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A Study on Bi HX Group

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ABSTRACT
In this paper we introduce the concept of Bi HX group and study some of its properties.

Keywords
Bi group, Sub-bi group, HX group, Bi HX group, HX group homomorphism, Bi HX group homomorphism.

INTRODUCTION
The theoretical needs of the set-value mappings lead the birth of some mathematical structures. Prof. Li Hongxing [1, 2] first introduced the concept of HX group which originated the study of HX group; moreover, some useful results are obtained. Since the operations in a HX group is based on the operations of some elements in the base algebra, it is worth to study how to represent directly these operations and to judge whether a subset of the power set are a certain algebraic structure. In this paper we introduce and discuss the properties of Bi HX group.

1. PRELIMINARIES
Definition 1.1
A set (G, +, o) with two binary operation '+' and 'o' is called a Bi group if there exists two proper subsets G_1 and G_2 of G such that,

(i) $(G_1,+)$ is a group,

(ii) (G_2, o) is a group,

(iii) $G = G_1 \cup G_2$

Definition 1.2. A non-empty subset H of a bi group (G, +, o) is called a sub- bi group, if H itself is a bi group under a operation '+' and 'o' defined on G.

Definition 1.3. [1] In 2^G-{ϕ} we define an algebraic operation: $AB = \{ab / a \in A, b \in B\}$ (I)

An nonempty set $\mathcal{g} \subset 2^G$-{ϕ} is called a HX group on G, if $\mathcal{g}$ is a group with respect to the operation (I), which its unit element is denoted by E.

Definition 1.4. Let $\mathcal{g}$ be a HX group on G. Let us defined that,

$G^* = \bigcup$ {A/A $\in \mathcal{g}$}

$$G^o = \{a \in G^* / a^{-1} \in G^*\}$$

Definition 1.5. A set ($\mathcal{g}$,+, o) with two binary operation "+" and "o" is called a Bi HX group if there exists two proper subsets $\mathcal{g}_1$ & $\mathcal{g}_2$ such that

(i) ($\mathcal{g}_1$, +) is a HX group

(ii) ($\mathcal{g}_2$, o) is a HX group

(iii) $\mathcal{g} = \mathcal{g}_1 \bigcup \mathcal{g}_2$

Definition 1.6. Let $\mathcal{g}$ and $\mathcal{g}^1$ be any two HX groups. A mapping f: $\mathcal{g} \rightarrow \mathcal{g}^1$ is called a HX group homomorphism if it satisfies the condition,
f(AB) = f(A) f(B) for all A,B $\in \mathcal{g}$.

Definition 1.7. Let ($\mathcal{g} = \mathcal{g}_1 \bigcup \mathcal{g}_2$, +, o) and ($\mathcal{g}^1 = \mathcal{g}_1^1 \bigcup \mathcal{g}_2^1$, +', o') be a Bi HX group. The map f: $\mathcal{g} \rightarrow \mathcal{g}^1$ is said to be an Bi HX group homomorphism if **f** is restricted to $\mathcal{g}_1$ **(i.e) f**/$\mathcal{g}_1$ is a HX group homomorphism from $\mathcal{g}_1$ to $\mathcal{g}_1^1$ and **f** is restricted to $\mathcal{g}_2$ **(i.e) f**/$\mathcal{g}_2$ is a HX group homomorphism from $\mathcal{g}_2$ to $\mathcal{g}_2^1$.

Definition 1.8. Let ($\mathcal{G}=\mathcal{G}_1 \bigcup \mathcal{G}_2$, +, o) be a Bi HX group of a bi group G.

Then define $G^* = G_1^* \bigcup G_2^*$

Where $G_1^* = \bigcup \{A / A \in \mathcal{G}_1\}$ and

$G_2^* = \bigcup \{A / A \in \mathcal{G}_2\}$

$G^o = G_1^o \bigcup G_2^o$ where

$G_1^o = \{a \in G_1^* / a^{-1} \in G_1^*\}$ and

$G_2^o = \{a \in G_2^* / a^{-1} \in G_2^*\}$

2. BASICS THEOREM ON HX GROUP:

Theorem2.1. [1] If $\mathcal{G}$ is a HX group on G, then

(i) $(\forall A \in \mathcal{G}) \left(|A| = |B|\right)$;

(ii)$(\forall A, B \in \mathcal{G})$

$\left(A \cap B \neq \phi \Rightarrow |A \cap B| = |E|\right)$

Proof. (i) In one respect we have

AE=A

$\Rightarrow (\forall a \in A)(aE \subset AE = A)$

$\Rightarrow |E| = |aE| \leq |A|$

In the other respect we have

$A A^{-1}=E$

$\Rightarrow \left(\forall b \in A^{-1}\right)\left(bA \subset A^{-1}A = E\right)$

$\Rightarrow |A| = |bA| \leq |E|$

(ii) First $|A \cap B| \leq |A| = |E|$

Second, $c \in A \cap B \Rightarrow cE \subset A \cap B \Rightarrow$

$|E| = |cE| \leq |A \cap B|$.

Theorem2.2. [1] Let H be a subgroup of G and E be a subset of G satisfying $E^2=E$. If $(\forall a \in H)(aE = Ea)$ then $\mathcal{G}=$ {aE /a$\in H$ } is a HX group on G its unit element just E.

Proof. Take the surjection f : H$\rightarrow \mathcal{G}$, $a \mapsto aE$.

f(ab)=(ab)E

=(ab)EE

=a(bE)E

=a(Eb)E

=(aE)(Be)

=f(a)f(b),

So, H~$\mathcal{G}$. This $\mathcal{G}$ is a group. Moreover, f(e)=eE=E. So E is the unit element of $\mathcal{G}$.

Theorem2.3. [1] Let $\mathcal{G}$ is a HX group on G. If E is a subgroup of G, then(i) $\mathcal{G}=$ {aE /a$\in G^*$};

(ii) G^* if a subgroup of G.

Proof. (i) $\forall A \in \mathcal{G}$, take a$\in$A. We have aE$\subset AE = A$. It can be proved that aE=A. If it is not true, then there exists b$\in A - aE$. Then we have $a^{-1}b \notin E$ because $b = ac \in aE$ if $a^{-1}b = c \in E$. If $d \in A^{-1}$ we have da and db$\in A^{-1}A = E$.

Thus $a^{-1}b = a^{-1}d^{-1}db$

$= (da)^{-1}(db) \in E.$

This is a contradiction with $a^{-1}b \notin E$. So aE$= A$. This means that $\mathcal{G}\subset$ {aE /a$\in G^*$}.

Conversely, $\forall\ a \in G^*$, $\exists A \in \mathcal{G}$, such that a$\in$A. So aE$= A \in \mathcal{G}$. Thus {aE /a$\in G^*$}$\subset\mathcal{G}$.

(ii) $\forall\ a \in G^*$, $\exists A \in \mathcal{G}$, such that a$\in$A. Noting $e \in E$ and $AA^{-1} = E$, then there exist b$\in A$, $b^{-1} \in A^{-1}$, such that $bb^{-1} = e$. From A=bE we have $c \in E$ such that a=bc. So,

$a^{-1} = (bc)^{-1}$

$= c^{-1}b^{-1} \in EA^{-1} = A^{-1} \subset G^*$

Theorem2.4. [1] Let f be a homomorphism from G to another group G'. We have

(i) If $\mathcal{g}$ is a HX group on G, then

$\mathcal{g}^1 = \{f(A)/A \in \mathcal{g}\}$ is a HX group on G' and $\mathcal{g} \sim \mathcal{g}^1$

(ii) Let f be a surjection. If $\mathcal{g}^1$ is a HX group on G', then

$\mathcal{g} = \{f^{-1}(A')/ A' \in \mathcal{g}^1\}$ is a HX group on G and $\mathcal{g} \sim \mathcal{g}^1$

Proposition 2.1. [1] Let $\mathcal{g}$ be a HX group on G, and $B \subset 2^G - \{\phi\}$ with $B^2=B$. If B satisfies the condition: $(\forall A \in \mathcal{g})\ (AB = BA)$, then

$\mathcal{g}_B = \{AB/A \in \mathcal{g}\}$ is a HX group on G and $\mathcal{g} - \mathcal{g}_B$.

The proof is straight.

3. SOME RESULTS ON Bi-HX GROUP:

Theorem 3.1. If $\mathcal{g}$ is a Bi HX group on a Bi group G, then

(i) $(\forall A \in \mathcal{g})\ (|A| = |B|)$

(ii) $(\forall A, B \in \mathcal{g})$

$$(A \cap B \neq \phi \Rightarrow |A \cap B| = |E|)$$

Proof. (i) $A \in \mathcal{g} = \mathcal{g}_1 \cup \mathcal{g}_2$

Let $A \in \mathcal{g}_1$ and $\mathcal{g}_1$ is a HX group on G_1.

Therefore, $|A| = |B|$

Similarly, $A \in \mathcal{g}_2$ and $\mathcal{g}_2$ is a HX group on G_2.

$|A| = |B|$

Hence $(\forall A \in \mathcal{g})\ (|A| = |B|)$

(ii) Let $A, B \in \mathcal{g}_1$. Since $\mathcal{g}_1$ is a HX group on G_1

$$|A \cap B| = |E|$$

Similarly, $A, B \in \mathcal{g}_2$. Since $\mathcal{g}_2$ is a HX group on G_2

Therefore $|A \cap B| = |E|$

Hence $(\forall A, B \in \mathcal{g})$

$$(A \cap B \neq \phi \Rightarrow |A \cap B| = |E|).$$

Theorem 3.2. Let $\mathcal{g} = (\mathcal{g}_1 \cup \mathcal{g}_2, +, o)$ be a Bi HX group of a bi group $G = (G_1 \cup G_2, +, o)$. Then

(a) G^* is a sub-bi group of G

(b) $G^o \neq \phi$ iff $e_1 \in G^o$ and $e_2 \in G^o$ where e_1 is the identity element of G_1 and e_2 is the identity element of G_2 respectively.

(c) $G^o \neq \phi$ iff G^o is a sub-bi group of G.

Proof. (a) Let $a, b \in G^*$. Then

(i) $a, b \in G_1^*$, then there exists $A, B \in \mathcal{g}_1$ such that $a \in A$ and $b \in B$.

$\Rightarrow a+b \in A+B \subset G_1^*$

$\Rightarrow (G_1^*, +)$ is a group.

(ii) $c, d \in G_2^*$, then there exists $C, D \in \mathcal{g}_2$ such that $c \in C$ and $d \in D$.

$\Rightarrow cod \in CoD \subset G_2^*$

$\Rightarrow (G_2^*, o)$ is a group.

Clearly $G^* = G_1^* \cup G_2^*$

Therefore, G^* is a sub-bi group of G.

(b) Obvious.

(c) Let $G^o \neq \phi$

Let $a, b \in G^o$

(i) Let $a, b \in G_1^o$

$\Rightarrow -a, -b \in G_1^*$

$\Rightarrow -(a+b) \in G_1^*$

$\Rightarrow a+b \in G_1^o$

Therefore $(G_1^o, +)$ is a group.

(ii) Let a, b $\in G_2^o$

$\Rightarrow a^{-1}, b^{-1} \in G_2^*$

$\Rightarrow (ab)^{-1} \in G_2^*$

$\Rightarrow ab \in G_2^o$

$\Rightarrow (G_2^o, +)$ is a group.

Therefore $G^o = G_1^o \cup G_2^o$ is a sub-bi group of G.

Theorem3.3. Let f be a bi group homomorphism from G to another bigroup G'. We have

(i) If $\mathcal{G}$ is a Bi HX group on a bi group G, then $\mathcal{G}_1$ = {f(A)/A $\in \mathcal{G}$} is a Bi HX group on G' and $\mathcal{G} \sim \mathcal{G}_1$

(ii) Let f be a surjection. If $\mathcal{G}_1$ is a Bi HX group on G', then $\mathcal{G}$ = {$f^{-1}(A')$/ $A' \in \mathcal{G}_1$ }is a Bi HX group on G and $\mathcal{G} \sim \mathcal{G}_1$.

4. CONCLUSIONS

Further work is in progress in order to develop the Fuzzy Bi-HX group and Anti Fuzzy Bi-HX group.

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