldots, (3n-2)$
and $i \neq n + (n/2) - 1$ and $2n + (n/2) - 1$ (Rule 3)
$f(n + n/2 - 1) = 5n - 3$ and
$f(2n + n/2 - 1) = 3n - 3$

Define f_+: $V(G) \rightarrow \{0, 1, 2, \ldots, (2k-1)\}$ by
$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v (Rule 4)

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in $\{0, 1, 2, \ldots, (2k-1)\}$. Hence the graph $S_2\square S_n$, for $n \equiv 2 \pmod 6$, is edge-odd graceful.

Subcase (b): $n \equiv 0, 4 \pmod 6$

The edges are given labeling as in the figure 4

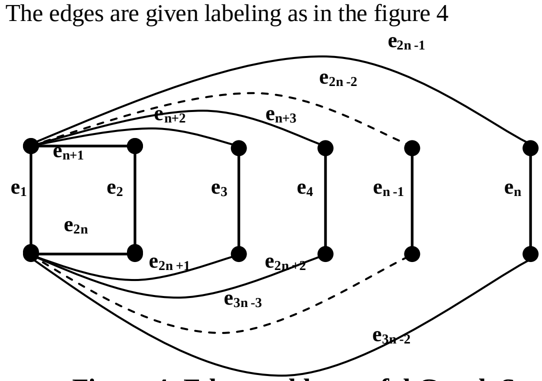

Figure 4: Edge – odd graceful Graph $S_2\square S_n$, for n = 0, 4 (mod 6)

Define f: $E(G) \rightarrow \{1, 3, \ldots, 2q-1\}$ by
$f(e_i) = (2i-1)$, for $i = 1, 2, \ldots, (3n-2)$
and $i \neq 1$ and $2n-1$ and

For $n \equiv 0 \pmod 6$
$f(1) = 1$ and $f(2n-1) = 4n - 3$ [Rule 5]

For $n \equiv 4 \pmod 6$
$f(1) = 4n - 3$ and $f(2n-1) = 1$

Define f_+: $V(G) \rightarrow \{0, 1, 2, \ldots, (2k-1)\}$ by
$f_+(v) \equiv \Sigma f(uv) \bmod (2k)$, where this sum run over all edges through v (Rule 6)

Hence the map f and the induced map f_+ provide labels as distinct odd numbers for edges and also the labelings for vertex set has distinct values in $\{0, 1, 2, \ldots, (2k-1)\}$. Hence the graph $S_2\square S_n$, for $n \equiv 0, 4 \pmod 6$, is edge-odd graceful.

Example 2.1: The connected graph $S_2\square S_7$ is edge – odd graceful.
The following graph (figure 5) is the book graph with 14 vertices and 19 edges, with some arbitrary edge-odd graceful labeling to its vertices and edges.

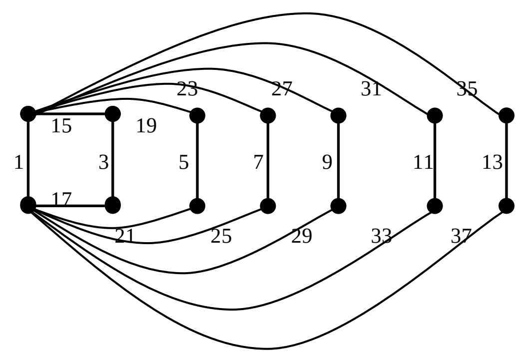

Figure 5: Edge – odd graceful Graph $S_2\square S_7$ for n odd

Example 2.2: The connected graph $S_2\square S_8$, for $n \equiv 2$ (mod 6) is edge – odd graceful.

The figure 6 is the book graph with 16 vertices and 22 edges, with some arbitrary edge-odd graceful labeling to its vertices and edges.

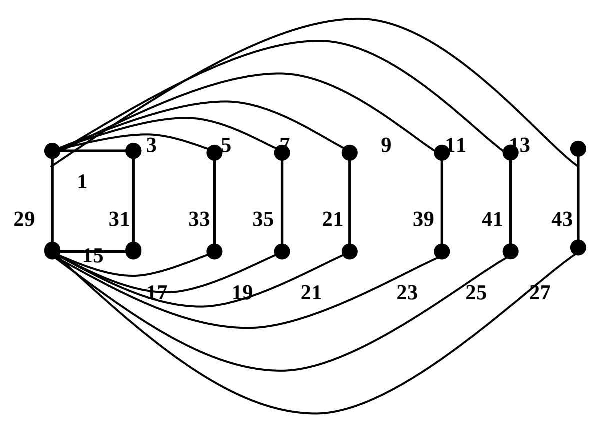

Figure 6: Edge – odd graceful Graph $S_2\square S_8$ ($n \equiv 2 \pmod 6$)

Example 2.3: The connected graph $S_2\square S_6$, $S_2\square S_{10}$ ($n \equiv 0, 4 \pmod 6$) is edge – odd graceful.

The following graph (figure 7) is the book graph $S_2\square S_6$ with 12 vertices and 16 edges, with some arbitrary edge-odd graceful labeling to its vertices and edges.

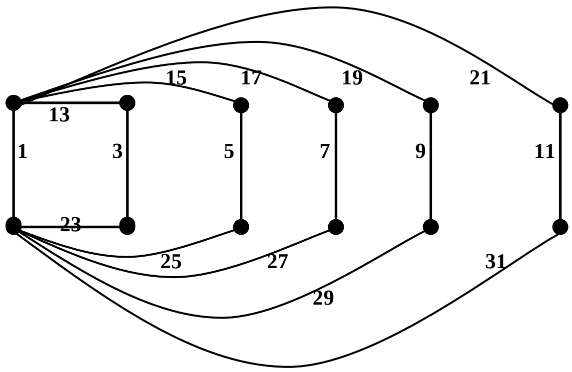

Figure 7: Edge – odd graceful Graph $S_2\square S_6$, ($n \equiv 0 \pmod 6$)

The following graph (figure 8) is the book graph $S_2\square S_{10}$ with 20 vertices and 28 edges, with some arbitrary edge-odd graceful labeling to its vertices and edges.

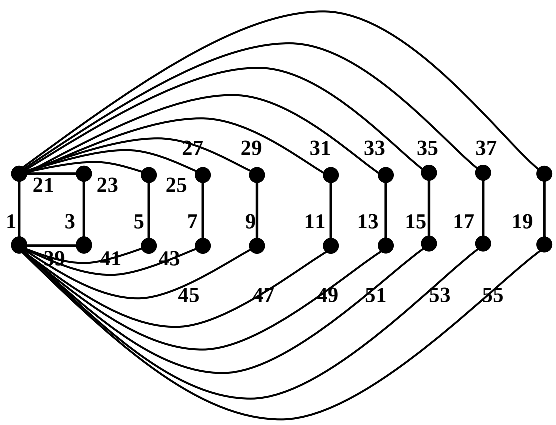

Figure 8: Edge – odd graceful Graph $S_2\square S_{10}$, ($n \equiv 4 \pmod 6$)

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