In path factorization, Ushio [1] gave the necessary and sufficient conditions for $P_k$-design when $k$ is odd. $P_{2p}$-factorization of a complete bipartite graph for $p$ an integer, was studied by Wang [2]. Further, Beiling [3] extended the work of Wang [2], and studied $P_{2k}$-factorization of complete bipartite multigraphs. For even value of $k$ in $P_k$-factorization the spectrum problem is completely solved [1, 2, 3]. However, for odd value of $k$ i.e. $P_3, P_5, P_7, P_9$ and $P_{(4k-1)}$, the path factorization have been studied by a number of researchers [4, 5, 6, 7, 8]. The necessary and sufficient conditions for the existence of $P_3$-factorization of symmetric complete bipartite digraph were given by Du B [9]. Earlier we have discussed the necessary and sufficient conditions for the existence of $P_5$ and $P_7$-factorization of symmetric complete bipartite digraph [10, 11]. Now, in the present paper, we give the necessary and sufficient conditions for the existence of $P_9$-factorization of symmetric complete bipartite digraph, $K_{(m,n)^*}$.

References

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**Index Terms**

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